KINETIC DESCRIPTION OF FERMION PRODUCTION IN THE OSCILLATOR REPRESENTATION

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We investigate the fermion creation in quantum kinetic theory by applying "oscillator representation" approach, which was earlier developed for bosonic systems. We show that in some particular cases (Yukawa-like interaction, fixed direction of external vector field) resulting Kinetic Equation (KE) reduces to KE obtained by time-dependent Bogoliubov transformation method. We conclude "oscillator representation" approach to be more universal for the derivation of quantum transport equations in strong space-homogeneous time-dependent fields. We discuss some possible applications of obtained KE to cosmology and particle production in strong laser fields.

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1. Introduction

Spontaneous particle production of pairs under the action of strong external fields attracts a lot of attention since it is a multi-particle non-perturbative effect which is still lacking experimental verification. According to the famous Schwinger formula [1] the probability of particle creation in the constant electromagnetic field becomes essential if the field reaches the critical value $E^{\rm c} = m^2/e$. Considering electron–positron pair creation in laser field we may conclude that one must have unattainable static field $E^{\rm c}_{e^+e^-} = 1.32 \times 10^{16}$ V/cm. However, theoretical and numerical results obtained in recent papers (*e.g.* [2–6]) show that the probability of particle creation in *time-dependent* fields can be gradually enhanced in contrast to static ones. Also recent developments in laser technology (method of chirped pulse amplification [7]) and underway construction of X-ray free electron lasers will allow an experimental verification of spontaneous particle creation in the nearest future.

Fermion production in external time-dependent fields was considered in a number of papers [8–11]. The method of the time-dependent Bogoliubov transformation[1] used there, faces difficulties if the external field has more than one non-zero component, thus KE was obtained only for the linear polarized (also called flux-tube geometry) field ($A_\mu = (0,0,0,A(t) = A_3(t))$). As was shown earlier for boson creation [15] in periodic laser field, pair production is efficient if the field has rather the circular polarisation than the linear one[2].

The approach presented in current paper resolves the problem of fermion creation in an arbitrary polarised field (*e.g.* in Hamilton gauge $A_\mu = (0, A_1(t), A_2(t), A_3(t))$). This result can be of great importance for the theoretical predictions about dilepton and photon yield in high-intensity laser experiments [13, 14].

As already mentioned, the classical approach to the description of the particle creation in the external fields or in curved space is based on the time-dependent Bogoliubov transformation [8, 9]. Concentrating on fermion fields we will briefly summarise this approach. First of all let us suppose that in the asymptotic states $t \to \pm\infty$ the external field vanishes. It means that for the limit $t \to \pm\infty$ the interpretation in terms of particles and antiparticles becomes possible. Thus we can introduce the orthogonal set of functions $\psi^{\pm}_{ks}(x)$, which represent the positive and negative frequency solution at $t \to -\infty$. The field function can be decomposed as

$$\psi(x) = \sum_s \int \frac{d^3k}{(2\pi)^{3/2}} \left(\psi^+_s(x;\boldsymbol{k}) a_s(\boldsymbol{k}) + \psi^+_s(x;\boldsymbol{k}) b^+_s(\boldsymbol{k})\right) , \tag{1}$$

where we perform the sum over spin indices s. $a_s(b^+_s)$ is the annihilation (creation) operator of the (anti)particle with the spin s and momentum $\boldsymbol{k}$. If we demand that the creation and annihilation operators introduced in this way satisfy general anticommutational relations, then the Fock space can be constructed with the vacuum state defined at $t \to -\infty$. However, following this procedure one can easily prove that Hamiltonian in the external time-dependent field will be non-diagonal (except the initial state $t \to -\infty$). To diagonalize it one can use Bogoliubov time-dependent transformation to redefine the particle creation $c_s(t)$ and annihilation $d^+_s(t)$ operators with nontrivial equations of motion for them. The particle number in

[1] The Bogoliubov transformations are named after N.N. Bogoliubov who introduced and applied these canonical transformations to determine integrals of motion for creation and annihilation operators [12].

[2] As was noted in [5] the linear polarised field is not appropriate for a quantitative description of the laser pulse. However, the linear polarised field was considered [2, 5, 6, 13] to obtain qualitative level results.

this formalism can be defined at every time moment as $N_{\boldsymbol{k}_s}(t) \equiv f_s(\boldsymbol{k},t) = \langle 0|c_s^+(\boldsymbol{k};t)c_s(\boldsymbol{k};t)|0\rangle$.

In this work, we will use another approach — the so-called "oscillator representation", which was earlier developed for bosonic systems [15, 16]. The main idea is to construct the field function and to define the time-dependent creation and annihilation operators so that the Hamiltonian immediately becomes diagonal. Moreover, it should take the form of the free field Hamiltonian at $t \to -\infty$. The solution of equations of motion derived from Dirac equation for introduced operators will provide the wanted distribution function $f_s(\boldsymbol{k},t)$.

