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ON GRAVITATIONAL REPULSION EFFECT AT EXTREME CONDITIONS IN GAUGE THEORIES OF GRAVITY

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Homogeneous isotropic gravitating models are discussed in the framework of gauge approach to gravitation. Generalized cosmological Friedmann equations without specific solutions are deduced for models filled with scalar fields and usual gravitating matter. Extreme conditions leading to gravitational repulsion effect are analyzed.

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1. Introduction

As it is well known, general relativity theory (GR) does not lead to restrictions for admissible energy densities in the case of gravitating systems with positive values of energy density and pressure satisfying energy dominance condition. As a result, the problem of gravitational singularities takes place in the frame of GR [1]. The appearance of non-physical state with divergent energy density limiting world lines in the past or in the future is characteristic feature of Friedmannian cosmological models (problem of cosmological singularity — PCS) and also of collapsing systems. From physical point of view, the problem of gravitational singularities is connected with the fact that gravitational interaction in the case of gravitating systems with positive energy densities and pressures in the frame of GR as well as Newton's theory of gravitation has the character of attraction but not repulsion. Note that the gravitational interaction in GR can have the repulsion character in the case of gravitating systems with negative pressure (for example systems including massive or nonlinear scalar fields). According to accepted opinion, it is possible cause of acceleration of cosmological expansion at present epoch. However, such effect does not permit to solve the PCS in the frame of GR [2]: all Friedmannian cosmological models of flat and open type, and the most part of closed models are singular.

There were many attempts to solve the problem of gravitational singularities and at first of all the PCS in the frame of GR and other classical theories of gravitation; a number of particular regular cosmological solutions was obtained (see [3] and references herein). In connection with this note that the solution of PCS means not only obtaining regular cosmological solutions, but also excluding singular solutions; as a result regular behavior of cosmological solutions has to be their characteristic feature. The most existent attempts to solve the PCS do not satisfy indicated conditions. According to wide known opinion, the solution of PCS and generally of the problem of gravitational singularities of GR has to be connected with quantum gravitational effects, which must be essential at Planckian conditions, when energy density is comparable with Planckian one. Previously some regular bouncing cosmological solutions were obtained in the frame of candidates to quantum gravitation theory — string theory/M-theory and loop quantum gravity (see, for example, [4–6]). From physical point of view, these solutions have some difficulties [7]. In the case of bouncing cosmological solutions built in the frame of string theory the condition of energy density positivity for gravitating matter is violated. In the case of bouncing solutions obtained in loop quantum gravity, where a bounce takes place for microscopic model having a volume comparable with Planckian one, there is the following problem. If one supposes that Universe at compression stage is macroscopic object, one has to explain the transformation of macro-universe into micro-universe before a bounce, this means one has to introduce some model inverse to inflation.

As it was shown in a number of papers (see [7, 16] and references herein), gauge theories of gravitation (GTG) and at, first of all, the Poincaré GTG which are natural generalization of GR by applying the gauge approach to gravitational interaction [8,9], offer an opportunity to solve the PCS. All homogeneous isotropic cosmological models including inflationary models are regular in metrics, Hubble parameter, its time derivative, if certain restriction on equation of state for gravitating matter at extreme conditions is valid. Unlike Friedmannian models of GR non-physical state with divergent energy density does not appear because of gravitational repulsion effect at extreme conditions, which takes place in GTG in the case of usual gravitating systems with positive energy densities and pressures satisfying energy dominance condition.

The present paper is devoted to analysis of gravitational repulsion effect at extreme conditions in the frame of GTG in the case of homogeneous isotropic gravitating systems. In Sec. 2 some important properties of homogeneous isotropic models in the frame of Poincaré GTG are briefly discussed. In Sec. 3 generalized cosmological Friedmann equations without specific solutions for such models filled with usual matter and scalar fields are introduced. In Sec. 4 extreme conditions leading to gravitational repulsion effect are analyzed.

