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ENERGY AND MOMENTUM OF BELL–SZEKERES SPACE-TIME IN MØLLER PRESCRIPTION

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This study is purposed to elaborate the problem of energy and momentum distribution of the Bell–Szekeres space time in general theory of relativity. In this connection, we use the energy-momentum definition of Møller and obtain that the energy momentum distributions (due to matter plus field) are vanishing everywhere. This results are exactly the same as viewpoint of Aygün *et al.* and agree with a previous work of Rosen, Saltı *et al.* and Johri *et al.* who investigated the problem of the energy in Friedmann–Robertson–Walker universe. The result that the total energy-momentum of the universe in these models are zero support the viewpoint of Tryon.

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1. Introduction

The subject of energy-momentum localization in General Relativity (GR) and Teleparallel Gravity (TG) continues to be an open one because there is no given yet a generally accepted expression for the energy-momentum density. For the solution of the problem many researchers have computed the energy as well as the momentum and angular momentum associated with various space-times. After Einstein [1] obtained an expression for the energy-momentum complexes many physicists, such as Landau and Lifshitz [2], Papapetrou [3], Tolman [4], Weinberg [5], Qadir–Sharif [6] and Bergmann and Thomson [7] had given different definitions for the energy-momentum complex. These definitions were restricted to evaluate energy distribution in quasi-cartesian coordinates. This motivated Møller [8] and many others, like Komar [9] and Penrose [10] to construct coordinate independent definitions. Møller proposed an expression which could be utilized to any coordinate systems. There have been several attempts to calculate energy-momentum prescriptions associated with different space-times [11, 12]. Virbhadra [13] showed that the definitions of Einstein, Tolman and Landau and Lifshitz give the same energy distribution for the Kerr–Newman metric. Later, Aguirregabiria *et al.* [14] proved that definitions of Einstein, Landau and Lifshitz, Weinberg and Papapetrou give the same result for any metric of Kerr–Schild class. Later, Virbhadra [15] emphasized that these complexes in fact coincide for space-times more general than the Kerr–Schild class. He also computed energy distribution for a general non-static spherically symmetric space-time of Kerr–Schild class and found that all these definitions give the same result as given by the Penrose quasi-local definition of energy. Vargas [16], by using teleparallel gravity analogs of Einstein and Landau–Lifshitz energy-momentum definitions, found that energy is zero in Friedmann–Robertson–Walker space-times. This result agrees with the previous works of Cooperstock–Israelit [17], Rosen [18], Banerjee–Sen [19] who investigated the problem of the energy in Friedmann–Robertson–Walker universe in Einstein's theory of general relativity. After this work, Saltı and Havare [20] considered Bergmann–Thomson's definition in both general relativity and teleparallel gravity for the viscous Kasner-type metric. Tryon [21] suggested that in our universe all conserved quantities have to vanish. Tryon's big bang model predicted a homogeneous, isotropic and closed universe including matter and anti-matter equally. Aygün *et al.* have investigated energy momentum-distributions of Marder universe for Einstein, Møller, Bergmann–Thomson, Landau–Lifshitz, Papapetrou, Qadir–Sharif and Weinberg's definitions in general relativity and teleparallel version of Einstein, Bergmann–Thomson and Landau–Lifshitz definitions and also the momentum four-vector (due to matter plus field) is found to be zero [22, 23].

The basic purpose of this paper is to obtain the total energy for Bell–Szekeres metric by using the energy-momentum expression of Møller in general relativity. We will proceed according to the following scheme. In Section 2, we give the Bell–Szekeres space-time and the features of gravitational and electromagnetic waves. Section 3 gives the energy-momentum definitions of Møller in general relativity and we calculate the total energy-momentum density for the Bell–Szekeres space-time. Finally, we summarize and discuss our results. Throughout this paper, the Latin indices $(i, j, \ldots)$ represent the vector number and the Greek $(\mu, \nu, \ldots)$ represent the vector components; all indices run 0 to 3. We use geometrized units where $G = 1$ and $c = 1$.

2. Bell–Szekeres space-time

It is known that exact *gravitational plane waves* are very simple time dependent plane symmetric solutions of Einstein's equations [24]. Nevertheless, they show two main nontrivial global features, namely: *(i)* the absence of a global Cauchy surface, which is a consequence of the focusing effect that the waves exert on null rays [25], *(ii)* the presence of a Killing–Cauchy horizon which may be physically understood as the caustic produced by the focusing of null rays [26]. The inverse of the focusing time is a measure of the strength of the wave. For an Einstein–Maxwell plane wave such inverse time equals the electromagnetic energy per unit surface of the wave. This makes exact plane waves very different from their linearized counterparts, which have no focusing points and admit a globally hyperbolic space-time structure. One expects that exact plane waves may be produced in the collision of black holes [27] or to represent traveling waves on strongly gravitating cosmic strings [28]. In recent years these waves have been used in classical general relativity to test some conjectures on the stability of Cauchy horizons [29], and in string theory to test classical and quantum string behaviour in strong gravitational fields [30]. Interest in them also stems from the fact that plane waves are a subclass of exact classical solutions to string theory [31]. In Einstein–Maxwell theory the particular class of plane symmetric waves are seen to contain only a non-null component of Ricci tensor and only a non-null component of the Weyl tensor. Depending on whether the Ricci component or the Weyl component is zero we will distinguish *pure gravitational plane waves* or *pure electromagnetic plane waves*, respectively. When we consider a plane wave collision, we should analyze separately the collision between pure gravitational waves, between pure electromagnetic waves or between mixed waves. Namely: *(i)* when two pure gravitational plane waves interact, the focusing effect of each wave distorts the causal structure of the space-time near the null horizons that these waves contain and either a spacelike curvature singularity or a new regular Killing–Cauchy horizon is created, *(ii)* when two pure electromagnetic plane waves interact, the situation is more subtle. In fact, in the full Einstein–Maxwell theory, Maxwell's equations remain linear indicating non direct electromagnetic interaction between the waves. However, there is a non-linear interaction of the waves through the gravitational field generated by their electromagnetic energy, which is similar to the magnitude of the interaction between pure gravitational waves. In that sense, the collision of two electromagnetic waves is seen to produce gravitational waves. *(iii)* in the case of mixed collisions, the pure electromagnetic wave is partially reflected by the incident pure gravitational wave. The gravitational wave, however, is not necessarily reflected.

