Vol. 37 (2006) ACTA PHYSICA POLONICA B No 8

NEUTRINOPRODUCTION OF THE RESONANCES: APPROACH WITH THE PHENOMENOLOGICAL FORM-FACTORS*

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(Received July 7, 2006)

We present general formulas for the production of the spin-3/2 and 1/2 resonances by neutrinos and then specialize to the first four resonances $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$. The production of the resonances is described by vector and axial form-factors. We show how some of them could be determined from the electroproduction data and from the theory. Then we calculate the cross section for neutrino reactions and compare the results with the experimental data.

PACS numbers: 14.20.Gk, 13.40.Gp

1. Introduction

In the new experiments studying neutrino oscillations there is a strong interest to go beyond the QE scattering and Δ resonance excitations [1,2]. In this region of invariant masses, three isospin 1/2 states, $P_{11}(1440)$, $D_{13}(1520)$ and $S_{11}(1535)$ contribute. They are known as the second resonance region. Existing data for neutrino excitation of these resonances are scarce and come from old bubble-chamber experiments on deuterium targets. In the on-going and coming experiments, heavy nuclear targets like carbon, oxygen, iron and lead are used. In obtaining neutrino–nucleus amplitudes, the knowledge of the relevant neutrino–nucleon amplitudes is a prerequisite and in this paper we describe them it terms of phenomenological vector and axial form-factors. For the Δ resonance after many years several of the form-factors and their Q^2 dependencies became accurately known and were found to deviate from the dipoles. For the higher resonances only the results of the Rein–Sehgal model [3], extended recently in [4,5], are used up

* Presented by O. Lalakulich at the XX Max Born Symposium "Nuclear Effects in Neutrino Interactions", Wrocław, Poland, December 7–10, 2005.

to now. At Dortmund University we analyze the second resonance region within the phenomenological model, in which the form-factors are determined from electroproduction data and from PCAC hypothesis. Some of our results are presented here, details can be found in [6].

Among the other approaches to the problem of Δ resonance excitation by neutrinos there are calculations based on dispersion relations [7], phenomenological models [8, 9], superscaling approach [10], as well as model incorporating mesonic states [11], including a cloud of pions.

2. General method to determine the vector and axial form-factors

We adopt the approach of determining the vector form-factors from helicity amplitudes of electroproduction data, which became recently available from the Jefferson Laboratory [12–14] and Mainz accelerators [15].

Let us denote the nucleon as $|N, J_z\rangle$ and the resonance as $|R, J_z\rangle$ with J_z the helicity. For the transition $\gamma N \to R$ there are three helicity amplitudes:

$$A_{1/2} = K \left\langle R, +\frac{1}{2} \left| J_{\rm em} \cdot \varepsilon^{(R)} \right| N, -\frac{1}{2} \right\rangle , \tag{2.1}$$

$$A_{3/2} = K \left\langle R, +\frac{3}{2} \left| J_{\rm em} \cdot \varepsilon^{(R)} \right| N, +\frac{1}{2} \right\rangle , \tag{2.2}$$

$$S_{1/2} = K \frac{q_z}{\sqrt{Q^2}} \left\langle R, +\frac{1}{2} \left| J_{\rm em} \cdot \varepsilon^{(S)} \right| N, +\frac{1}{2} \right\rangle , \tag{2.3}$$

where $K = \sqrt{\pi\alpha/[m_N(W^2 - m_N^2)]}$ and $\varepsilon^{(i)}$ is the photon polarization vector.

We write these amplitudes in terms of the electromagnetic form-factors and use the resulting formulas to fit the experimental data. This allows us to extract the form-factors, which are then related to the weak vector form-factors that we use in neutrino reactions. This approach is general and can be applied to any resonance provided that the corresponding electroproduction data is available.

The axial form-factors are more difficult to determine. For each resonance we appeal to PCAC which relates two form-factors, and one coupling is determined making use of the partial decay widths $R \to \pi N$. The signs of the axial form-factors are chosen in such a way that the structure functions W_3 for all resonances are positive, as indicated or suggested by the data. As a consequence the neutrino induced cross sections are larger than the corresponding antineutrino cross sections.