This paper is organised as follows. In Sec. 2 the oscillator representation formalism for fermions is given. In Sec. 3 the kinetic equation is derived. The applications of the model and particularly the fermion production in the early Universe are considered in Sec. 4. Sec. 5 summarises the article. Appendix contains the KE in the convenient form for numerical calculations.

2. Oscillator (holomorphic) representation

We start from the Lagrangian for electrodynamics

$$L = \bar{\psi}(iD_\mu\gamma^\mu - m(t))\psi\,, \tag{2}$$

where D_α is the covariant derivative

$$D_\alpha = \partial_\alpha + ieA_\alpha\,. \tag{3}$$

Let us assume that the function $m(t)$ is an arbitrary smooth function of time. We also demand that the electric field is classical and space-homogeneous with a continuous $A_\alpha(t)$. The classical approximation is supported by the leading order of a large N approximation, see *e.g.* [10].

In order to derive the correct oscillator representation of the fermion field one can use the Foldy–Wouthuysen transformation [17]:

$$\psi(x) \;=\; \int \frac{d^3p}{(2\pi)^{3/2}}\, e^{i\,\boldsymbol{px}}\, e^{iS\,(\boldsymbol{p},t)} \left(a_s(\boldsymbol{p},t)u_s(0) + b_s^+(-\boldsymbol{p},t)v_s(0)\right), \tag{4}$$

$$\bar{\psi}(x) \;=\; \int \frac{d^3p}{(2\pi)^{3/2}}\, e^{-i\,\boldsymbol{px}} \left(a_s^+(\boldsymbol{p},t)\bar{u}_s(0) + b_s(-\boldsymbol{p},t)\bar{v_s}(0)\right) e^{iS\,(\boldsymbol{p},t)}\,, \tag{5}$$

where summation over spinor indices ($s = 1,2$) is implied, and spinors $u(0), v(0)$ are defined as a solution of free Dirac equation in the particle rest frame:

$$(\gamma_0 - 1)u(0) \;=\; 0\,, \tag{6}$$

$$(\gamma_0 + 1)v(0) \;=\; 0\,. \tag{7}$$

The unitary operator $e^{iS\,(\boldsymbol{p},t)}$ can be dependent both on time and momentum. According to the scheme of oscillator representation $e^{iS\,(\boldsymbol{p},t)}$ is defined so that the Hamiltonian density after the field decomposition

$$\begin{aligned} H(\boldsymbol{p},t) \;=\; & \left(a_s^+(\boldsymbol{p},t)\bar{u}_s(0) + b_s(-\boldsymbol{p},t)\bar{v}_s(0)\right) e^{iS(\boldsymbol{p},t)}(P_i\gamma_i + m)e^{iS(\boldsymbol{p},t)} \\ & \times \left(a_s(\boldsymbol{p},t)u_s(0) + b_s^+(-\boldsymbol{p},t)v_s(0)\right) , \end{aligned} \tag{8}$$

becomes diagonal, *i.e.*

$$\begin{aligned} H(\boldsymbol{p},t) \;=\; & \left(a_s^+(\boldsymbol{p},t)\bar{u}_s(0) + b_s(-\boldsymbol{p},t)\bar{v}_s(0)\right), \omega(\boldsymbol{P},t) \\ & \times \left(a_s(\boldsymbol{p},t)u_s(0) + b_s^+(-\boldsymbol{p},t)v_s(0)\right) , \end{aligned} \tag{9}$$

with quasiparticle energy $\omega(\boldsymbol{P},t) = \sqrt{\boldsymbol{P}^2 + m^2}$ defined by kinetic momentum $\boldsymbol{P} = \boldsymbol{p} - e\boldsymbol{A}$.

This condition can be rewritten as

$$e^{iS(\boldsymbol{p},t)}(P_i\gamma_i + m)e^{iS(\boldsymbol{p},t)} = \omega(\boldsymbol{P},t)\,, \tag{10}$$

with the following solution

$$e^{iS(\boldsymbol{p},t)} = \frac{\omega(\boldsymbol{P},t) + m + P_i\gamma^i}{\sqrt{2\omega(\boldsymbol{P},t)(m + \omega(\boldsymbol{P},t))}}\,. \tag{11}$$

One can write down the explicit expression for S in a close form

$$S(\boldsymbol{p},t) = i\frac{P_i\gamma_i}{|\boldsymbol{P}|}\phi\,, \tag{12}$$

where ϕ is given by

$$\sin(2\phi) = \frac{|\boldsymbol{P}|}{\omega(\boldsymbol{P},t)}\,, \qquad \cos(2\phi) = \frac{m}{\omega(\boldsymbol{P},t)}\,. \tag{13}$$

