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ANALYSIS OF THE REIN–SEHGAL MODEL IN THE CONTEXT OF THE QUARK–HADRON DUALITY*

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An analysis of the Rein–Sehgal model in the context of the quark–hadron duality hypothesis is presented. The resonance region structure functions reconstructed from the Rein–Sehgal model at different values of $Q^2_{\rm RES}$ are compared with the DIS structure functions calculated at higher $Q^2_{\rm DIS}$. The ratios of corresponding integrals in the Nachtman variable are also calculated and presented as functions of $Q^2_{\rm RES}$. The obtained functions are approximately flat for $Q^2_{\rm RES} > 0.5\,{\rm GeV}^2$ but the quark–hadron duality is not observed.

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1. Introduction

New long baseline experiments such as T2K will be able to measure neutrino oscillation parameters with higher precision. The experimental data is always compared with the outcome of Monte Carlo (MC) simulations, therefore, the accuracy of the experimental analysis strongly depends on the accuracy of the theoretical description of neutrino interactions, which are implemented in the MC codes. During last five years a lot of effort has been devoted to obtain a better description of neutrino cross section in the few GeV energy region [1].

The GeV neutrinos interact with nucleons quasi-elastically (elastically) and inelastically. The single pion production (SPP) is usually distinguished from the more inelastic channels. It is described by a resonance model (RES). More inelastic processes are described by the deep inelastic scattering (DIS) formalism. A combination of the RES with the DIS formalisms is one

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of the main practical problems in constructing MC codes. It is necessary to find the DIS structure functions which lead to the smooth gluing with the RES description.

In neutrino experiments a typical target is composed of nuclei. The presence of nuclear effects is expected to smear out the resonance peaks. Therefore, it seems that to describe properly the neutrino–nucleus interaction it is enough to apply an approach which reconstructs the cross sections on average.

The quark–hadron (QH) duality of Bloom & Gilman [2] might give a hint how to obtain the structure functions for scattering of the few GeV neutrinos. The duality phenomenon was discovered in the electron–proton scattering [3]. It was observed that the F_2 resonance data is averaged by the DIS scaling curve.

The typical resonance data is collected in the region of high x and small Q^2. The deep inelastic scattering region is characterized by low x and high Q^2. Based on the duality hypothesis one can try to enrich the description of the resonance region by knowledge which comes from the deep inelastic measurements. However, because of the lack of experimental data, it is not known if the duality occurs also for neutrino scattering. It is currently impossible to verify it. But it is expected that the MINERνA experiment will be able to give answer for this question.

It is important to investigate how theoretical models, which describe the RES region and which are implemented in MC schemes work in comparison with the predictions of the deep inelastic formalism.

The Rein–Sehgal (RS) model is implemented in many MC generators (NUANCE, NUET, NeuGen). It describes single pion production in neutrino –nucleon scattering. The aim of this paper is to compare the RS model with the DIS predictions in the context of the QH duality hypothesis. This is a part of the analysis which has been already presented in Ref. [4].

2. The Rein–Sehgal model

The Rein–Sehgal model [5] describes single pion production in the charged (CC) and the neutral (NC) current neutrino scattering. The pions are produced by excitations of 18 resonances. The kinematical region covered by the model is restricted in hadronic invariant mass $W < 2\,\mathrm{GeV}$. The model is based on the Feynman, Ravndal and Kislinger (FKR) approach [6], which was developed to describe the photoelectric meson production. The resonances are identified as a $\mathrm{SU}(6) \supset \mathrm{SU}(3)_{\mathrm{quark}} \otimes \mathrm{SU}(2)_{\mathrm{spin}}$ multiplet members. The FKR model is an example of a relativistic quark oscillator model. The presence of the oscillator potential gives rise to the additional quantum number — the angular momentum.

The electroweak interaction is introduced into the RS model by the minimal coupling scheme. Both vector and axial currents are computed. It is assumed that like in the FKR model the vector and the axial currents are multiplied by appropriate form factors. The form factors are assumed to have a dipole form and are described by two parameters: axial ($M_{\rm A}$) and vector ($M_{\rm V}$) masses.

The scattering amplitude for the single pion production is given by the coherent sum of the amplitudes for production of the resonances. The production amplitude of a given resonance is accompanied by terms which describe branching ratios of resonance decay to a single pion in the final state (the Breit–Wigner term is multiplied by the resonance elasticity and the decay sign). The cross section is expressed as a sum of helicity cross sections ($\sigma_{\rm L,T,S}$). A simple recalculation allows to find three (since the lepton mass is neglected) structure functions ($F^{\rm RS}_{1,2,3}$) which are given by linear combinations of $\sigma_{\rm L,T,S}$.