3. Form-factors for the first four resonances

3.1. $S_{11}(1535)$ resonance

For spin-1/2 resonances the parametrization for the weak vertex of the resonance production is similar to the parametrization for quasi-elastic scattering. The matrix elements of the S_{11} production can be written as:

$$\langle S_{11}|J^\nu|N\rangle = \bar{u}(p')\left[\frac{g_1^{\rm V}}{\mu^2}(Q^2\gamma^\nu + \not{q}q^\nu)\gamma_5 + \frac{g_2^{\rm V}}{\mu} i\sigma^{\nu\rho}q_\rho\gamma_5 - g_1^{\rm A}\gamma^\nu - \frac{g_3^{\rm A}}{m_N}q^\nu\right]u(p)\,, \tag{3.1}$$

where we use the standard notation for the σ-matrices $\sigma^{\nu\rho} = \frac{i}{2}[\gamma^\nu, \gamma^\rho]$ and $\mu = m_N + M_R$ as normalization factor.

The matrix element of the electromagnetic current has the same structure as the vector part in Eq. (3.1)

$$\langle S_{11}|J^\nu_{\rm em}|N\rangle = \bar{u}(p')\left[\frac{g_1^{\rm em}}{\mu^2}(Q^2\gamma^\nu + \not{q}q^\nu) + \frac{g_2^{\rm em}}{\mu} i\sigma^{\nu\rho}q_\rho\right]\gamma_5 u(p)\,. \tag{3.2}$$

The two form-factors $g^{\rm em}_{1,2}$ are different for protons and neutrons because the electromagnetic current has isovector and isoscalar components. In the difference

$$a^{1/2}_{\gamma n\to R^0} - a^{1/2}_{\gamma p\to R^+} = a^{1/2(iV)}_{(W^+ n\to R^+)}$$

the isoscalar component of the current drops out and the isovector part coincide with the vector part of the weak amplitude. Furthermore, since the S_{11} has spin 1/2, only two helicity amplitudes contribute. Substituting the current (3.2) into expressions (2.3) and using explicit representation of the spinors for the nucleon and resonance in the laboratory frame yields the amplitudes

$$A^{S_{11}}_{1/2} \;=\; \sqrt{2N}\left[\frac{g_1^{\rm em}}{\mu^2}Q^2 + \frac{g_2^{\rm em}}{\mu}(M_R - m_N)\right]\,, \tag{3.3}$$

$$S^{S_{11}}_{1/2} \;=\; \sqrt{N}q_z\left[-\frac{g_1^{\rm em}}{\mu^2}(M_R - m_N) + \frac{g_2^{\rm em}}{\mu}\right]\,, \tag{3.4}$$

where $N = \pi\alpha_{\rm em}/[m_N(W^2 - m_N^2)]\,2m_N(p'^0 + M_R)$.

For the $S_{11}(1535)$ the helicity amplitudes were measured only for protons. To obtain the neutron form-factors we assume the relation $A^n_{1/2} \approx -A^p_{1/2}$, which implies that the contribution from the isoscalar photon is small. We use recent experimental data [14,16] to make a numerical fit, which is shown in Fig. 1. This fit yields the following form-factors

$$g_1^p = \frac{2.0/D_{\rm V}}{1+Q^2/1.2M_{\rm V}^2}\left[1+7.2\ln\left(1+\frac{Q^2}{1\,{\rm GeV}^2}\right)\right],$$
$$g_2^p = \frac{0.84}{D_{\rm V}}\left[1+0.11\ln\left(1+\frac{Q^2}{1\,{\rm GeV}^2}\right)\right], \tag{3.5}$$

where $D_{\rm V}=(1+Q^2/M_{\rm V}^2)^2$ denotes the dipole function with the vector mass parameter $M_{\rm V}=0.84\,{\rm GeV}$. Notice, that one of the form-factors, at least for $Q^2<3.5\,{\rm GeV}^2$, falls down slower than the dipole function. The weak vector form-factors $g_i^{\rm V}$ reduce in our case to $g_i^{\rm V}\approx -2g_i^p\approx 2g_i^n$.

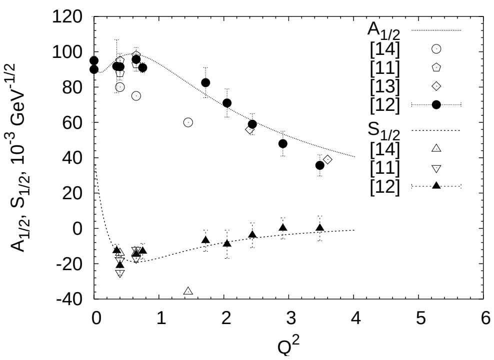

Fig. 1. Helicity amplitudes for the $S_{11}(1535)$ resonance, calculated with the form-factors from Eq. (3.5).