It is easy to convince that

$$u(\boldsymbol{p} \to \boldsymbol{P}) \;=\; e^{iS(\boldsymbol{p},t)}u(0)\,, \tag{14}$$

$$v(-(\boldsymbol{p} \to \boldsymbol{P})) \;=\; e^{iS(\boldsymbol{p},t)}v(0)\,, \tag{15}$$

where $u(\boldsymbol{p}), v(\boldsymbol{p})$ are the solutions of free Dirac equation. Taking into account these expressions (14), (15) we can rewrite the decomposition of Eqs. (4), (5) in the form

$$\psi(x) \;=\; \int \frac{d^3p}{(2\pi)^{3/2}}\, e^{i\,\boldsymbol{p}\boldsymbol{x}} \left\{a_s(\boldsymbol{p},t)u_s(\boldsymbol{P}) + b_s^+(-\boldsymbol{p},t)v_s(-\boldsymbol{P})\right\} , \tag{16}$$

$$\bar{\psi}(x) \;=\; \int \frac{d^3p}{(2\pi)^{3/2}}\, e^{-i\,\boldsymbol{p}\boldsymbol{x}} \left\{a_s^+(\boldsymbol{p},t)\bar{u}_s(\boldsymbol{P}) + b_s(-\boldsymbol{p},t)\bar{v}_s(-\boldsymbol{P})\right\} . \tag{17}$$

Thus, the formal substitution $\boldsymbol{p} \to \boldsymbol{P}$ in the free field decomposition results in (4), (5).

The equations of motion for the creation and annihilation operators can be obtained from Dirac equations (dots denote the derivative with respect to time):

$$\dot{a}_{s'}(\boldsymbol{p},t) \;=\; -i\omega a_{s'}(\boldsymbol{p},t) - \Theta^u_{s's} a_s(\boldsymbol{p},t) - \Xi^u_{s's} b^+_s(-\boldsymbol{p},t)\,, \tag{18}$$

$$\dot{b}^+_{s'}(-\boldsymbol{p},t) \;=\; i\omega b^+_{s'}(-\boldsymbol{p},t) - \Theta^v_{s's} b^+_s(-\boldsymbol{p},t) - \Xi^v_{s's} a_s(\boldsymbol{p},t)\,, \tag{19}$$

or in matrix form

$$\dot{a}(\boldsymbol{p},t) \;=\; -i\omega a(\boldsymbol{p},t) - \Theta^u a(\boldsymbol{p},t) - \Xi^u b^+(-\boldsymbol{p},t)\,, \tag{20}$$

$$\dot{b}^+(-\boldsymbol{p},t) \;=\; i\omega b^+(-\boldsymbol{p},t) - \Theta^v b^+(-\boldsymbol{p},t) - \Xi^v a(\boldsymbol{p},t)\,, \tag{21}$$

where $\Theta^{u,v}$, $\Xi^{u,v}$ are given by

$$\Theta^u_{s's} \;=\; u^+_{s'}(0)e^{-iS}\partial_0 e^{iS} u_s(0)\,, \tag{22}$$

$$\Theta^v_{s's} \;=\; v^+_{s'}(0)e^{-iS}\partial_0 e^{iS} v_s(0)\,, \tag{23}$$

$$\Xi^u_{s's} \;=\; u^+_{s'}(0)e^{-iS}\partial_0 e^{iS} v_s(0)\,, \tag{24}$$

$$\Xi^v_{s's} \;=\; v^+_{s'}(0)e^{-iS}\partial_0 e^{iS} u_s(0)\,. \tag{25}$$

The straightforward calculation gives

$$\Theta^u \;=\; \Theta^v = ie\,\frac{[\boldsymbol{P}\times\boldsymbol{E}]}{2\omega(\omega+m)}\,\boldsymbol{\sigma}\,, \tag{26}$$

$$\Xi^u \;=\; -\Xi^v = \frac{\dot{\omega}+\dot{m}}{2\omega(\omega+m)}\,\boldsymbol{P}\boldsymbol{\sigma} - \frac{e}{2\omega}\,\boldsymbol{E}\boldsymbol{\sigma}\,, \tag{27}$$

where σ_k are Pauli matrices and $\boldsymbol{E} = -\dot{\boldsymbol{A}}$ is the field strength.