2. Homogeneous isotropic gravitating models in Poincaré GTG

In the framework of gauge approach to gravitation the Poincaré GTG is of the greatest interest [8,7]. The role of gravitational field variables in the Poincaré GTG plays the tetrad (translational gauge field) and anholonomic Lorentz connection (Lorentz gauge field), corresponding field strengths are torsion and curvature tensors, and physical space-time is 4-dimensional Riemann–Cartan continuum. As sources of gravitational field in the Poincaré GTG are energy-momentum and spin tensors. Unlike gauge Yang–Mills fields, for which the Lagrangian is quadratic in the gauge field strengths, gravitational Lagrangian of the Poincaré GTG can include also linear in curvature term (scalar curvature), which is necessary to satisfy the correspondence principle with GR. By using sufficiently general expression for gravitational Lagrangian, homogeneous isotropic gravitating models were investigated in the frame of Poincaré GTG in a number papers (see for example [7, 10, 16]). Because of high spatial symmetry gravitational equations depend weakly on the structure of quadratic part of gravitational Lagrangian, that permits to obtain conclusions of general character. Below some important relations describing homogeneous isotropic models in the Poincaré GTG will be given.

Spatial symmetries of Riemann–Cartan space are defined by a set of linearly independent Killing's vectors, with respect to which the Lie derivatives of metric and torsion tensors vanish (see [1] Chapter 2, [17]). Homogeneous isotropic models possess 6 linearly independent Killing's vectors and metric tensor in co-moving system of reference has the form of Robertson–Walker metrics [17]

$$g_{\mu\nu} = \mathrm{diag}\left(1, -\frac{R^2\,(t)}{1-kr^2},\ -R^2\,(t)\,r^2,\ -R^2\,(t)\,r^2\sin^2\vartheta\right),$$

where $R(t)$ is the scale factor, $k = +1, 0, -1$ for closed, flat, open models respectively (the light velocity $c = 1$), spatial spherical coordinates are used. Then the torsion tensor $S^{\lambda}{}_{\mu\nu} = -S^{\lambda}{}_{\nu\mu}$ satisfying symmetry conditions can have the following non-vanishing components [10]: $S^1{}_{10} = S^2{}_{20} = S^3{}_{30} = S(t)$, $S_{123} = S_{231} = S_{312} = \tilde{S}(t)(R^3 r^2/\sqrt{1-kr^2})\sin\theta$, where $S(t)$ and $\tilde{S}(t)$ are functions of time. The functions S and $\tilde{S}$ have different properties with respect to transformations of spatial inversions, namely, the function $\tilde{S}(t)$

has pseudoscalar character. By supposing $\tilde{S}=0$, we obtain for the curvature tensor $F^{\mu\nu}{}_{\sigma\rho}=-F^{\nu\mu}{}_{\sigma\rho}=-F^{\mu\nu}{}_{\rho\sigma}$ the following non-vanishing components: $F^{01}{}_{01}=F^{02}{}_{02}=F^{03}{}_{03}\equiv A$ and $F^{12}{}_{12}=F^{13}{}_{13}=F^{23}{}_{23}\equiv B$ with

$$A=\frac{\left(\dot{R}-2RS\right)^{\cdot}}{R}\,, \qquad B=\frac{k+\left(\dot{R}-2RS\right)^{2}}{R^{2}}\,, \tag{1}$$

and Bianchi identities in this case are reduced to the only relation

$$\dot{B}+2H\left(B-A\right)+4AS=0\,, \tag{2}$$

where $H=\dot{R}/R$ is the Hubble parameter, and a dot denotes differentiation with respect to time.