The determination of the axial form-factors relies on PCAC, which allows to express one of the axial form-factors via the strong πNR coupling $g_{\rm S}$.

$$g_3^{\rm A}(Q^2)=\frac{m_N(M_R-m_N)}{Q^2+m_\pi^2}g_1^{\rm A}(Q^2)\,,\quad g_1^{\rm A}(0)=-\sqrt{\frac{2}{3}}\frac{g_{\rm S}f_\pi}{M_R-m_N}=-0.21. \tag{3.6}$$

The coupling $g_{\rm S}$ is in turn defined through the elastic resonance width

$$\mathit{\Gamma}_{S11\to\pi N}=\frac{g_{\rm S}^2}{8\pi M_R^2}\Big[(M_R+m_N)^2-m_\pi^2\Big]|p_\pi|\,. \tag{3.7}$$

With the experimental value $\mathit{\Gamma}_{S11\to\pi N}(W=M_R)=\mathit{\Gamma}_0^{(S)}=0.4\times 0.150\,{\rm GeV}$ we obtain $g_{\rm S}=1.12$. For details see [6].

The Q^2 dependence of the form-factors can be determined either experimentally (provided that the data is available) or in a specific theoretical model. Motivated by the results on P_{33} resonance, we suppose, that the form-factor $g_1^{\rm A}$ behaves as

$$g_1^{\mathrm{A}} = \frac{-0.21/D_{\mathrm{A}}}{1+Q^2/3M_{\mathrm{A}}^2}\,, \tag{3.8}$$

where $D_{\mathrm{A}} = (1+Q^2/M_{\mathrm{A}}^2)^2$ denotes the dipole function with the axial mass parameter $M_{\mathrm{A}} = 1.05\,\mathrm{GeV}$.

For the running width of the resonance we use

$$\Gamma^{(S)}(W) = \Gamma_0^{(S)} \frac{p_\pi(W)}{p_\pi(M_R)}$$

which is consistent with Eq. (3.7).

The above described methods for determining the vector and axial form-factors can be used for any resonance. The Q^2 dependencies of $g_1^{\mathrm{A}}(Q^2)$ and $g_3^{\mathrm{A}}(Q^2)$ must be checked and determined from neutrino experiments. The analysis of the other resonances is similar and we give here numerical values for the couplings and the functional form of the form-factors. As a final step we should provide the structure functions $\mathcal{W}_1, \ldots \mathcal{W}_5$ that enter the differential cross section in terms of the form-factors. The corresponding expressions are given in [6].

3.2. $P_{11}(1440)$ *resonance*

For the P_{11} resonance we define the vertex by equation

$$\langle P_{11}|J^\nu|N\rangle = \bar{u}(p')\left[\frac{f_1^{\mathrm{V}}}{\mu^2}(Q^2\gamma^\nu + \not{q}q^\nu) + \frac{f_2^{\mathrm{V}}}{\mu} i\sigma^{\nu\rho}q_\rho - f_1^{\mathrm{A}}\gamma^\nu\gamma_5 - \frac{f_3^{\mathrm{A}}}{m_N}q^\nu\gamma_5\right]u(p)\,, \tag{3.9}$$

For this resonance there are experimental data for the helicity amplitudes on proton target shown in Fig. 2. For the fit of the form-factors we use only [14, 16], which (for $Q^2 < 3.5\,\mathrm{GeV}^2$) results in

$$f_1^p = \frac{2.3/D_{\mathrm{V}}}{1+Q^2/4.3M_{\mathrm{V}}^2}\,, \qquad f_2^p = \frac{-0.76}{D_{\mathrm{V}}}\left[1 - 2.8\ln\left(1+\frac{Q^2}{1\,\mathrm{GeV}^2}\right)\right]. \tag{3.10}$$

The weak vector form-factors are calculated in analogy to the previous resonance as $f_i^{\mathrm{V}} \approx -2f_i^p$, which again ignores the isoscalar contribution.

From PCAC we determine the axial couplings

$$f_3^{\mathrm{A}}(Q^2) = \frac{m_N(M_R+m_N)}{Q^2+m_\pi^2} f_1^{\mathrm{A}}(Q^2)\,, \qquad f_1^{\mathrm{A}}(0) = -0.51\,.$$

The Q^2 dependence of f_1^{A} is arbitrary taken as

$$f_1^{\mathrm{A}}(Q^2) = \frac{f_1^{\mathrm{A}}(0)/D_{\mathrm{A}}}{1+Q^2/3M_{\mathrm{A}}^2}\,. \tag{3.11}$$

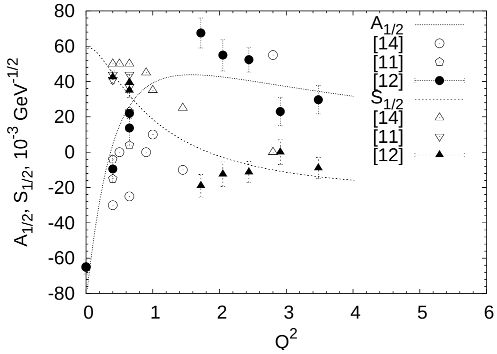

Fig. 2. Helicity amplitudes for the $P_{11}(1440)$ resonance, calculated with the form-factors from Eq. (3.10).