The Rein–Sehgal approach takes into consideration also the nonresonant background but in the rather ad hock way.

3. Resonance structure functions

The RS model describes the single pion production, therefore, to compare the RS and DIS structure functions for the inclusive cross sections either the DIS or the RS model must be modified. We modify the RS structure functions with the help of what we call 1-pion functions.

The 1-pion functions are defined for each channel of single pion production separately as probabilities that at a given value of W the final hadronic state is that of SPP:

$$f_{1\pi}(W) \equiv \frac{d\sigma^{\rm SPP}}{dW} \Big/ \frac{d\sigma^{\rm DIS}}{dW}. \tag{1}$$

These functions were obtained from the Monte Carlo simulation based on the LUND algorithm [7]. Therefore, they are defined by the the fragmentation and hadronization routines implemented there.

The plots of the 1-pion functions have been shown in Ref. [4]. These functions are implemented in WROCLAW neutrino MC generator [7].

We assume that the following procedure provides the structure functions in the resonance region defined as $W < 2\,{\rm GeV}$:

$$F^{\rm RES}_{j=1,2,3}(x,Q^2) = \frac{F^{\rm RS}_j(x,Q^2)}{f_{1\pi}(W(x,Q^2))}\,. \tag{2}$$

We also assume that all structure functions are rescaled by the same factor.

4. Duality

We say that the quark–hadron duality is present on the quantitative level if the following relation between the resonance and scaling structure functions holds:

$$\int_{\xi_{\min}}^{\xi_{\max}} d\xi F_i^{\mathrm{RES}}(\xi, Q^2_{\mathrm{RES}}) \approx \int_{\xi_{\min}}^{\xi_{\max}} d\xi F_i^{\mathrm{DIS}}(\xi, Q^2_{\mathrm{DIS}})\,, \tag{3}$$

where ξ is the Nachtmann variable, which is introduced to compensate the target mass corrections.

The above equation should hold for different values of Q^2_{RES} characteristic for the resonance production and for a fixed value of Q^2_{DIS}. The region of integration, RES region, is defined to be identical with the resonance region of the RS model: $W_{\min} = M + m_\pi$ and $W_{\max} = 2\,\mathrm{GeV}$ which is then translated into the appropriate region in ξ:

$$\xi_{\min} = \xi(W_{\max}, Q^2_{\mathrm{RES}})\,, \qquad \xi_{\max} = \xi(W_{\min}, Q^2_{\mathrm{RES}})\,. \tag{4}$$

To measure the deviations from the equality (3) we define the ratio of two integrals over the resonance region:

$$\mathcal{R}\left(F_{\mathrm{RES}}, Q^2_{\mathrm{RES}}; F_{\mathrm{DIS}}, Q^2_{\mathrm{DIS}}\right) \equiv \frac{\int_{\xi_{\min}}^{\xi_{\max}} d\xi\, F_{\mathrm{RES}}(\xi, Q^2_{\mathrm{RES}})}{\int_{\xi_{\min}}^{\xi_{\max}} d\xi\, F_{\mathrm{DIS}}(\xi, Q^2_{\mathrm{DIS}})}\,. \tag{5}$$

Using the above quantity, we define the functions:

$$\mathcal{R}_2(Q^2_{\mathrm{RES}}, Q^2_{\mathrm{DIS}}) \equiv \mathcal{R}\left(F_2^{\mathrm{RES}}, Q^2_{\mathrm{RES}}; F_2^{\mathrm{DIS}}, Q^2_{\mathrm{DIS}}\right) \tag{6}$$

and

$$\mathcal{R}_{1,3}(Q^2_{\mathrm{RES}}, Q^2_{\mathrm{DIS}}) \equiv \mathcal{R}\left(xF_{1,3}^{\mathrm{RES}}, Q^2_{\mathrm{RES}}; xF_{1,3}^{\mathrm{DIS}}, Q^2_{\mathrm{DIS}}\right)\,. \tag{7}$$

It is understood that $Q^2_{\mathrm{RES}} \ll Q^2_{\mathrm{DIS}}$.

The QH duality for F_i structure function is said to be present if $\mathcal{R}_i \simeq 1$.

5. Numerical results

We investigate the original Rein–Sehgal model as it is defined in Ref. [5]. Since the resonance region is characterized by small four-momentum transfer we restrict our analysis to $Q^2_{\rm RES} < 3\,{\rm GeV}^2$. The DIS structure functions are obtained by applying the GRV94 parton distribution functions (PDF) [8].

