# MODEL-INDEPENDENT CONSTRAINTS FOR PARITY-VIOLATING DIS AT LOW $Q^2$*

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The positivity constraints for hadronic tensor lead to the bounds for $\tau$ lepton polarization in Parity Violating DIS. Another model-independent constraints are provided by the QCD duality which may be described in a similar way to QCD sum rules method. The Parity Conserving spin-dependent case is considered in more detail, while several specific comments on Parity Violating case are made. The observation is made that $\Delta(1232)$ resonance should be excluded from duality consideration and possible reason for that is offered.

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## 1. Introduction

The $\nu_\mu \to \nu_\tau$ oscillation hypothesis can be tested by means of $\tau$ production via $\nu_\tau$ scattering through charged current interactions, namely

$$\nu_\tau(\bar{\nu}_\tau) + N \to \tau^-(\tau^+) + X \quad , \tag{1.1}$$

where $N$ is a nucleon target. This process will be studied with underground neutrino telescopes, such as: AMANDA, ANTARES, NESTOR and BAIKAL [4], as well as long-baseline neutrino oscillation experiments, such as: ICARUS, MINOS, MONOLITH and OPERA [5].

This process at typical $Q^2 \sim 1$ GeV$^2$ corresponds to transition region of QCD and represents therefore a serious problem for theoretical analysis. It is highly desirable to apply the methods which are less sensitive to various model assumptions. Here we present two such methods: positivity of density matrix and exploration of Bloom–Gilman duality, stressing the more fundamental aspects of the latter.

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## 2. Density matrix positivity and scattered lepton polarization

The positivity of density matrix, which emerged from Max Born interpretation of quantum mechanics, is known to provide the powerful method of analysis of various spin-dependent processes [6]. Here we will review its application [7] for Parity Violating DIS.

In lepton nucleon deep inelastic scattering all the observables involve the hadronic tensor of the nucleon $W_{\mu\nu}(p,q)$, where $p$, $k$ and $k^{'}$ are the four momenta of the nucleon, incoming $\nu_\tau(\bar{\nu}_\tau)$ and produced $\tau^-$ $(\tau^+)$, respectively, and $q=k-k^{'}$ is the momentum transfer:

$$\begin{aligned} W_{\mu\nu}(p,q) \;&=\; -g_{\mu\nu}W_1(\nu,q^2)+\frac{p_\mu p_\nu}{M^2}\,W_2(\nu,q^2)-i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{2M^2}\,W_3(\nu,q^2)\\ &+\frac{q_\mu q_\nu}{M^2}\,W_4(\nu,q^2)+\frac{p_\mu q_\nu+q_\mu p_\nu}{2M^2}\,W_5(\nu,q^2)\,. \end{aligned} \tag{2.1}$$

All structure functions, which are made dimensionless by including appropriate mass factors, depend on two Lorentz scalars $\nu = p\cdot q/M$ and $q^2=-Q^2$ $(Q^2>0)$, where $M$ is the nucleon mass. In the laboratory frame, let us denote by $E_\nu$, $E_\tau$ and $p_\tau$ the neutrino energy, $\tau$ energy and momentum, respectively and $\theta$ the scattering angle. We then have $\nu = E_\nu - E_\tau$ and $Q^2 = 2E_\nu[E_\tau - p_\tau\cos\theta]-m_\tau^2$, where $m_\tau = 1.777$ GeV is the $\tau$ mass. Finally, the Bjorken variable $x$ is defined as $x=Q^2/2p\cdot q$ and the physical region is $x_{\min}\le x\le 1$, where $x_{\min}=m_\tau^2/2M(E_\nu-m_\tau)$.

The unpolarized cross sections for deep inelastic scattering (1.1), are expressed as

$$\frac{d\sigma^\pm}{dE_\tau d\cos\theta}=\frac{G_F^2}{2\pi}\frac{M_W^4 p_\tau}{(Q^2+M_W^2)^2}R_\pm\;, \tag{2.2}$$

where $G_{\rm F}$ is the Fermi constant and $M_W$ is the $W$-boson mass. Here

$$\begin{aligned} R_\pm \;&=\; \frac{1}{M}\left\{\left(2W_1+\frac{m_\tau^2}{M^2}\,W_4\right)(E_\tau-p_\tau\cos\theta)+W_2\,(E_\tau+p_\tau\cos\theta)\right.\\ &\left.\pm\frac{W_3}{M}\left(E_\nu E_\tau+p_\tau^2-(E_\nu+E_\tau)p_\tau\cos\theta\right)-\frac{m_\tau^2}{M}\,W_5\right\}\,, \end{aligned} \tag{2.3}$$

where the $\pm$ signs correspond to $\tau^\mp$ productions.