The Bell–Szekeres solution [32] represents the collision of two electromagnetic plane shock waves followed by trailing radiation. The interaction region is isometric to the Bertotti–Robinson universe [33], which is the static conformally flat solution of Einstein–Maxwell equations with a uniform electric field. Such a geometry is similar to the throat of Reissner–Nordstrom solution for the special case $M = Q$ [34].

The Bell–Szekeres (BS) metric is given by

$$ds^2 = 2dudv + e^{-U}\left(e^{V}dx^2 + e^{-V}dy^2\right)\,, \tag{1}$$

where the metric functions U and V depend on the null coordinates u and v. The complete solution of the Einstein–Maxwell equations is

$$U = -\log(f(u)+g(v)), \quad V = \log(rw - pq) - \log(rw + pq), \tag{2}$$

where

$$\begin{aligned} r &= \left(\frac{1}{2}+f\right)^{1/2}, \qquad p = \left(\frac{1}{2}-f\right)^{1/2}, \\ w &= \left(\frac{1}{2}+g\right)^{1/2}, \qquad q = \left(\frac{1}{2}-g\right)^{1/2}, \end{aligned} \tag{3}$$

with

$$f = \frac{1}{2} - \sin^2 P, \qquad g = \frac{1}{2} - \sin^2 Q\,. \tag{4}$$

Here $P = au\Theta(u)$, $Q = bv\Theta(v)$, where Θ is the Heaviside unit step function, a and b are arbitrary constants.

3. Møller energy-momentum in General Relativity

The energy-momentum complex of Møller [35] is given by

$$M_\mu^\nu = \frac{1}{8\pi}\chi^{\nu\alpha}_{\mu,\alpha} \tag{5}$$

satisfying the local conservation laws:

$$\frac{\partial M_\mu^\nu}{\partial x^\nu} = 0\,, \tag{6}$$

where the antisymmetric super-potential $\chi_\mu^{\nu\alpha}$ is

$$\chi_\mu^{\nu\alpha} = \sqrt{-g}[g_{\mu\beta,\gamma} - g_{\mu\gamma,\beta}]g^{\nu\gamma}g^{\alpha\beta}\,. \tag{7}$$

The locally conserved energy-momentum complex M_μ^ν contains contributions from matter and non-gravitational fields. M_0^0 is the energy density and M_α^0 are the momentum density components. The momentum four-vector of Møller is given by

$$P_\mu = \int\int\int M_\mu^0 dxdydz\,. \tag{8}$$

Using Gauss's theorem, this definition transforms into

$$P_\mu = \frac{1}{8\pi}\int\int \chi_\mu^{\nu\alpha}\mu_\alpha dS\,, \tag{9}$$

where μ_α is the outward unit normal vector over the infinitesimal surface element dS. P_i give momentum components P_1, P_2, P_3 and P_0 gives the energy. The required components of $\chi_\mu^{\nu\alpha}$ are

$$\begin{aligned}\chi_2^{02} &= -2(\cos(au\Theta(u))\sin(au\Theta(u)) \\ &\quad + \sin(bv\Theta(v))\cos(bv\Theta(v)))(\Theta(u) + u\Theta_u)a\,, \end{aligned}\tag{10}$$

$$\begin{aligned}\chi_3^{03} &= -2(\cos(au\Theta(u))\sin(au\Theta(u)) \\ &\quad - \sin(bv\Theta(v))\cos(bv\Theta(v)))(\Theta(u) + u\Theta_u)a\,, \end{aligned}\tag{11}$$

where u indices describe the derivative with respect to u. From Eqs. (10), (11) and (5) we get

$$M_0^0 = M_\mu^0 = 0\,. \tag{12}$$

Substituting Eq. (12) into Eq. (9) we easily see that Møller energy and momentum in the Bell–Szekeres space-time is

$$P_\mu = 0\,. \tag{13}$$

4. Summary and discussion

A large number of researchers are interested in studying the energy and momentum contents of universe in various space- time models. Rosen, using Einstein's energy momentum complex, studied the total energy a homogeneous isotropic universe described by FRW metric and obtained zero. With the Landau–Lifshitz definition of energy, Johri *et al.* demonstrated that the total energy of FRW spatially closed universe is zero at all times irrespective of equations of state of the cosmic fluid and the total energy enclosed within any finite volume of spatially flat FRW universe is zero at all times.

In this paper, for the Bell–Szekeres space-time, using the energy-momentum complex of Møller we found that the total energy momentum distribution is vanishing. In addition, the results that total energy and momentum distribution are vanishing in Bell–Szekeres space-time agree with the previous work of Aygün *et al.*, Saltı *et al.*, Rosen and Johri *et al.* and support the viewpoint of Tryon.

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