We will use gravitational Lagrangian in the form

$$\begin{aligned}\mathcal{L}_{\mathrm{G}} \;=\;& f_0F+F^{\alpha\beta\mu\nu}\left(f_1F_{\alpha\beta\mu\nu}+f_2F_{\alpha\mu\beta\nu}+f_3F_{\mu\nu\alpha\beta}\right)+F^{\mu\nu}\left(f_4F_{\mu\nu}+f_5F_{\nu\mu}\right)\\ &+f_6\,F^2+S^{\alpha\mu\nu}\left(a_1\,S_{\alpha\mu\nu}+a_2\,S_{\nu\mu\alpha}\right)+a_3\,S^{\alpha}{}_{\mu\alpha}S_{\beta}{}^{\mu\beta}\,,\end{aligned}$$

where $F_{\mu\nu}=F^{\alpha}{}_{\mu\alpha\nu}$, $F=F^{\mu}{}_{\mu}$, f_i $(i=1,2,\ldots,6)$, a_k $(k=1,2,3)$ are indefinite parameters, $f_0=(16\pi G)^{-1}$, G is Newton's gravitational constant. Gravitational equations for homogeneous isotropic gravitating models (with $\tilde{S}=0$) are reduced to the system of 3 equations [10], which by using (2) can be written as

$$\begin{aligned}6f_0B-12f\left(A^2-B^2\right)-3a\left(H-S\right)S \;&=\; \rho\,,\\ 2f_0\left(2A+B\right)+4f\left(A^2-B^2\right)-a\left(\dot{S}+HS-S^2\right) \;&=\; -p\,,\\ f\left(\dot{A}+\dot{B}\right)+\left[f_0+\frac{1}{8}a+4f\left(A+B\right)\right]S \;&=\; 0\,.\end{aligned} \tag{3}$$

where $f=f_1+\frac{1}{2}f_2+f_3+f_4+f_5+3f_6$, $a=2a_1+a_2+3a_3$, ρ is the energy density, p is the pressure and the average of spin distribution of gravitating matter is supposed to be equal to zero. The system of equations (3) leads to cosmological equations without high derivatives if $a=0$ [10] (see below). Then we find from (3) the curvature functions A and B and the torsion S in the following form

$$\begin{aligned}A \;&=\; -\frac{1}{12f_0}\,\frac{\rho+3p-\alpha\left(\rho-3p\right)^2/2}{1+\alpha\left(\rho-3p\right)}\,,\\ B \;&=\; \frac{1}{6f_0}\,\frac{\rho+\alpha\left(\rho-3p\right)^2/4}{1+\alpha\left(\rho-3p\right)}\,,\\ S(t) \;&=\; -\frac{1}{4}\frac{d}{dt}\ln\left|1+\alpha(\rho-3p)\right|\,,\end{aligned} \tag{4}$$

where indefinite parameter $\alpha = f/(3f_0{}^2)$ has inverse dimension of energy density. Note, that in the case $\tilde{S} \neq 0$ Bianchi identities for homogeneous isotropic models are reduced to two relations and system of gravitational equations of the Poincaré GTG includes 4 equations. However, these gravitational equations have the solution (4) together with $\tilde{S} = 0$ always, if $a = 0$ [18].

3. Generalized cosmological Friedmann equations in GTG

By using expressions (1) of curvature functions for homogeneous isotropic gravitating models and the solution (4) of gravitational equations of the Poincaré GTG we obtain the following generalized cosmological Friedmann equations (GCFE)

$$\frac{k}{R^2} + \left\{ \frac{d}{dt} \ln \left[R\sqrt{|1 + \alpha\,(\rho - 3p)|} \right] \right\}^2 = \frac{8\pi G}{3} \frac{\rho + \frac{\alpha}{4}\,(\rho - 3p)^2}{1 + \alpha\,(\rho - 3p)}\,, \tag{5}$$

$$\frac{1}{R}\frac{d}{dt}\left[\frac{dR}{dt} + R\frac{d}{dt}\left(\ln\sqrt{|1+\alpha\,(\rho-3p)|} \right) \right] = -\frac{4\pi G}{3} \frac{\rho + 3p - \frac{\alpha}{2}\,(\rho-3p)^2}{1 + \alpha\,(\rho-3p)}\,. \tag{6}$$