Because of time reversal invariance, the polarization vector $\overrightarrow{P}$ of the $\tau$ in its rest frame, lies in the scattering plane defined by the momenta of the incident neutrino and the produced $\tau$. It has a component $P_{\rm L}$ along the direction of $\overrightarrow{p_\tau}$ and a component $P_{\rm P}$ perpendicular to $\overrightarrow{p_\tau}$. In addition, it is convenient to introduce also the degree of polarization defined as $P=\sqrt{P_{\rm P}^2+P_{\rm L}^2}$. As previously the $\pm$ signs correspond to $\tau^\mp$ productions and it is clear that if $W_3 = 0$, one has $R_+ = R_-$ and $\tau^+$ and $\tau^-$ have opposite polarizations.

From Eq. (2.1) clearly the hadronic tensor $W_{\mu\nu}(p,q)$ is semi-positive:

$$a^*_\mu W_{\mu\nu}(p,q) a_\nu \geq 0 \;, \tag{2.4}$$

for *any* complex 4-vector $a_\mu$. The $4\times 4$ matrix representation of $W_{\mu\nu}(p,q)$ in the laboratory frame, where $p = (M,0,0,0)$ and $q = (\nu, \sqrt{\nu^2+Q^2}, 0, 0)$ reads $\begin{pmatrix} M_1 & 0 \\ 0 & M_0 \end{pmatrix}$, where $M_1$ and $M_0$ are the following $2\times 2$ Hermitian matrices:

$$M_1 = \begin{pmatrix} -W_1 + W_2 + \frac{\nu^2}{M^2}W_4 + \frac{\nu}{M}W_5 & \frac{\sqrt{\nu^2+Q^2}}{M}\left(\frac{\nu}{M}W_4 + \frac{1}{2}W_5\right) \\ \frac{\sqrt{\nu^2+Q^2}}{M}\left(\frac{\nu}{M}W_4 + \frac{1}{2}W_5\right) & W_1 + \frac{\nu^2+Q^2}{M^2}W_4 \end{pmatrix}, \tag{2.5}$$

and

$$M_0 = \begin{pmatrix} W_1 & \frac{-i\sqrt{\nu^2+Q^2}}{2M}W_3 \\ \frac{+i\sqrt{\nu^2+Q^2}}{2M}W_3 & W_1 \end{pmatrix}. \tag{2.6}$$

The *necessary and sufficient conditions* for $W_{\mu\nu}(p,q)$ to satisfy inequality (2.4) are that all the principal minors of $M_1$ and $M_0$ should be positive definite. So for the diagonal elements we have three inequalities linear in the $W_i$'s namely

$$W_1 \;\geq\; 0\,, \tag{2.7}$$

$$-W_1 + W_2 + \frac{\nu^2}{M^2}W_4 + \frac{\nu}{M}W_5 \;\geq\; 0\,, \tag{2.8}$$

$$W_1 + \frac{\nu^2+Q^2}{M^2}W_4 \;\geq\; 0\,, \tag{2.9}$$

and from the $2\times 2$ determinants of $M_0$ and $M_1$ we get two inequalities quadratic in the $W_i$'s namely

$$W_1^2 \;\geq\; \frac{\nu^2+Q^2}{4M^2}W_3^2\,, \tag{2.10}$$

or equivalently

$$W_1 \;\geq\; \frac{\sqrt{\nu^2+Q^2}}{2M}|W_3|\,, \tag{2.11}$$

and

$$\left(-W_1 + W_2 + \frac{\nu^2}{M^2}W_4 + \frac{\nu}{M}W_5\right)\left(W_1 + \frac{\nu^2+Q^2}{M^2}W_4\right)$$
$$\geq \frac{\nu^2+Q^2}{M^2}\left(\frac{\nu}{M}W_4 + \frac{1}{2}W_5\right)^2 . \tag{2.12}$$

By imposing the last condition, only one of the two inequalities (2.8) or (2.9) is needed, the other follows automatically.