In infinite past $t \to -\infty$, the operators a_s and b_s should satisfy the classical anticommutation relations. However, one can see that the second terms in the right-hand side of the equations of motion (20, 21) can modify the classical anticommutation relations in the presence of the time-dependent field. Indeed let us assume that

$$\{a_s(\boldsymbol{k},t), a^+_{s'}(\boldsymbol{k'},t)\} = \Pi_{ss'}(\boldsymbol{k},t)\delta^3(\boldsymbol{k}-\boldsymbol{k'})\,, \tag{28}$$

where the equation for unknown time-dependent matrix $\Pi_{ss'}$ is derived by using (20, 21)

$$\dot{\Pi}(\boldsymbol{k},t) = [\Pi(\boldsymbol{k},t), \Theta^u(\boldsymbol{k},t)]\,. \tag{29}$$

However, taking into account trivial symmetries of the matrix $\Pi_{ss'}$

$$\Pi_{ss'} = \Pi_{s's} = \Pi^*_{s's} \tag{30}$$

and initial conditions in infinite past $\Pi_{ss'}(\boldsymbol{k}, t \to -\infty) = \delta_{ss'}$ one easily proves that $\Pi_{ss'}(t) = \text{const} = \delta_{ss'}$, *i.e.* canonical commutation relations are valid even in time-dependent external field.

3. Distribution function and kinetic equation

In this section we will derive the equation of motion (or KE) for the one-particle correlator

$$f_{ss'}(\boldsymbol{p}, t) = \langle 0| a^+_{s'}(\boldsymbol{p}, t) a_s(\boldsymbol{p}, t)|0\rangle\,. \tag{31}$$

The distribution function is the diagonal components of $f_{ss'}(\boldsymbol{p}, t)$ [9, 10, 18]. Further calculations will show that $f_{ss'}$ is coupled to one-particle correlator describing the antiparticle distribution $g_{ss'}$ and two anomalous correlators $y^{\pm}_{ss'}$:

$$g_{ss'}(\boldsymbol{p}, t) = \langle 0| b_{s'}(-\boldsymbol{p}, t) b^+_s(-\boldsymbol{p}, t)|0\rangle\,, \tag{32}$$

$$y^-_{ss'}(\boldsymbol{p}, t) = \langle 0| b_{s'}(-\boldsymbol{p}, t) a_s(\boldsymbol{p}, t)|0\rangle\,, \tag{33}$$

$$y^+_{ss'}(\boldsymbol{p}, t) = \langle 0| a^+_{s'}(\boldsymbol{p}, t) b^+_s(-\boldsymbol{p}, t)|0\rangle\,. \tag{34}$$

KE is obtained by differentiating (31)–(34) with respect to time and subsequent substituting the equations of motion (20), (21) and definitions (31)–(34). After some trivial transformation one gets:

$$\dot f = i[f, \Theta] - y^- \Xi - \Xi y^+\,, \tag{35}$$

$$\dot y^- = -2i\omega y^- + i[y^-, \Theta] + f\Xi - \Xi g\,, \tag{36}$$

$$\dot y^+ = 2i\omega y^+ + i[y^+, \Theta] - g\Xi + \Xi f\,, \tag{37}$$

$$\dot g = i[g, \Theta] + y^- \Xi + \Xi y^+\,, \tag{38}$$

where $\Theta = \Theta^u/i = \Theta_i \sigma_i$ and $\Xi = \Xi^u = \Xi_i \sigma_i$ are hermitian matrices. It is more convenient to introduce two hermitian matrices

$$r^+ = \frac{1}{2}(y^+ + y^-)\,, \tag{39}$$

$$r^- = \frac{i}{2}(y^+ - y^-)\,, \tag{40}$$

and decompose $\chi = f,\ g,\ r^{\pm}$ into U(2) basis (I is a unitary matrix)

$$\chi \ = \ \chi_0 I + \chi_i\sigma_i\,, \qquad i = 1,2,3; \tag{41}$$

$$\chi_0 \ = \ \frac{1}{2}\mathrm{Tr}\chi\,, \qquad \chi_i = \frac{1}{2}\mathrm{Tr}\,(\chi\sigma_i)\,. \tag{42}$$

Finally in terms of (41), (42) KE (35)–(38) takes the following form:

$$\dot{f}_0 \ = \ -2\boldsymbol{\Xi}\boldsymbol{r}^+\,, \tag{43}$$

$$\dot{\boldsymbol{f}} \ = \ -2\left[\boldsymbol{f}\times\boldsymbol{\Theta}\right] + 2\left[\boldsymbol{r}^-\times\boldsymbol{\Xi}\right] - 2r_0^+\boldsymbol{\Xi}\,, \tag{44}$$

$$\dot{g}_0 \ = \ 2\boldsymbol{\Xi}\boldsymbol{r}^+\,, \tag{45}$$

$$\dot{\boldsymbol{g}} \ = \ -2\left[\boldsymbol{g}\times\boldsymbol{\Theta}\right] - 2\left[\boldsymbol{r}^-\times\boldsymbol{\Xi}\right] + 2r_0^+\boldsymbol{\Xi}\,, \tag{46}$$

$$\dot{r}_0^+ \ = \ 2\omega r_0^- + \boldsymbol{\Xi}\boldsymbol{f} - \boldsymbol{\Xi}\boldsymbol{g}, \tag{47}$$