The difference of (5), (6) from Friedmannian cosmological equations of GR is connected with terms containing the parameter α. These terms arise from quadratic in the curvature tensor part of gravitational Lagrangian, which unlike metric theories of gravitation does not lead to high derivatives in cosmological equations[1]. The conservation law in GTG has usual form

$$\dot{\rho} + 3H\,(\rho + p) = 0\,. \tag{7}$$

Now by using GCFE (5), (6) we will consider homogeneous isotropic models filled with non-interacting scalar field ϕ minimally coupled with gravitation and gravitating matter with equation of state in general form $p_{\rm m} = p_{\rm m}(\rho_{\rm m})$, where values of gravitating matter are denoted by means of index "m". (The generalization for the case with several scalar fields can be made directly). Then the energy density ρ and the pressure p take the form

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + \rho_{\rm m} \quad (\rho > 0)\,, \qquad p = \frac{1}{2}\dot{\phi}^2 - V + p_{\rm m}\,, \tag{8}$$

where $V = V(\phi)$ is a scalar field potential. By using the scalar field equation in homogeneous isotropic space

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} \tag{9}$$

[1] As it was shown in [11,12] generalized cosmological Friedmann equations (5), (6) can be deduced also in the frame of the most general GTG-affine-metric GTG [9].

we obtain from (7)–(9) the conservation law for gravitating matter

$$\dot{\rho}_{\rm m} + 3H\left(\rho_{\rm m} + p_{\rm m}\right) = 0\,. \qquad (10)$$

By using (8)–(10) the GCFE (5), (6) can be transformed to the following form

$$\left\{H\left[Z + 3\alpha\left(\dot{\phi}^2 + \frac{1}{2}Y\right)\right] + 3\alpha\frac{\partial V}{\partial\phi}\dot{\phi}\right\}^2 + \frac{k}{R^2}\,Z^2$$
$$= \frac{8\pi G}{3}\left[\rho_{\rm m} + \frac{1}{2}\dot{\phi}^2 + V + \frac{1}{4}\alpha\left(4V - \dot{\phi}^2 + \rho_{\rm m} - 3p_{\rm m}\right)^2\right] Z\,, \qquad (11)$$

$$\dot{H}\left[Z + 3\alpha\left(\dot{\phi}^2 + \frac{1}{2}Y\right)\right] Z + H^2\Bigg\{\Bigg[Z - 15\alpha\dot{\phi}^2 - 3\alpha Y$$
$$- \frac{9\alpha}{2}\left(\frac{dp_{\rm m}}{d\rho_{\rm m}}Y + 3\left(\rho_{\rm m} + p_{\rm m}\right)^2\frac{d^2p_{\rm m}}{d\rho_{\rm m}^2}\right)\Bigg] Z - 18\alpha^2\left(\dot{\phi}^2 + \frac{1}{2}Y\right)^2\Bigg\}$$
$$- 12\alpha H\frac{\partial V}{\partial\phi}\dot{\phi}\left[Z + 3\alpha\left(\dot{\phi}^2 + \frac{1}{2}Y\right)\right]$$
$$+ 3\alpha\left[\frac{\partial^2 V}{\partial\phi^2}\dot{\phi}^2 - \left(\frac{\partial V}{\partial\phi}\right)^2\right] Z - 18\alpha^2\left(\frac{\partial V}{\partial\phi}\right)^2\dot{\phi}^2$$
$$= \frac{8\pi G}{3}\left[V - \dot{\phi}^2 - \frac{1}{2}\left(\rho_{\rm m} + 3p_{\rm m}\right) + \frac{1}{4}\alpha\left(4V - \dot{\phi}^2 + \rho_{\rm m} - 3p_{\rm m}\right)^2\right] Z\,, \qquad (12)$$

where $Z = 1 + \alpha\left(4V - \dot{\phi}^2 + \rho_{\rm m} - 3p_{\rm m}\right)$ and $Y = \left(\rho_{\rm m} + p_{\rm m}\right)\left(3(dp_{\rm m}/d\rho_{\rm m}) - 1\right)$.

