LOW-LYING DIPOLE STRENGTHS IN ^{162,164}Dy NUCLEI

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Collective 1^+ scissors mode states of ^{162,164}Dy isotopes are investigated in the framework of the rotational-invariant QRPA model. Results are compared with the ones from both the nuclear resonance fluorescence (NRF) and inelastic neutron scattering experiments. The $M1$ strengths as well as the total $M1$ in these well deformed isotopes in 2–4 MeV are calculated to be in a general agreement with especially the NRF values.

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1. Introduction

Observation of a new class of rather collective isovector magnetic dipole excitation at the excitation energy of ~ 3 MeV in the heavy deformed rare-earth nucleus ^{156}Gd in high resolution electron scattering experiment at the Darmstadt electron linear accelerator [1] and its first report [2] opened a new area in nuclear spectroscopy. This predominantly orbital mode was originally predicted in the two-rotor model [3] and it is now referred to as the scissors mode, where the neutrons and protons are assumed to behave as rigid deformed bodies making scissors-like oscillations against each other and around a common axis. The first nuclear resonance fluorescence experiment at the Stuttgart Dynamitron [4] confirmed these strong $M1$ excitations in ^{156}Gd and in the neighboring Gd isotopes ^{158,160}Gd. Since then there have been numerous studies on the subject, both theoretical and experimental (see, for instance, [6–9]). Various collective models and microscopic calculations in which the isovector $M1$ excitations are explained microscopically by the quasiparticle random phase approximation (QRPA) have been included in the theoretical approaches [3, 5, 6, 10].

Many data have been collected on scissors mode excitations for the rare-earth nuclei. Accurate determination of the probability of the $M1$ transitions has been shown to be very important in order to understand the structure of these excitations [11].

^{162}Dy and ^{164}Dy have been experimentally investigated at the Stuttgart high resolution photon scattering facility [12], and have been observed to exhibit interesting characteristics. Among them are the large $B(M1)\uparrow$ at 2.9 MeV in ^{162}Dy, more 1^+ states in ^{164}Dy compared to ^{162}Dy, which could be associated with the "deformed" shell effects, and anomalously large summed $M1$ strengths compared to the neighboring nuclei. In addition, the transition probability of these well deformed nuclei have been shown to be concentrated around 3 MeV and there exist three 1^+ states with very large $B(M1)\uparrow$ around 3.1 MeV in ^{164}Dy. The strengths extracted from more recent NRF experiments on ^{162,164}Dy [13] have been in perfect agreement with those of [12]. The inelastic neutron scattering (INS) reaction, combined with the Doppler-shift attenuation method (DSAM), has also been used in order to study the 1^+ scissors mode states [14]. Even though the $M1$ strengths determined from [14] are in general agreement with those measured, for instance, in [12] and [13], in ^{164}Dy a significant discrepancy is observed for the states above 3.1 MeV for which the $B(M1)\uparrow$ strengths are noticeably lower than the NRF values.

In this work, the collective $I^\pi K = 1^+1$ scissors mode states of two even-even dysprosium nuclei ^{162}Dy and ^{164}Dy are studied in the framework of the rotational-invariant QRPA model. This model has been shown to describe for instance the fragmentation of the scissors mode and the δ^2 dependence of the summed $B(M1)\uparrow$ strength of the Sm [21], Ce and Nd [18] isotopes satisfactorily.

2. Description of 1^+ states

In the quasiparticle representation, the model Hamiltonian of a system with nucleons interacting via pairing forces in the axially symmetric average field can be written as

$$H_{sqp} = \sum_{s\tau} \varepsilon_s(\tau)\Big(\alpha_s^+(\tau)\alpha_s(\tau) + \alpha_{\tilde{s}}^+(\tau)\alpha_{\tilde{s}}(\tau)\Big)\,, \tag{1}$$

where ε_s is the single quasiparticle energy, τ is n for neutrons and p for protons, $\alpha_s^+(\alpha_s)$ is the quasiparticle creation(annihilation) operator, and $|\tilde{s}\rangle$ is the time-reversal of the single particle state $|s\rangle$. This Hamiltonian, which represents the quasiparticle excitations, is not invariant under rotational transformations due to the axially symmetric isoscalar and isovector terms of the average field.