Isospin symmetry implies that the charged current amplitude for the excitations of the $\Delta(1232)$ resonance is for the proton $\sqrt{3}$ times bigger than for the neutron interactions. In the case of the DIS predictions the situation is the opposite — the neutrino–neutron cross section (and thus also structure functions) are bigger than the proton one. Therefore, it is difficult to observe the duality for both proton and neutron simultaneously [9]. In order to bypass this problem we will consider the isoscalar target.

In Fig. 1 the qualitative comparison of the Rein–Sehgal (not rescaled) and the DIS (both F_2 and xF_3) CC structure functions is presented. The RS structure functions are computed for several values of $Q^2_{\rm RES}$, which are characteristic for the resonance region. They are compared with the scaling curves at $Q^2_{\rm DIS} = 10\,{\rm GeV}^2$. For both CC and NC structure functions the sliding of the $\Delta(1232)$ peaks along scaling curves is observed. However, the strength of the Δ peaks for xF_2 seem to be too small to get the QH duality.

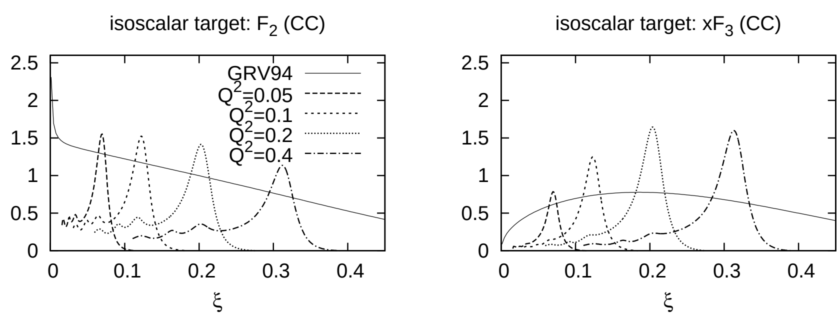

Fig. 1. Comparison of the Rein–Sehgal structure functions at $Q^2 = 0.05$, 0.1, 0.2 and $0.4\,{\rm GeV}^2$ with the appropriate scaling functions at $Q^2_{\rm DIS}{=}10\,{\rm GeV}^2$. In the left figure the plots of F_2 and in the right figure the plots of xF_3 structure functions for CC neutrino-isoscalar target scattering are presented. The Rein–Sehgal structure functions are not rescaled.

In order to perform more quantitative analysis the RS structure functions are rescaled by the 1-pion functions (Eq. 2). The deviations from the duality are measured by the ratios defined in Eqs. (6) and (7). The ratios for xF_1, F_2 and xF_3 are shown in Fig. 2. Both CC and NC structure functions are discussed and again the results are presented for the isoscalar target. All ratios are visibly smaller than one. It could be seen that the $\mathcal{R}_1$ functions reach the biggest values (about 0.7). But it is important to remark that the xF_1 structure functions were computed from the naive Callan–Gross relation.

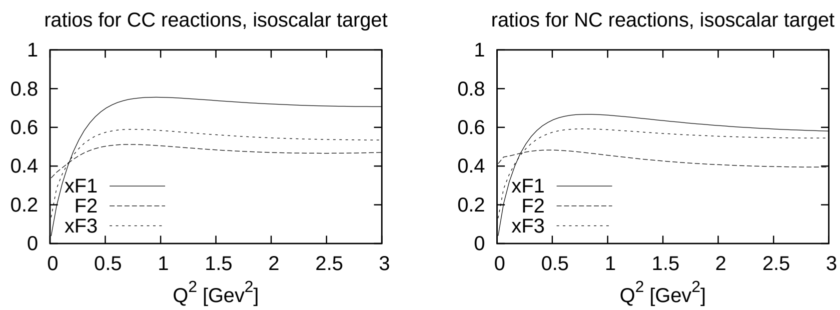

Fig. 2. The functions $\mathcal{R}_{1,2,3}$ (solid, dashed and dotted lines correspondingly, see Eqs. (6) and (7)) for isoscalar target. Both the CC (left figure) and NC reactions (right figure) are considered.

It is interesting to remark that all the ratios saturate for approximately $Q^2 > 0.5\,\mathrm{GeV}^2$. It coincides with the Close and Insgur [10] remark that the QH duality should be observed only for $Q^2 > 0.5\,\mathrm{GeV}^2$, when the scattering amplitude is dominated by the magnetic contribution.