By transformation of GCFE (5), (6) to the form (11), (12) these equations were multiplied by Z. As a result (11), (12) have specific solutions when $Z = 0$ [13,7]. In fact, by using the expression of the Hubble parameter following from (11)

$$H_\pm = \frac{-3\alpha\,\dfrac{\partial V}{\partial\phi}\,\dot{\phi} \pm \sqrt{D}}{Z + 3\alpha\left(\dot{\phi}^2 + \dfrac{1}{2}Y\right)}, \qquad (13)$$

where

$$D = \frac{8\pi G}{3}\left[\rho_{\rm m} + \frac{1}{2}\dot{\phi}^2 + V + \frac{1}{4}\alpha\left(4V - \dot{\phi}^2 + \rho_{\rm m} - 3p_{\rm m}\right)^2\right] Z - \frac{k}{R^2}Z^2\,,$$

it is easy to show that (12) is satisfied, if $Z = 0$. It is because the terms in (12), which do not include Z as multiplier, vanish by virtue of (13).

Excluding these terms and dividing obtained equation on Z we will have, instead of (12), the following equation without specific solutions

$$\dot{H}\left[Z+3\alpha\left(\dot{\phi}^2+\frac{1}{2}Y\right)\right]+3H^2\left[Z-\alpha\dot{\phi}^2+\alpha Y\right.$$
$$\left.-\frac{3\alpha}{2}\left(\frac{dp_{\mathrm{m}}}{d\rho_{\mathrm{m}}}Y+3\left(\rho_{\mathrm{m}}+p_{\mathrm{m}}\right)^2\frac{d^2p_{\mathrm{m}}}{d\rho_{\mathrm{m}}^2}\right)\right]+3\alpha\left[\frac{\partial^2V}{\partial\phi^2}\dot{\phi}^2-\left(\frac{\partial V}{\partial\phi}\right)^2\right]$$
$$=8\pi G\left[V+\frac{1}{2}\left(\rho_{\mathrm{m}}-p_{\mathrm{m}}\right)+\frac{1}{4}\alpha\left(4V-\dot{\phi}^2+\rho_{\mathrm{m}}-3p_{\mathrm{m}}\right)^2\right]-\frac{2k}{R^2}Z\,. \quad (14)$$

The system of equations (11) and (14) is equivalent to GCFE (5), (6) in considered case of models filled by usual matter and scalar fields. If the interaction between scalar fields and usual gravitating matter is not neglecting, the GCFE (11), (14) have to be generalized. By taking into account the interaction by means of scalar field potentials, which in this case depend also on the energy density of gravitating matter ρ_{m}, namely $V=V(\phi,\rho_{\mathrm{m}})$ [7], we can obtain the generalization of equations (11), (14) by similar way. The form of (11) does not change, but the value of Y in this case is defined as

$$Y=\frac{\rho_{\mathrm{m}}+p_{\mathrm{m}}}{1+\frac{\partial V}{\partial\rho_{\mathrm{m}}}}\left(\frac{d}{d\rho_{\mathrm{m}}}\left(3p_{\mathrm{m}}-4V\right)-1\right).$$

The equation (11) is generalized as

$$\dot{H}\left[Z+3\alpha\left(\dot{\phi}^2+\frac{1}{2}Y\right)\right]+3H^2\left[Z-\alpha\dot{\phi}^2+\frac{5\alpha}{2}Y-\frac{3\alpha}{2}\frac{Y}{1+\frac{\partial V}{\partial\rho_{\mathrm{m}}}}\right.$$
$$\left.\times\left(1+\frac{dp_{\mathrm{m}}}{d\rho_{\mathrm{m}}}+\frac{\rho_{\mathrm{m}}+p_{\mathrm{m}}}{1+\frac{\partial V}{\partial\rho_{\mathrm{m}}}}\frac{\partial^2V}{\partial\rho_{\mathrm{m}}^2}\right)-\frac{3\alpha}{2}\frac{\left(\rho_{\mathrm{m}}+p_{\mathrm{m}}\right)^2}{\left(1+\frac{\partial V}{\partial\rho_{\mathrm{m}}}\right)^2}\frac{d^2}{d\rho_{\mathrm{m}}^2}\left(3p_{\mathrm{m}}-4V\right)\right]$$
$$-\frac{27}{2}\alpha H\dot{\phi}\,\frac{\rho_{\mathrm{m}}+p_{\mathrm{m}}}{\left(1+\frac{\partial V}{\partial\rho_{\mathrm{m}}}\right)^2}\frac{\partial^2V}{\partial\phi\partial\rho_{\mathrm{m}}}\left[1+\frac{1}{3}\frac{d}{d\rho_{\mathrm{m}}}\left(p_{\mathrm{m}}+2V\right)\right]$$
$$+3\alpha\left[\frac{\partial^2V}{\partial\phi^2}\dot{\phi}^2-\left(\frac{\partial V}{\partial\phi}\right)^2\right]$$
$$=8\pi G\left[V+\frac{1}{2}\left(\rho_{\mathrm{m}}-p_{\mathrm{m}}\right)+\frac{1}{4}\alpha\left(4V-\dot{\phi}^2+\rho_{\mathrm{m}}-3p_{\mathrm{m}}\right)^2\right]-\frac{2k}{R^2}Z\,. \quad (15)$$

