A DEFINITION FOR FINE TUNING IN ANALOGY TO THE CHAOS

Zsolt Hetesi†

Eötvös University, Department of Astronomy
H-1518 Budapest, P.O.Box 32., Hungary

László Végh‡

Institute of Nuclear Research of the Hungarian Academy of Sciences
H-4001 Debrecen, Pf. 51, Hungary

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Anthropic principles were grown from the problem of fine tuning. Although anthropic principles have been discussed in cosmology for years there is no exact definition for fine tuning. Starting from the supposed similarity in the topologies of chaotic and fine tuned regions of the proper phase spaces, we introduce an alternative Lyapunov indicator for the measure of fine tuning. This fine-tuning indicator expresses the decrease of life-bearing potentiality of a universe with the increase of the difference from the physical constants of the Universe with maximum life-bearing potentiality.

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The Universe and life are connected. Man exists because the Universe is governed by laws that have led to high levels of organization of matter. It seems there are apparent coincidences in the dimensionless basic constants of the nature. These basic numbers of the nature not only allow the existence of stable atoms from which matter is built but also lead to the formation of galaxies and stars and even more complex structures all are necessary for the existence of life. A small change in the basic constants would result a universe without these life-supporting conditions. Therefore we can say that the Universe is finely tuned for life.

† `zs.hetesi@astro.elte.hu`
‡ `vl@atomki.hu`

Anthropic principles address the question why our Universe has the fine-tuning property [5]. Is this a fortunate condition, inevitable or it is expected? The weak anthropic principle stresses that we, intelligent observers, may observe only very special properties being compatible with our existence. The strong anthropic principle claims that the Universe must have those features which are necessary for life to develop at some stage of its history. The immediate consequence of the strong anthropic principle is that the physical laws and constants must be such as to allow the emergence of life. There exist several different versions or supplements of strong anthropic principle. The most known is the design argument which states that life can occur because of some purposive design. That is the values of physical constants were selected purposively. For more specific anthropic definitions see Refs. [1, 2].

To state that the physical constants of the Universe derive specific values from an ensemble of different values, we have to suppose the *(i)* conceptual or *(ii)* real existence of the numerical ensemble. The design argument has chosen the first solution *(i)*, when the other possibilities exist only as possibilities in the mind of the Designer. The many-worlds-hypothesis represents the second version *(ii)*, supposing that an ensemble of other different existing universes is necessary for the existence of our Universe [2]. Both design argument and many-worlds-hypothesis are created to explain the fine tuning of physical constants. There exist some alternative definitions [7,8] but their common feature is the lack of well-based probability interpretation for the fine tuning, see the paper of Manson [6].

Life depends on the presence of the proper chemical elements as building blocks and the existence of stars which can radiate enough energy for long time for evolution of life. If the abundance of carbon and other essential elements are lower or the number of properly radiating stars are smaller then we have a smaller probability for the evolution of intelligent life in a universe. To define fine tuning in a more quantitative way we can study the probability function for intelligent life of a universe. The form of this probability function can be regarded as an anthropic adaptation of the Drake equation [4]. Physical constants are regarded here as variables. Therefore, we can present a probability distribution of life-bearing potentiality for universes as a function of the basic constants of the physics.

A mathematical expression of fine tuning may stimulate the discussion of anthropic arguments. Our aim is to construct a simple mathematical definition of fine tuning analogous to the Lyapunov-indicator in the chaos theory. (Such a definition can lead to testable statements therefore arguments of Smolin [8] about the unfalsificability of anthropic principles are avoidable.)

Discussing the chaotic behavior in the phase space there are regions with increased sensitivity for the parameter values. This sensitivity in an extreme case is characterized by the Lyapunov indicator γ of the given region [9]. It gives a number as result if the motion is chaotic and gives zero if not. Let us consider two trajectories not far from each other. The initial distance is d_0 and after t the final distance is d. If the distance is growing exponentially, *i.e.* $d(t) = d_0 \exp(\alpha t)$ is true, then the Lyapunov indicator γ is equal to α. Generally the system is chaotic, if in the expression

$$\gamma = \frac{\ln d/d_0}{\Delta t} \to \alpha \,, \tag{1}$$

$\gamma > 0$. If the increase of distance is smaller, *e.g.* $d(t) = d_0 \beta t$ then γ tends to zero.

We suppose that fine tuned region(s) has/have similar topology as chaotic regions. That is, there are regions of increased sensitivity in the parameter space describing universes for the life bearing universes and these regions have similar character to a chaotic region. In order to characterize fine tuning we introduce a quantity like to the one used in the description of chaotic behavior.

In the extreme limit of fine tuning, the probability function can be represented by a Dirac-delta function taken at the point of the parameter space of physical constants which corresponds to the single life-bearing universe. The Dirac-delta function $\delta(Q - Q_0)$ can be represented as the $n \to \infty$ limit of the Gaussian function

$$\delta(Q - Q_0) = \lim_{n\to\infty} \frac{n}{\sqrt{\pi}} \exp\left(-n^2 (Q - Q_0)^2\right) \tag{2}$$

see [10]. Now we assume that if there is a fine tuning in the physical parameter Q around the maximum probability at Q_0 then the probability function for the life-bearing of universes can be approximated by the Gaussian form:

$$p_{\mathrm{life}}(q) = \frac{n}{\sqrt{\pi}} \exp(-n^2 q^2) \,, \tag{3}$$

where $q = |Q - Q_0|$. The measure of fine tuning can be defined as

$$\gamma_{\mathrm{fine-tuning}} = -\frac{(\ln p/p_0)^{1/2}}{\Delta Q} \to n \,. \tag{4}$$

According to this definition, we have fine tuning, if $\gamma_{\mathrm{fine-tuning}} > 0$. The larger $\gamma_{\mathrm{fine-tuning}}$ the stronger fine-tuning. Working with dimensionless parameters this definition does not depend on the parametrization.

Drawing a parallel between chaos theory and fine tuning is not an arbitrary assumption in this case. As chaos appears when Lyapunov indicator is not zero fine tuning means that there exist life-bearing islands in the parameter space of physical constants.

Discussing the production of the carbon and oxigen in the Universe, the position of the famous resonance in C_{12} nuclei plays a crucial role. According to [3] outside a narrow window of 0.5% and 4% of the values of the strong and nuclear forces, respectively, the stellar production of carbon or oxigen is reduced by factors of 30 to 1000. These production functions express Gaussian-like form. We can guess that the probability to find planets with a proper life-bearing mass would have even a higher decrease. Now the $\gamma_{\mathrm{fine-tuning}}$ indicator of Eq. (4) has a definite nonzero value.

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