Our approach is based on the 1-pion function which are not accurately known. It is important to compare our results with other methods of obtaining the RES region structure functions. The simplest choice is to use the RS model in which all the resonance elasticities are equal to one. In Fig. 3 such plots of ratios computed for F_2 and xF_3 CC structure functions are presented. We compare the ratios obtained for the rescaled RS structure functions with the ratios computed for the RS approach with elasticities

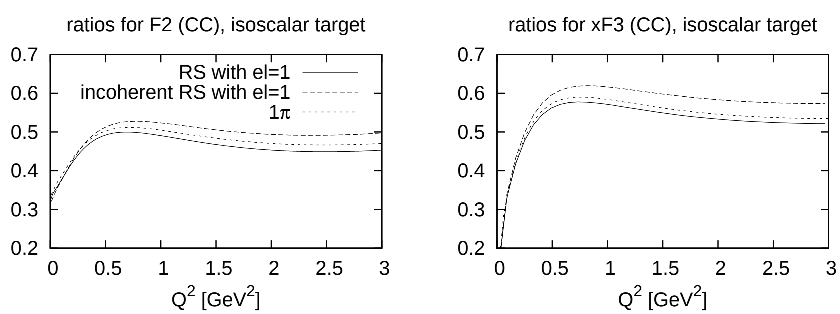

Fig. 3. $\mathcal{R}_2$ ratios computed for three cases: *(i)* the RS model with elasticities equal to 1 (solid line); *(ii)* the RS model with elasticities equal to 1 but with amplitudes summed incoherently (dashed line). *(iii)* the RS model with structure functions rescaled by the 1-pion function (dotted line).

equal to one (we distinguish two cases: the amplitudes can be summed incoherently or coherently). It could be seen that the ratio for the rescaled RS model lies between the ratios of the coherent and the incoherent versions of the RS approach.

Our analysis caries uncertainties connected with the inaccuracy of the Rein–Sehgal model (*e.g.* the description of the nonresonant background, the axial and vector masses). Some of the parameters of the RS approach could be updated. The axial mass is one of them [11]. The way in which the change of $M_{\rm A}$ influences the ratios is illustrated in Fig. 4, where the $\mathcal{R}_2$'s for different values of $M_{\rm A}$ are plotted. It is evident that raising the axial mass by 5% increases the ratio by about 10%. This effect is rather big but still too small to improve the quark–hadron duality.

In Fig. 4 the dependence of the $\mathcal{R}_2$ on the number of resonances is also shown. The ratios are computed by taking into account: four ($W_{\rm max} = 1.6\,{\rm GeV}$), eight ($W_{\rm max} = 1.8\,{\rm GeV}$) and the 18 resonances in the complete RS model ($W_{\rm max} = 2\,{\rm GeV}$). It could be seen that inclusion of the large number of resonances makes ratios flattened.

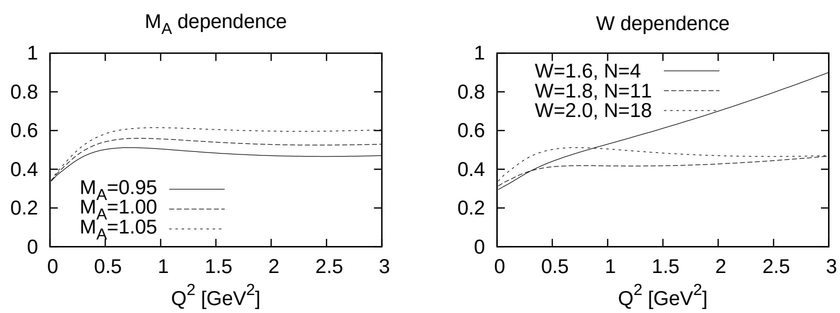

Fig. 4. In the left figure the dependence of $\mathcal{R}_2$ on $M_{\rm A}$ is shown. The ratios are plotted for: $M_{\rm A} = 0.95\,{\rm GeV}$ (solid line), $1.00\,{\rm GeV}$ (dashed line) and $1.05\,{\rm GeV}$ (dotted line). In the right figure dependence of $\mathcal{R}_2$ on $W_{\rm max}$ is shown. The ratios are plotted for: $W_{\rm max} = 1.6\,{\rm GeV}$ (solid line), $1.8\,{\rm GeV}$ (dashed line) and $2\,{\rm GeV}$ (dotted line). N is the number of resonances which are included in calculations.

6. Summary

The analysis of the Rein–Sehgal structure functions modified by the 1-pion functions was presented. It was shown that it is difficult to obtain the quark–hadron duality even for isoscalar target. However, our analysis has some uncertainties which might be treated as degrees of freedom of the approach. Therefore, more detailed studies are required. In particular, a better description of the nonresonant background might influence the final conclusion.

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