3.3. $D_{13}(1520)$ *resonance*

We define form-factors in analogy to those of the P_{33} resonance [18]. By fitting the data from Ref. [14] for proton and predictions from Ref. [13] for neutron we found the following form factors

$$\begin{aligned} C_3^{(p)} &= \frac{2.95/D_{\rm V}}{1+Q^2/8.9M_{\rm V}^2}\,, \quad C_4^{(p)} = \frac{-1.05/D_{\rm V}}{1+Q^2/8.9M_{\rm V}^2}\,, \quad C_5^{(p)} = \frac{-0.48}{D_{\rm V}}\,, \\ C_3^{(n)} &= \frac{-1.13/D_{\rm V}}{1+Q^2/8.9M_{\rm V}^2}\,, \quad C_4^{(n)} = \frac{0.46/D_{\rm V}}{1+Q^2/8.9M_{\rm V}^2}\,, \quad C_5^{(n)} = \frac{-0.17}{D_{\rm V}}\,. \end{aligned} \tag{3.12}$$

To give an impression, how good this parametrization is, we plot in Fig. 3 the helicity amplitudes for the proton. The vector form-factors are determined in analogy to those of other isospin-1/2 resonances as $C_i^V = C_i^{(n)} - C_i^{(p)}$.

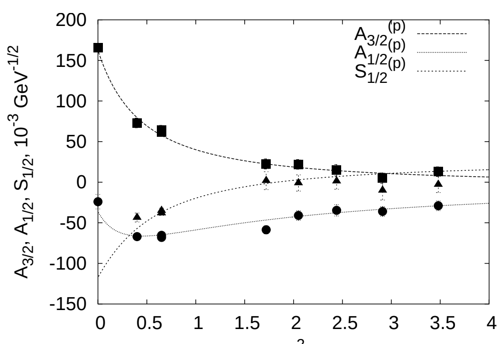

Fig. 3. Helicity amplitudes for the $D_{13}(1520)$ resonance, calculated with the form-factors from Eq. (3.12).

For the axial form-factors we get with the help of PCAC

$$C_6^{\mathrm{A}}(Q^2) = m_N^2 \frac{C_5^{\mathrm{A}}(Q^2)}{m_\pi^2 + Q^2}, \quad C_5^{\mathrm{A}}(Q^2) = \frac{-2.1/D_{\mathrm{A}}}{1 + Q^2/3M_{\mathrm{A}}^2}. \tag{3.13}$$

We also use $C_3^{\mathrm{A}} = C_4^{\mathrm{A}} = 0$. Further investigation is presented in [6].

3.4. $P_{33}(1232)$ *resonance*

As mentioned in Introduction, Δ resonance has been studied extensively and is understood better than the others. The form-factors have been determined from comparison with electroproduction data and PCAC. Recent data from Mainz accelerator [15] give helicity amplitudes as functions of Q^2, which are shown in Fig. 4. Fitting these amplitudes, we determine the form-factors

$$C_3^{(p)} = \frac{2.13/D_{\mathrm{V}}}{1 + Q^2/4M_{\mathrm{V}}^2}, \quad C_4^{(p)} = \frac{-1.51/D_{\mathrm{V}}}{1 + Q^2/4M_{\mathrm{V}}^2}, \quad C_5^{(p)} = \frac{-0.48/D_{\mathrm{V}}}{1 + Q^2/0.776M_{\mathrm{V}}^2}. \tag{3.14}$$

The form-factors $C_3^{(p)}$ and $C_4^{(p)}$ agree with those, obtained within the magnetic dipole dominance approximation, with accuracy 5%. For this resonance isospin relations give $C_i^{(p)} = C_i^{(n)} = C_i^{(V)}$.