The system of (11), (15) does not have specific solutions (with $Z=0$) unlike (9), (10) of reference [7]. By using obtained equations (11), (14), (15) the repulsion gravitational effect can be analyzed in the case of homogeneous isotropic gravitating systems in the frame of GTG.

4. Gravitational repulsion effect in GTG

As it was noted above, the difference of GCFE from Friedmannian cosmological equations of GR is connected with terms containing the parameter α. The value of $|\alpha|^{-1}$ determines the scale of extremely high energy densities. Solutions of GCFE (5), (6) coincide practically with corresponding solutions of GR, if the energy density is small $|\alpha(\rho - 3p)| \ll 1$ ($p \neq \frac{1}{3}\rho$). The difference between GR and GTG can be essential at extremely high energy densities $|\alpha(\rho - 3p)| \gtrsim 1$. Ultra-relativistic matter ($p = \frac{1}{3}\rho$) and gravitating vacuum ($p = -\rho$) with constant energy density are two exceptional systems, because GCFE (5), (6) are identical to Friedmannian cosmological equations of GR in these cases independently on values of energy density, and non-einsteinian space-time characteristics vanish. Properties of solutions of equations (5), (6) at extreme conditions depend on the sign of parameter α and certain restriction on equation of state of gravitating matter. The study of inflationary models including scalar fields shows, that GCFE (5), (6) lead to acceptable restriction for scalar field variables at extreme conditions if $\alpha > 0$ [7,16]. In the case $\alpha > 0$ all cosmological solutions have regular bouncing character, if at extreme conditions $p > \frac{1}{3}\rho$. There are physical reasons to assume, that the restriction $p > \frac{1}{3}\rho$ is valid for gravitating matter at extreme conditions [14]. Really in the case of perfect gas of fermions at zeroth absolute temperature, the pressure tends to the value $\rho/3$ from below, if density of gas increases. Then we have for nuclear matter at extreme conditions $p > \frac{1}{3}\rho$ because of strong nucleon interaction [14]. We will suppose below, that the condition $p > \frac{1}{3}\rho$ is valid for gravitating matter at extreme conditions and, in particular, at the beginning of cosmological expansion[2]. Note, that this condition is valid for so-called stiff equation of state $p = \rho$ used in the theory of the early Universe (Ya.B. Zeldovich and others).

The GCFE lead to restrictions on admissible values of energy density. In fact, if energy density ρ is positive and $\alpha > 0$, from equation (5) in the case $k = +1$, 0 follows the relation:

$$Z \equiv 1 + \alpha\,(\rho - 3p) \geq 0\,. \tag{16}$$

The condition (16) is valid not only for closed and flat models, but also for cosmological models of open type ($k = -1$) [7]. In the case of models filled with usual gravitating matter without scalar fields the equation $Z = 0$ determines limiting (maximum) energy density, and regular transition from compression to expansion (bounce) takes place for all cosmological solutions