$$\dot{\boldsymbol{r}}^+ \ = \ 2\omega\boldsymbol{r}^- - 2\left[\boldsymbol{r}^+\times\boldsymbol{\Theta}\right] + \boldsymbol{\Xi}(f_0 - g_0)\,, \tag{48}$$

$$\dot{r}_0^- \ = \ -2\omega r_0^+, \tag{49}$$

$$\dot{\boldsymbol{r}}^- \ = \ -2\omega\boldsymbol{r}^+ - 2\left[\boldsymbol{r}^-\times\boldsymbol{\Theta}\right] + \left[\boldsymbol{f}\times\boldsymbol{\Xi}\right] + \left[\boldsymbol{g}\times\boldsymbol{\Xi}\right]. \tag{50}$$

The KE (43)–(50) allows some further simplifications if we assume zero initial conditions in infinite past $t\to-\infty$ for $f_0, \boldsymbol{f} = 0$ (absence of particles), $r_0^{\pm}, \boldsymbol{r}^{\pm} = 0$ (anomalous correlators turns to zero at in-vacuum), and $g_0 = 1, \boldsymbol{g} = 0$ (neutrality condition and absence of antiparticles). Summing (43) and (45), (44) and (46) one finds the following integrals of motion:

$$f_0 \ = \ 1 - g_0\,, \tag{51}$$

$$\boldsymbol{f} \ = \ -\boldsymbol{g}\,. \tag{52}$$

Taking them into account KE equation (43)–(50) is rewritten as

$$\dot{f}_0 \ = \ -2\boldsymbol{\Xi}\boldsymbol{r}^+\,, \tag{53}$$

$$\dot{\boldsymbol{f}} \ = \ -2\left[\boldsymbol{f}\times\boldsymbol{\Theta}\right] + 2\left[\boldsymbol{r}^-\times\boldsymbol{\Xi}\right] - 2r_0^+\boldsymbol{\Xi}\,, \tag{54}$$

$$\dot{r}_0^+ \ = \ 2\omega r_0^- + 2\boldsymbol{\Xi}\boldsymbol{f}\,, \tag{55}$$

$$\dot{\boldsymbol{r}}^+ \ = \ 2\omega\boldsymbol{r}^- - 2\left[\boldsymbol{r}^+\times\boldsymbol{\Theta}\right] + \boldsymbol{\Xi}(2f_0 - 1)\,, \tag{56}$$

$$\dot{r}_0^- \ = \ -2\omega r_0^+\,, \tag{57}$$

$$\dot{\boldsymbol{r}}^- \ = \ -2\omega\boldsymbol{r}^+ - 2\left[\boldsymbol{r}^-\times\boldsymbol{\Theta}\right]. \tag{58}$$

4. Applications

4.1. Scalar-like interaction

If the vector field vanishes $A_i(t) = 0, E_i(t) = 0$, we obtain

$$\boldsymbol{\Theta} \ = \ 0\,, \tag{59}$$

$$\boldsymbol{\Xi} \ = \ \frac{\dot{m}}{2\omega^2}\,\boldsymbol{p}\,. \tag{60}$$

For this case the KE (43)–(50) allows some simplification. First of all, the vector components $\boldsymbol{r}^{\pm}$, $\boldsymbol{f}$, $\boldsymbol{g}$ will be collinear to $\boldsymbol{p}$. Thus let us redefine

$$\boldsymbol{r}^{\pm} = \mp v^{\pm} \frac{\boldsymbol{p}}{|\boldsymbol{p}|} \tag{61}$$

and assuming zero initial conditions in the asymptotic past rewrite the KE in the form

$$\begin{aligned} \dot{f}_0 &= 2Wv^{+} \,, & (62)\\ \dot{v}^{+} &= W(1-2f_0) - 2\omega v^{-} \,, & (63)\\ \dot{v}^{-} &= 2\omega v^{+} \,, & (64) \end{aligned}$$

where $W = \dot{m}/(2\omega^2)\,|\boldsymbol{p}|$ and in the above equations the neutrality condition $g_0 = 1 - f_0$ was used. Applying zero initial conditions to f_0 and $\boldsymbol{f}$ we can conclude that the vector components of the distribution function for fermions and anti-fermions are identical zero and the evolution of f_0 component is given by (62)–(64).

So far we have not discussed any origin of the time dependence of the mass $m(t)$. However, any fermion interaction of the Yukawa type $\Delta L = g\phi\bar{\psi}\psi$ in the mean field approximation for the scalar field ϕ results in the time-dependent effective fermion mass $m(t) = m + g\,\langle\phi\rangle$. Such a type of interactions is presented in the Higgs model, sigma model and can appear in some effective field theories (*e.g.* Walecka model [19]).

Here we will concentrate on another important example where time-dependence of the effective fermion mass can be generated, namely, conformal field theory.