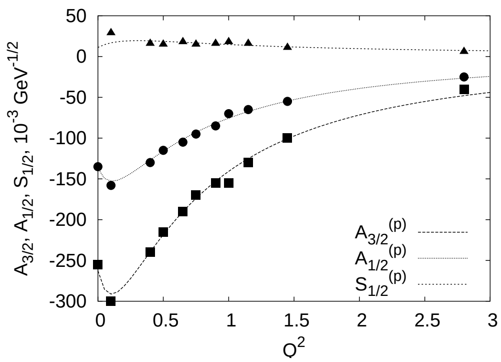

Fig. 4. Helicity amplitudes for the $P_{33}(1232)$ resonance, calculated with the form-factors from Eq. (3.14).

For the axial form-factors we adopt the functional form

$$\begin{aligned} C_6^{\mathrm{A}}(Q^2) &= m_N^2 \frac{C_5^{\mathrm{A}}(Q^2)}{m_\pi^2 + Q^2}, \qquad & C_5^{\mathrm{A}} &= \frac{1.2/D_{\mathrm{A}}}{1 + Q^2/3M_{\mathrm{A}}^2}, \\ C_4^{\mathrm{A}} &= -\frac{1}{4} C_5^{\mathrm{A}}, & C_3^{\mathrm{A}} &= 0, \end{aligned}$$

as discussed earlier [7]. The differential cross section $d\sigma/dQ^2$ for the reaction $\nu p \to \mu^- R^{++} \to \mu^- p\pi^+$ is calculated in our previous article [18]. Special attention was devoted to the muon mass effects, which decrease the differential cross section at low Q^2 and thereby bring better agreement with the data.

4. Cross sections in the second resonance region

Now, that the form-factors are known, we calculate the cross section for the channels $\nu n \to R \to p\pi^0$ and $\nu n \to R \to n\pi^+$, where both $I = 3/2$ and $I = 1/2$ resonances contribute. The experimental data, available in this region, show that the BNL [19] points are consistently higher than those of ANL [20, 21] and SKAT [22]. This is also evident in earlier compilations of the data. For instance, Sakuda [23] used the BNL data and his cross sections are larger that those of Paschos *et al.* [24] where ANL and SKAT data were used. The error bars in these early experiments are rather large and it should be the task of the next experiments to improve them and settle the issue.

The solid curves in Fig. 5 show the theoretically calculated cross sections with the cut $W < 2.0$ GeV and the dashed curve with the cut $W < 1.6$ GeV. For $p\pi^0$ the solid curve goes through most of the experimental points except for those of the BNL experiment, which are consistently higher than data of the two other experiments.

For the $n\pi^+$ channel our curve is a little lower than the experimental points. This means that there are contributions from higher resonances or additional axial form factors. Another possibility is to add a smooth background which grows with energy. An incoherent isospin-1/2 background of approximately $5\times 10^{-40}(E_\nu/1\,\text{GeV}-0.28)^{1/4}\,\text{cm}^2$ would be sufficient to fit the data, as it is shown in Fig. 5 by a double-dashed curve. This background

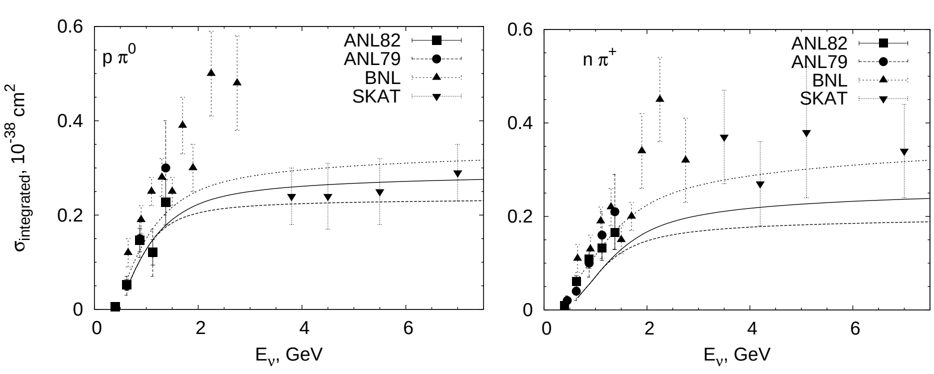

Fig. 5. Integrated cross section for the $\mu^- p\pi^0$ and $\mu^- n\pi^+$ final states.

may originate from various sources. By isospin conservation it will give one half of this value to the $p\pi^0$ channel. Since experimental points are not consistent with each other, it is premature for us to speculate on the additional terms.

OL thanks the organizers of the Max Born Symposium for the opportunity to present our results and for the financial support. The support of the BMBF under contract 05HT4PEA/9 is gratefully acknowledged.

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