A satisfactory description of the scissors mode 1^+ states is possible by restoration of the broken invariance [19, 20], and the rotational invariance of the quasiparticle Hamiltonian in Eq. 1 can be restored by including the effective isoscalar (h_0) and isovector (h_1) forces. These terms are selected in such a way that the new Hamiltonian commutes with the J_ν components of the total angular momentum

$$[H_{sqp} + h_0 + h_1, J_\nu] = 0\,, \tag{2}$$

where $\nu = \pm$. Detailed definitions of h_0 and h_1 are given in [21]. Considering these restoring forces and the isovector spin–spin interactions, which generate the 1^+ states, one obtains the model Hamiltonian of the system as

$$H = H_{sqp} + h_0 + h_1 + V_{\sigma\tau}\,, \tag{3}$$

where

$$V_{\sigma\tau} = \frac{1}{2}\kappa_{\sigma\tau}\sum_{i\neq j}\vec{\sigma}_i\vec{\sigma}_j\vec{\tau}_i\vec{\tau}_j \tag{4}$$

with $\vec{\sigma}$ and $\vec{\tau}$ being the Pauli spin and isospin matrices, respectively. Several works have been published without restoring the broken rotational invariance (see, for instance, [16] and [17]).

In RPA, the excited collective 1^+ states are defined as single-phonon states described by

$$|\psi_i\rangle = Q_i^+|\psi_0\rangle = \frac{1}{\sqrt{2}}\sum_{ss',\tau}[\psi^i_{ss'}(\tau)C^+_{ss'}(\tau) - \phi^i_{ss'}(\tau)C_{ss'}(\tau)]|\psi_0\rangle\,, \tag{5}$$

where Q_i^+ represents the phonon creation operator, $|\psi_0\rangle$ is the phonon vacuum, $C^+_{ss'}(C_{ss'})$ are the two-quasiparticle creation (annihilation) operators with $C_{ss'} = \{\alpha_s\alpha_{s'}\}_{I^\pi K=1^+1}$, and $\psi^i_{ss'}$ and $\phi^i_{ss'}$ are two-quasiparticle amplitudes satisfying the normalization condition

$$\sum_{ss',\tau}\left[\psi^{i^2}_{ss'}(\tau) - \phi^{i^2}_{ss'}(\tau)\right] = 1\,. \tag{6}$$

The dispersion equation for the excitation energy of 1^+ states

$$\omega_i^2 J_{\rm eff}(\omega_i) = 0 \tag{7}$$

is obtained solving the equations of motion

$$[H_{sqp} + h_0 + h_1 + V_{\sigma\tau}, Q_i^+] = \omega_i Q_i^+\,, \tag{8}$$

and using the conventional procedure of RPA. In Eq. (7), ω_i represent the energies and

$$J_{\rm eff} = J - 8\kappa_{\sigma\tau}\frac{X^2}{D_\sigma} + \frac{\omega_i^2}{\gamma_1 - F_1}J_1^2 - 8\kappa_{\sigma\tau}\frac{\omega_i^2}{\gamma_1 - F_1}\frac{JX_1^2 - 2J_1XX_1}{D_\sigma}\,, \tag{9}$$

with the four parts on the right side of the equation representing contribution of the restoring isoscalar (h_0) and the isovector forces (h_1), the spin forces and the interference of all the three forces, respectively. Definition of the individual symbols in Eq. (9) as well as the neutron–neutron and proton–proton two-quasiparticle amplitudes are explicitly given in [21].