4.1.1. Fermion production in the Early Universe

For the sake of consistency let us recall some general and well-known facts of general relativity.

The Lagrangian of the massive fermion field in general relativity is written as

$$L = \sqrt{-g}\left[i\bar{\psi}(x)\gamma^{n}(x)\Delta_n\,\psi(x) - m\bar{\psi}(x)\psi(x)\right]\,, \tag{65}$$

where

$$\begin{aligned} \gamma^{n}(x) &= h^{n}_{(\alpha)}\gamma^{\alpha}\,, & (66)\\ \Delta_n &= \partial_n + \frac{1}{4}C_{\mu\nu\rho}h^{(\rho)}_{n}\gamma^{\nu}\gamma^{\mu}\,, & (67) \end{aligned}$$

The rotation coefficients can be defined by means of tetrad as (for details, refer to [9, 20])

$$C_{\mu\nu\rho} = \left(\Delta_m h^{n}_{(\mu)}\right)h_{(\nu)n}h^{m}_{(\rho)}\,. \tag{68}$$

We will consider conformally static space-time. In this case the metric is conformally related to a static space-time. For a particular case of Robertson–Walker metric we have

$$g_{\mu\nu} = a^2(\eta) h_{\mu\nu}\,, \tag{69}$$

$$ds^2 = a^2(\eta)(d\eta^2 - d\boldsymbol{x}^2)\,, \tag{70}$$

where a is the conformal factor, $\eta = \int dt\ a(t)$ is the conformal time, $h_{\mu\nu}$ is the Minkowski metric. Introducing new field variables $\psi_c = a^{1/2}(\eta)\psi$ we obtain

$$L = a^2(\eta)\left[i\bar{\psi}_c(x)\gamma^\mu\partial_\mu\psi_c(x) - m(\eta)\bar{\psi}_c(x)\psi_c(x)\right]\,, \tag{71}$$

where we define the effective fermion mass $m(\eta) \equiv ma(\eta)$.

The cosmological creation of particles is considered in terms of the so-called conformal Universe [21], where the volume of the Universe does not increase, while all masses, including the Planck mass, are scaled by the cosmic factor $a(\eta)$. We assume a stiff state scenario, where the mass $m(\eta)$ is given by

$$m(\eta) = m_{\rm I}\sqrt{1 + 2H_I\eta}\,, \tag{72}$$

where $H_{\rm I}$ is initial data at the matter production instant.

The kinetic equation transformed into convenient form for numerical calculations is given in Appendix. In Fig. 1 the evolution of the fermion number density (g_f is the degeneracy factor)

$$n = 2g_f \int \frac{d^3p}{(2\pi)^3} f(\boldsymbol{p}, t)\,, \tag{73}$$

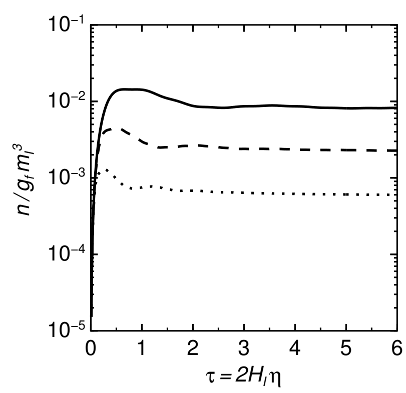

Fig. 1. The time dependence of the number density for fermions with different masses $m_{\rm I}/(2H_{\rm I}) = 0.5$, 1, 2 (from top to bottom).

for different ratios $m_{\rm I}/(2H_{\rm I})$ is shown. The final spectra of produced particles are far from exponential as seen from Fig. 2. The time evolution of specific entropy S/n is shown in Fig. 3, where entropy S is given by

$$S = 2g_f \int \frac{d^3p}{(2\pi)^3} \left[f(\boldsymbol{p},t) \ln f(\boldsymbol{p},t) + (1 - f(\boldsymbol{p},t)) \ln(1 - f(\boldsymbol{p},t)) \right] . \quad (74)$$

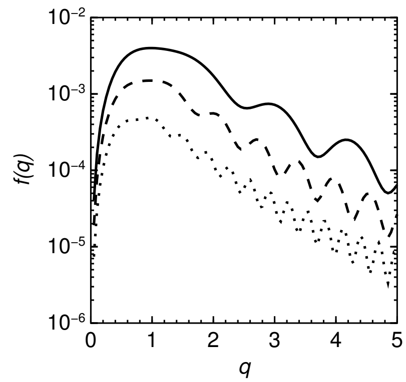

Fig. 2. The momentum dependence of the distribution function for fermions with different masses $m_{\rm I}/(2H_{\rm I}) = 0.5$, 1, 2 (from top to bottom) at the fixed moment of time $\tau \equiv 2H_{\rm I}\, t = 6$; $q \equiv |\boldsymbol{q}|$ is dimensionless momentum $\boldsymbol{q} = \boldsymbol{p}/m_{\rm I}$.