[2] In the case $\alpha < 0$ GCFE, (5), (6) lead to regular bouncing solutions, if at extreme conditions $p < \frac{1}{3}\rho$ [10, 3]. However, in this case GCFE have also singular solutions for some hypothetical superdense systems [15, 3].

by reaching limiting energy density. Near limiting energy density the gravitational interaction has the repulsion character. In the case of systems including also scalar fields a bounce takes place in points of so-called "ounce surfaces" in space of variables $(\phi, \dot{\phi}, \rho_{\rm m})$ [7]. Near bounce surfaces as well as bounds $Z = 0$ gravitational interaction has the character of repulsion, but not attraction. By using GCFE in the form (11), (14) we will find below, by what conditions gravitational repulsion effect takes place.

(a) At first, we will consider gravitating systems filled with gravitating matter without scalar fields. Then the acceleration $a = \ddot{R}/R = \dot{H} + H^2$ from (13), (14) takes the following form

$$a \;=\; \left(Z+\frac{3}{2}\alpha Y\right)^{-3}\left\{\left[4\pi G\left(\rho_{\rm m}-p_{\rm m}+\frac{\alpha}{2}(\rho_{\rm m}-3p_{\rm m})^2\right)-\frac{2k}{R^2}Z\right]\left(Z+\frac{3}{2}\alpha Y\right)^2 - 2D\left[Z+\frac{3}{4}\alpha Y-\frac{9\alpha}{4}\left(\frac{dp_{\rm m}}{d\rho_{\rm m}}Y+3(p_{\rm m}+\rho_{\rm m})^2\frac{d^2p_{\rm m}}{d\rho_{\rm m}^2}\right)\right]\right\}. \tag{17}$$

Obviously the repulsion $(a > 0)$ will take place, if the expression in figured parentheses in (17) is positive. The domain of admissible energy densities, by which the repulsion effect takes place, depends on equation of state of gravitating matter at extreme conditions and on the value of parameter α. In particular, in the case of flat models $(k = 0)$ with linear equation of state $p_{\rm m} = w\rho_{\rm m}$ $(w = \text{const} > \frac{1}{3})$ we obtain from (17) the following condition for energy densities corresponding to repulsion effect

$$y(x) \equiv \frac{1}{8}(9w^2-1)x^3 + \frac{3(3w^2-1)}{3w+1}x^2 + \frac{3(9w+5)}{2(3w+1)}x - 1 > 0\,, \tag{18}$$

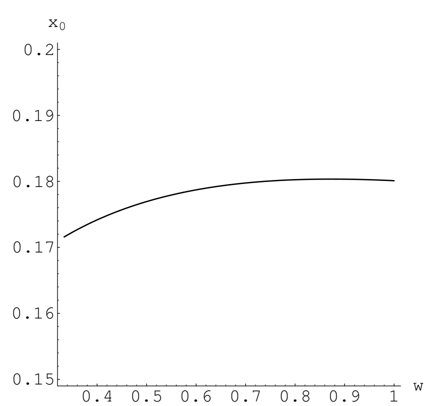

Fig. 1. The root x_0 as function of parameter w.

where $x = \alpha\rho_{\rm m}(3w-1) > 0$. Moreover, from condition $Z = 1-x \geq 0$ follows, that $x \leq 1$. By this restriction the cubic equation $y(x) = 0$ has the only real root x_0 and inequality (18) is fulfilled at $x > x_0$. Numerical solution of equation $y(x)=0$ gives the dependence of value x_0 on parameter w (see Fig. 1). The gravitational repulsion effect in considered case takes place at energy densities defined by the following condition $x_0 < x \leq 1$, and the value $x = 1$ corresponds to limiting energy density.