In order to test the usefulness of these constraints to restrict the allowed domains for $P_{\mathrm{P}}$ and $P_{\mathrm{L}}$, we proceed by the following method, without refer-

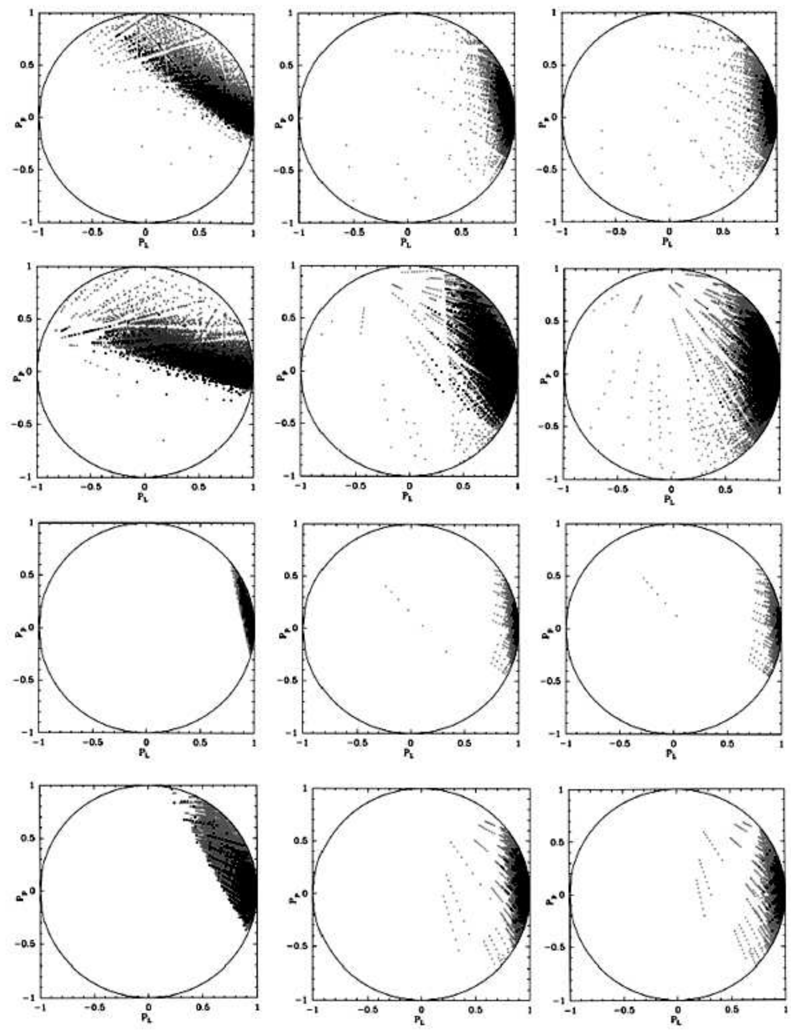

Fig. 1. For $\tau^+$ production, $P_{\mathrm{P}}$ *versus* $P_{\mathrm{L}}$ in a domain limited by $R_+ \geq 0$, $P \leq 1$ (grey area) plus non trivial positivity constraints (black area). From top to bottom and left to right, $E_\nu = 10$ GeV, $Q^2 = 1$ GeV$^2$, $x = 0.25, 0.6, 0.9$, $E_\nu = 10$ GeV, $Q^2 = 4$ GeV$^2$, $x = 0.4, 0.6, 0.9$, $E_\nu = 20$ GeV, $Q^2 = 1$ GeV$^2$, $x = 0.25, 0.6, 0.9$, $E_\nu = 20$ GeV, $Q^2 = 4$ GeV$^2$, $x = 0.25, 0.6, 0.9$.

ring to a specific model for the $W_i$'s. We generate randomly the values of the $W_i$'s, in the ranges $[0,+1]$ for $W_1$ and $W_2$, which are clearly positive and $[-1,+1]$ for $i=3,4,5$. The most trivial positivity constraints are $R_\pm \geq 0$, but in fact they are too weak and do not imply the obvious requirements $|P_\mathrm{L}| \leq 1$ and $|P_\mathrm{P}| \leq 1$ or $P \leq 1$ [1]. So we first impose $R_\pm \geq 0$ and $P \leq 1$ for different values of $E_\nu$, $Q^2$ and $x$ and as shown in Fig. 1, for $\tau^+$ production, the points which satisfy these constraints are represented by grey dots inside the disk, $P_\mathrm{L}^2 + P_\mathrm{P}^2 \leq 1$.

If we now add the non trivial positivity constraints Eqs. (10)–(15), which also guarantee that $P \leq 1$, we get the black dots, giving a much smaller area. In Fig. 1, the top row corresponds to $E_\nu = 10$ GeV and $Q^2 = 1$ GeV$^2$, the row below to $E_\nu = 10$ GeV and $Q^2 = 4$ GeV$^2$ and the next two rows to $E_\nu = 20$ GeV and $Q^2 = 1, 4$ GeV$^2$. Going from left to right $x$ increases from a value close to its minimum to 0.9. It is interesting to note that the black allowed area increases with $Q^2$ and becomes smaller for increasing incident energy and increasing $x$. For $\tau^-$ production, the corresponding areas are obtained by symmetry with respect to the center of the disk. For increasing $x$, since $P_\mathrm{L}$ is more and more restricted to values close to $+1$ for $\tau^+$ ($-1$ for $\tau^-$), it is striking to observe that the non trivial positivity constraints lead to a situation where the $\tau^+$ ($\tau^-$) is almost purely right-handed (left-handed), although it has a non zero mass.