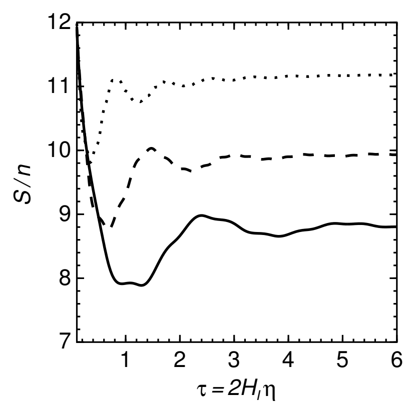

Fig. 3. The time dependence of the specific entropy S/n (74) for fermions with different masses $m_{\rm I}/(2H_{\rm I}) = 0.5$, 1, 2 (from bottom to top).

4.2. Vector interaction

In this section we consider some application of the model to the interaction of the type $\Delta L = eA_\mu \bar{\psi}\gamma^\mu\psi$.

4.2.1. Particle production in the external field with fixed direction

Here we consider another simple example, particle production in the external time-dependent field with fixed direction. As in the previous subsection, let us demand zero initial conditions for the particle fields in the asymptotic past. The vector potential is given by time-dependent amplitude and constant vector $\boldsymbol{n}$:

$$\boldsymbol{A} = A(t)\boldsymbol{n}\,. \tag{75}$$

The KE in this case also can be reduced[3] to the system of three equations

$$\dot{f}_0 \;=\; 2Wv^+\,, \tag{76}$$

$$\dot{v}^+ \;=\; W(1-2f_0) - 2\omega v^-\,, \tag{77}$$

$$\dot{v}^- \;=\; 2\omega v^+\,, \tag{78}$$

with W, $v^\pm$ defined by

$$W \;=\; |\boldsymbol{\Xi}| = \frac{e|\boldsymbol{E}|\sqrt{p_\perp^2 + m^2}}{2\omega^2}\,, \tag{79}$$

$$v^\pm \;=\; \mp\left(\frac{\Xi_\parallel}{|\boldsymbol{\Xi}|}r_\parallel^\pm + \frac{\Xi_\perp}{|\boldsymbol{\Xi}|}r_\perp^\pm\right)\,, \tag{80}$$

where the vector decomposition was used:

$$\boldsymbol{P} \;=\; P_\parallel \boldsymbol{n} + \frac{\boldsymbol{p}_\perp}{|\boldsymbol{p}_\perp|}p_\perp\,, \tag{81}$$

$$\boldsymbol{r}^\pm \;=\; r_\parallel^\pm \boldsymbol{n} + \frac{\boldsymbol{p}_\perp}{|\boldsymbol{p}_\perp|}r_\perp^\pm\,. \tag{82}$$

The set of equations (76)–(78) coincides with pioneering results obtained by Bialynicki-Birula, Gornicki, Rafelski [22] in the framework of the equal-time Wigner function. Also, for the one-component external field $\boldsymbol{A} = (0, 0, A_3(t))$, the derivation of this KE in the framework of Bogoliubov quasiparticles can be found in [9, 11]; some applications to particle production in strong laser fields [13, 14] as well as in the ultrarelativistic heavy-ion collision [4, 23] in the Abelian dominance approximation [24] were also considered.

[3] We leave the proof to the reader.

5. Summary

In the present paper, we use the oscillator representation [15,16] to derive the most general kinetic equation for fermions in time-dependent external electro-magnetic field of arbitrary direction. Also particle production due to time dependence of fermion mass (as a result of Yukawa-like interactions in the meanfield approximation or conformally static space-time) is considered.

The derived KE in the special cases (time-dependent mass, fixed direction of the external field) coincides with KE obtained by time-dependent Bogoliubov transformation approach. However, for the latter there is an unsolved problem of particle production in the vector fields of alternating direction. As shown in the present paper, the oscillator representation overcomes this difficulty and helps us to derive KE for fermions created in field of general polarization $\boldsymbol{A} = (A_1(t), A_2(t), A_3(t))$.

Some applications of derived KE are discussed. The detailed investigation and numerical calculations are done for fermion creation in conformal cosmology. The solution of KE in this case has shown that the obtained momentum distribution function is far from equilibrium, therefore can be an argument in favour that the introduction of temperature is impossible without taking into account collision processes in created fermion gas.

The consistent description of particle creation in the strong laser fields includes the effects of the field direction alteration. So far as the simplest model of the laser field only the linear polarised one is considered (*e.g.* [2, 5, 6, 13]). Our approach based on quantum field theory allows numerical calculation of particle production in the field of arbitrary polarisation. In this case the simplifications assumed in Sec. 4 are no longer applicable and one has to deal with the general system of equation (53)–(58). Preliminary numerical results on this subject are promising but out of the scope of the current paper and will be printed elsewhere.