(b) In the case of systems including non-interacting gravitating matter and scalar field the condition of gravitational repulsion obtained from (11) and (14) is

$$\begin{aligned}&\left\{8\pi G\left[V+\frac{1}{2}\left(\rho_{\rm m}-p_{\rm m}\right)+\frac{\alpha}{4}(4V-\dot{\phi}^2+\rho_{\rm m}-3p_{\rm m})^2\right]\right.\\&\left.-3\alpha\left[\frac{\partial^2 V}{\partial\phi^2}\dot{\phi}^2-\left(\frac{\partial V}{\partial\phi}\right)^2\right]-\frac{2k}{R^2}Z\right\}\left[Z+3\alpha\left(\dot{\phi}^2+\frac{1}{2}Y\right)\right]^2\\&+\left(-3\alpha\frac{\partial V}{\partial\phi}\dot{\phi}\pm\sqrt{D}\right)^2\left[-2Z+6\alpha\dot{\phi}^2-\frac{3}{2}\alpha Y\right.\\&\left.+\frac{9\alpha}{2}\left(\frac{dp_{\rm m}}{d\rho_{\rm m}}Y+3(p_{\rm m}+\rho_{\rm m})^2\frac{d^2p_{\rm m}}{d\rho_{\rm m}^2}\right)\right]>0\,.\end{aligned}\qquad(19)$$

Inequality (19) together with condition $Z \geq 0$ determine the domain of variables ϕ, $\dot{\phi}$ and $\rho_{\rm m}$ at extreme conditions near bounds ($Z = 0$) and bounce surfaces in space of these variables, where gravitational repulsion effect appears. This domain is different for H_+- and H_--solutions of (11) and (14) corresponding to two values of the Hubble parameter (13) [7]. Note that (19) describes also repulsion effect at small energy densities like to GR, when the pressure is negative because of contribution of scalar field. In the case of systems filled with interacting gravitating matter and scalar field the generalization of condition (19) can be obtained from (11) and (15). Note that in the case of closed and open models conditions for gravitational repulsion effect include the term with scale factor R depending on 3-space curvature also, this means that the repulsion effect for homogeneous isotropic systems depends on global structure of gravitating model.

5. Conclusion

The analysis of gravitational repulsion effect at extreme conditions in GTG presented above shows that this effect takes place near limiting energy density, near a bounce, and it depends on content and properties of gravitating matter at extreme conditions (equation of state for gravitating matter, the form of scalar field potentials *etc.*) and also on the scale of extremely high energy densities defined by the parameter α. If the value of limiting energy density is essentially less than the Planckian one, gravitational repulsion effect appears at classical conditions, when the quantum gravitational corrections according to widely known opinion are not essential[3]. In the case of systems including scalar fields the value of limiting energy density is different for different solutions and can be essentially greater than α^{-1}, but the appearance of gravitational repulsion effect does not depend on this fact [16]. If limiting energy density is comparable with the Planckian one, quantum gravitational corrections have to be taken into account, although classical GTG ensures satisfactory non-singular behavior of gravitating systems. Because GCFE in the case of gravitating systems with small energy densities lead to consequences similar to that of GR, gravitational repulsion effect at such conditions can appear in the case of gravitating models with negative pressure (dark energy) like to GR. In particular, if GCFE (5), (6) include cosmological constant, the value of which corresponds to dark energy density at present epoch, regular cosmological solutions of (5), (6) will contain the stage of accelerating cosmological expansion by dominating of vacuum energy (cosmological constant).

The study of gravitating systems in the frame of GTG shows, that important features of gravitational interaction at extreme conditions depend essentially on properties of gravitating matter, which are determined by other fundamental physical interactions. To describe the evolution of gravitating models we have to know, how corresponding properties of gravitating matter (at first of all its equation of state) change by model evolution. This conclusion obtained by investigation of homogeneous isotropic models has sufficiently general character, because the form of used GCFE (5), (6) does not depend on detailed structure of quadratic in the curvature part of gravitational Lagrangian. At the same time the dynamics of gravitating systems depends on the structure of quadratic part of gravitational Lagrangian of GTG, if the homogeneity and isotropy are broken. In connection with this, the search of GTG leading to the most satisfactory consequences in general case of inhomogeneous anisotropic models is of direct physical interest.

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[3] Probably, in the frame of considered GTG the quantum properties of gravity can be important near the bound $Z = 0$, where values of the torsion function S are extremely large.

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