## 3. Bloom–Gilman duality in QCD and $\Delta(1232)$

Let us discuss the quantitative description of Bloom–Gilman (BG) duality which requires the account [8] for the large $x$ enhanced higher twist (HT) terms behaving like $(M^2/(1-x)Q^2)^n \approx (M^2/s)^n$. The quantitative analysis of these terms may be performed by exploring the technique of Borel sum rules (SR), which is very popular when vacuum power corrections are considered [9] and was recently applied [10] for studies of BG duality.

As soon as only these enhanced power corrections are considered, the Borel SR in the variable $s$ is especially convenient. Comparing the analysis of BG duality to the most simple case of static meson characteristics one may see the two complications. Namely, the BG problem contains two scales, $s$ and $Q^2$, and the sum rule contains the contributions from $s$ and $u$ channels. However, keeping of the leading (in $1/(1-x)$) power corrections allows to avoid both complications. Calculating the Compton amplitude in the asymmetric point in the non-physical region close to the $s$ threshold, one may keep only the enhanced power corrections and neglect the contribution of $u$ channel. As a result, the Borel sum rule is completely similar to the

[1] Note that in the trivial case where $W_3 = W_4 = W_5 = 0$, $R \geq 0$ implies $P \leq 1$.

case of meson characteristics. Assuming the ansatz for spectral density

$$\rho(s) = \theta(s - s_0)\rho^{\mathrm{pert}}(s) + \theta(s_0 - s)\rho^{\mathrm{Res}}(s)\,, \tag{3.1}$$

where $s_0$ is the duality interval, and putting the Borel parameter to infinity, which leads to the disappearance of the power corrections, one gets

$$\int\limits_{s_{\min}}^{s_0} ds(\rho^{\mathrm{pert}}(s) - \rho^{\mathrm{Res}}(s)) = 0\,, \tag{3.2}$$

which is just BG duality.

Note that the calculation of $s_0$ from QCD, which is the real problem of QCD SR, would require the explicit account for the enhanced HT terms. This problem was already studied in detail [11] in the case of spin-averaged structure function $F_2$. One may worry, whether this analysis, assuming very large $x$, is applicable for studies of BG duality, corresponding to lower $x$. The positive answer may result from the following simple reasoning. The large $x$ behavior is governed by the power dependence $(1-x)^b$ and the exponent $b$, once established at very large $x$, should govern also the behavior at lower ones, relevant for BG duality. The similar reasoning may explain, why BFKL asymptotics, requiring very small $x$, may be successfully applied (see [12] and references therein) to describe data at much larger $x$. Namely, the Regge behavior $x^a$, once established at very low $x$, should also be applicable, with reasonable numerical accuracy, to much larger ones. The general success of the parametrization in the form $x^{-a}(1-x)^b$ may, in turn, be related to the *convexity* of parton distributions, which is preserved by the specific kinetic term of DGLAP equation [13].

It is important that some structure functions are better suited for application of duality. In the PC spin-dependent case this is the structure function $g_{\mathrm{T}}$ [10, 14]. The related property of this function is that it is free from the contribution of $\Delta(1232)$-resonance.

One may ask how general is that coincidence. The possible reason for such a generality may be the following. It is known since the seminal paper of Close and Isgur [15] that only parity doublets may fit BG duality. In (chiral-invariant) pQCD the states with opposite parity have the same mass. At the same time, it is not so in full QCD due to the Spontaneous Chiral Symmetry Breaking. Therefore, the opposite parity states become essentially different, and this difference is especially pronounced when one of the doublet components corresponds to strong low mass resonance, like $\Delta(1232)$. Therefore, one may just drop this strong component of parity doublet from consideration, with the hope that the remaining weak one will not essentially spoil the duality.

This universality of $\Delta(1232)$ exclusion seems to be supported by the talk of Lalakulich [16], as $\Delta$ relative contribution to $p+n$ is smaller than to $p-n$. Moreover, one may try to investigate the $\Delta(1232)$-free combination $3n-p$.

Another implication of the presented approach to BG duality for PV case may be the expected similarity of the contributions of vector and axial *transition* form factors, as they contribute identically to quark handbag diagram.

## 4. Conclusions

The positivity of density matrix provides the important constraints for scattered lepton polarization. There are arguments that $\Delta(1232)$ resonance should be excluded from the consideration of BG duality. Further analysis of this issues is highly desirable.

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