The most characteristic quantity of 1^+ states is the reduced $M1$ transition probability in terms of the nuclear magneton squared (μ_{N}^2) is given by

$$B(M1, 0^+ \to 1_i^+) = \frac{3}{4\pi}\Big|R_p(\omega_i) + \sum_{\tau}(g_s^{\tau} - g_l^{\tau})R_{\tau}(\omega_i)\Big|^2 \mu_{\mathrm{N}}^2\,, \tag{10}$$

where g_s is the spin and g_l is the orbital gyromagnetic ratios of the free nucleons

$$R_p(\omega_i) = \sum_{ss'}^{(p)} \varepsilon_{ss'} L_{ss'} j_{ss'} (\psi_{ss'}^i + \phi_{ss'}^i)\,, \tag{11}$$

and

$$R_{\tau}(\omega_i) = \sum_{ss'}^{(\tau)} \varepsilon_{ss'} L_{ss'} s_{ss'} (\psi_{ss'}^i + \phi_{ss'}^i)\,, \tag{12}$$

where $\varepsilon_{ss'}$ are two-quasiparticle energies, $s_{ss'}$ and $j_{ss'}$ are the single particle matrix elements of the spin and angular momentum operators, respectively. $L_{ss'}$ are expressed through the u and v parameters of the Bogolyubov canonical transformation as $L_{ss'} = u_s v_{s'} - u_{s'} v_s$.

3. Results and discussion

The collective 1^+ scissors mode states of two even–even dysprosium nuclei ^{162}Dy and ^{164}Dy are studied in the framework of QRPA. Calculations have been performed using both the non-rotational and the rotational-invariant model. For both calculations, the spin–spin interaction constant is chosen as $\kappa = 21/A$ MeV where A is the mass number, the average field deformation parameter δ is calculated according to [22] in the first order using the deformation parameters β_2 defined from the experimental quadrupole moments [23], and the ground state neutron and proton pairing energies are calculated based on [24]. Results from the non-rotational model have indicated that it overestimates the summed $B(M1)\uparrow$ sometimes twice the experimental value. Furthermore, comparison between the non-rotational and the rotational-invariant model has shown that separation of the rotational branch from the 1^+ states increases the fragmentation and collectivization of the 1^+ states, consistent with the comments in [21]. As a consequence, only the rotational model results are given here.

Results of the calculation as well as of the experimental data for the two nuclei of interest are illustrated in Fig. 1, where energies of 1^+ states are plotted against the reduced transition probability $B(M1)\!\uparrow$ in the energy interval 2–4 MeV. Experimental data points are taken from [13] (filled circles) and [14] (blank circles). The arrow at 2.9 MeV in ^{162}Dy (upper plot in Fig. 1) indicates that INS measurement of the $B(M1)\!\uparrow$ at this energy is larger than the indicated value (see [14]). The ^{164}Dy states with tentative parity assignments based on $K = 1$ from [13] are not included in the figure (bottom plot).

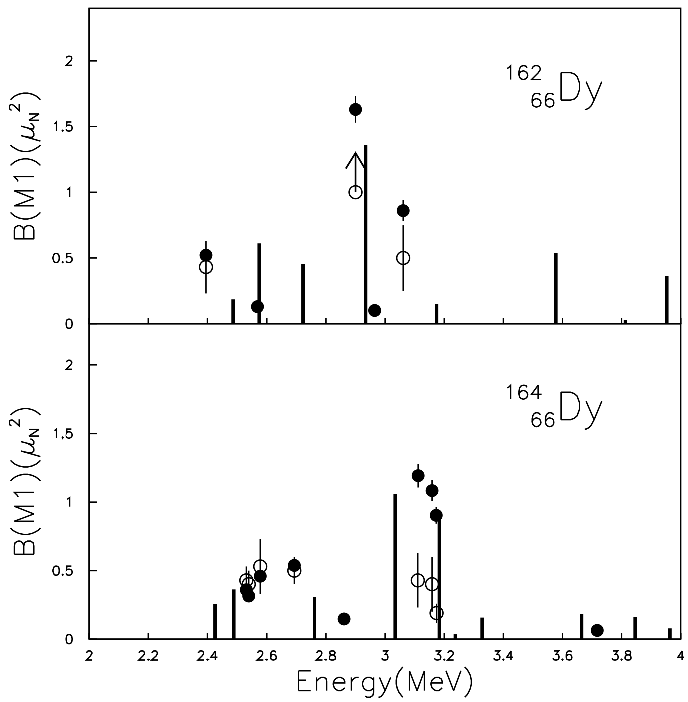

Fig. 1. Energy and $B(M1)\uparrow$ strengths for each 1^+ state for ^{162}Dy (top plot) and ^{164}Dy (bottom plot) from NRF (filled circles), INS (open circles) and rotational-invariant QRPA (verticle lines).