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Appendix A

We introduce dimensionless variables of the time τ, momentum q and the parameter $\gamma_{\rm I}$ according to the following formulas:

$$\tau \;=\; 2H_{\rm I}\eta, \tag{A.1}$$

$$\boldsymbol{q} \;=\; \frac{\boldsymbol{p}}{m_{\rm I}}\,, \tag{A.2}$$

$$\gamma_{\rm I} \;=\; \frac{m_{\rm I}}{2H_{\rm I}}\,. \tag{A.3}$$

The kinetic equation (62-64) in this variables takes the form

$$\frac{d}{d\tau}f_0 \;=\; 2\tilde{W}v^+, \tag{A.4}$$

$$\frac{d}{d\tau}v^+ \;=\; \tilde{W}(1-2f_0) - 2\tilde{\omega}\gamma_{\rm I}v^-, \tag{A.5}$$

$$\frac{d}{d\tau}v^- \;=\; 2\tilde{\omega}\gamma_{\rm I}v^+, \tag{A.6}$$

where

$$\tilde{W} \;=\; \frac{1}{4}\frac{|\boldsymbol{q}|}{(\boldsymbol{q}^2+1+\tau)\sqrt{1+\tau}}\,, \tag{A.7}$$

$$\tilde{\omega} \;=\; \sqrt{\boldsymbol{q}^2+1+\tau}\,. \tag{A.8}$$

REFERENCES

[1] J.S. Schwinger, *Phys. Rev.* **82**, 664 (1951).

[2] V.S. Popov, *JETP Lett.* **74**, 133 (2001) [*Pisma Zh. Eksp. Teor. Fiz.* **74**, 151 (2001)].

[3] A. Ringwald, *Phys. Lett.* **B510**, 107 (2001) [hep-ph/0103185].

[4] A.V. Prozorkevich, S.A. Smolyansky, V.V. Skokov, E.E. Zabrodin, *Phys. Lett.* **B583**, 103 (2004) [nucl-th/0401056].

[5] D.B. Blaschke, A.V. Prozorkevich, S.A. Smolyansky, A.V. Tarakanov, physics/0410114.

[6] D.B. Blaschke, A.V. Prozorkevich, C.D. Roberts, S.M. Schmidt, S.A. Smolyansky, nucl-th/0511085.

[7] S.V. Bulanov, T. Esirkepov, T. Tajima, *Phys. Rev. Lett.* **91**, 085001 (2003).

[8] M.S. Marinov, V.S. Popov, *Fortsch. Phys.* **25**, 373 (1977).

[9] A.A. Grib, S.G. Mamaev, V.M. Mostepanenko, *Quantum Effects in Intensive External Fields*, Energoatomizdat, Moscow 1988.

[10] Y. Kluger, J.M. Eisenberg, B. Svetitsky, F. Cooper, E. Mottola, *Phys. Rev. Lett.* **67**, 2427 (1991).

[11] S.A. Smolyansky *et al.*, hep-ph/9712377.

[12] N.N. Bogoliubov, *J. Phys.* **11**, 23 (1947).

[13] R. Alkofer, M.B. Hecht, C.D. Roberts, S.M. Schmidt, D.V. Vinnik, *Phys. Rev. Lett.* **87**, 193902 (2001) [nucl-th/0108046].

[14] A. Ringwald, hep-ph/0304139.

[15] V.N. Pervushin, V.V. Skokov, A.V. Reichel, S.A. Smolyansky, A.V. Prozorkevich, *Int. J. Mod. Phys.* **A20**, 5689 (2005) [hep-th/0307200].

[16] V.N. Pervushin, V.I. Smirichinski, *J. Phys. A: Math. Gen.*, **32**, 6191 (1999).

[17] C. Itzykson, J.-B. Zuber, *Quantum Field Theory* , McGraw-Hill Book Co., 1985.

[18] D.V. Vinnik, V.A. Mizerny, A.V. Prozorkevich, S.A. Smolyansky, V.D. Toneev, *Phys. Atom. Nucl.* **64**, 775 (2001) [*Yad. Fiz.* **64**, 836 (2001)].

[19] J.D. Walecka, *Ann. Phys.* **83**, 491 (1974).

[20] N.D. Birrel, P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, 1982.

[21] V.N. Pervushin, D.V. Proskurin, A.A. Gusev, *Grav. Cosmol.* **8**, 181 (2002).

[22] I. Bialynicki-Birula, P. Gornicki, J. Rafelski, *Phys. Rev.* **D44**, 1825 (1991).

[23] V.V. Skokov, P. Levai, *Phys. Rev.* **D71**, 094010 (2005) [hep-ph/0410339].

[24] M. Gyulassy, A. Iwasaki, *Phys. Lett.* **B165**, 157 (1985).