Calculation results are also given in Table I. Energies with $B(M1)\uparrow < 0.025\mu_{\rm N}^2$ are excluded from both the figure and the tables. The summed $B(M1)\!\uparrow$ strengths from the experiments and the theory are given in Table II, where the experimental errors have been obtained by quadratically adding up the individual errors from the contributing energy levels in the interval 2–4 MeV. In the following, theory results and their comparison with the data are discussed for ^{162}Dy and ^{164}Dy separately.

TABLE I

Calculation results for energy and $B(M1)\uparrow$ for ^{162}Dy and ^{164}Dy.

^{162}Dy		^{164}Dy	
$E^{\rm theo}$(MeV)	$B(M1)\uparrow(\mu_{\rm N}^2)$	$E^{\rm theo}$(MeV)	$B(M1)\uparrow(\mu_{\rm N}^2)$
2.486	0.185	2.425	0.257
2.575	0.612	2.489	0.362
2.723	0.452	2.762	0.308
2.935	1.361	3.034	1.059
3.174	0.152	3.184	0.882
3.578	0.539	3.238	0.037
3.813	0.027	3.328	0.158
3.953	0.363	3.665	0.182
		3.846	0.164
		3.963	0.079

TABLE II

Experimental and theoretical summed $B(M1)\uparrow$ strengths for ^{162}Dy and ^{164}Dy in the energy interval 2–4 MeV.

	$\sum B(M1)\uparrow(\mu_{\rm N}^2)$	
	^{162}Dy	^{164}Dy
NRF	3.24±0.13	5.061±0.149
INS	$> 1.93 \pm (> 0.32)$	2.88±0.39
Theory	3.691	3.488

3.1. Results for ^{162}Dy

Data from [12–14] indicate a few 1^+ states with a large $B(M1)\uparrow$ at 2.9 MeV. The theory almost perfectly estimates this state with a noticeably large $B(M1)\uparrow$ comparable with the experimental data. Calculation results are also somewhat consistent with the data in terms of the location of the cluster at low energies. Since the INS data determine just the lower limit for the strength at 2.9 MeV, calculation results for the summed $B(M1)\uparrow$ are compared with the NRF data only. Even though the calculated summed strength is a little larger than that of the NRF data, which could be expected since the experimental data may not have detected or may not sometimes be certain about every 1^+ state, they still could be considered to be in a good agreement.

3.2. Results for ^{164}Dy

As mentioned in Section 1, this nucleus has much larger fragmentation than the ^{162}Dy isotope and has a very large summed $B(M1)\uparrow$ compared to the neighboring nuclei. Although the NRF data [12, 13] give consistent results, the INS data agree with the NRF data only in the energy below 3.1 MeV (bottom plot in Fig. 1). Above that, although both the NRF and INS data give three states, the $B(M1)\uparrow$ values measured from the former are nearly a factor of 3 larger than those obtained from the latter. The theory successfully predicts two states around 3.1 MeV with a $B(M1)\uparrow$ consistent with the NRF data (see the bottom plot in Fig. 1). It also estimates several states with relatively smaller $B(M1)\uparrow$ at lower energies, which is also in good agreement with the data, both the NRF and INS. Meanwhile, although the summed $B(M1)\uparrow$ predicted by the theory is larger than that of the INS as expected, it is smaller than the value measured by NRF. However, it should be noted that the theory prediction is still consistent with the NRF within the statistical uncertainty.

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