Vol. 38 (2007) *ACTA PHYSICA POLONICA B* No 10

RELATION BETWEEN THE GRAVITATIONAL AND THE COSMOLOGICAL CONSTANT, II

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(Received March 14, 2007)

Considerations concerning the possibility to associate some "phenomenological" quantities that describe the field of point charges and the "particle" characteristics of point sources are presented. Relationships between the potential of the charges and the flux density of particles of the sources and relations among cross sections and some fundamental constants are also presented in this paper.

PACS numbers: 02.90+p

1. Introduction

In the paper [1], some relations between the gravitational and the "Seeliger's cosmological constant" were derived on the basis of the diffusion theory. The whole topic still allows completion. With the development of the knowledge, there is endeavour to explain the phenomena, *i.e.* to amend their "phenomenological" description on the basis of further structures and mechanisms. It is known, that there is "equivalency" between the theory of the potential and the Brown's motion and, therefore, also the diffusion theory, see [2]. Analogously, "the diffusion problem can be mapped onto the motion of a quantum particle in a random potential", see for example [3]. In other words, the diffusion or stochastic approach to quantum theory can be used to derive probabilistic or stochastic processes, see for example [4, 16]. It is also possible to introduce a relation between the diffusion coefficient and the Planck's constant, see *e.g.* [4]. If the quantum or particle structure of the gravitational field is anticipated, this approach may also be applied to gravitational interactions and to equations of gravitation. If we suppose that the gravitational charge is the source of the gravitational quanta (gravitons) and generally any charge is the source of the corresponding quanta of some field and that the scattering of the quanta of this field can occur, we may obtain some new information. The aim of this paper is also to indicate a connection between the theory of gravitation and the diffusion theory or transport theory.

The assumptions in this paper are as follows:

1. The gravitational field has a quantum structure and the gravitational charge is the source of the gravitational quanta (gravitons).
2. The physical vacuum has a "particle" (quantum) structure.
3. The scattering of these gravitational quanta occurs in this vacuum.

(These assumptions can also be applied to some other fields and other charges.)

At a certain level the "basic" quantities of the phenomenological description of a physical field are, *e.g.*, the potential, the field intensity and the force acting on an object in this field. From the point of view of the quantum structure of the field, among the basic concepts and quantities belong the space density of particles (quanta) of this field, the flux density of quanta, the current of particles, the cross sections of corresponding interactions and another quantities. If we anticipate the quantum structure of the gravitational field, then these quantities can also be used for the gravitational interactions. At the same time a natural question arises: "What are the relations between the classical phenomenological quantities and the equations and relations which describe the quantum structure of this field"?

From the mathematical point of view, we can lay the question: What are the operators which transform these quantities on each other and what are the relations between the equations with these quantities? For the sake of simplicity, we consider stationary monoenergetic sources of particles and a stationary Newton's field of point gravitational charge. We shall consider this Newton's gravitational charge as a source of gravitational quanta, which form its gravitational field. There exist historically many modifications of the Newton's law, see, *e.g.*, [7, 8] and for new versions, see, *e.g.*, [9]. There are also many formulae and relations for the flux density of the particles of point sources in dependence on the surrounding environment and on the kind and spectrum of emitted particles. The sources of particles, elementary particle physics and quantum field theory are described in many applications and books, *e.g.*, [10–15, 26–28]. From these applications, we use only some basic relations and quantities, the validity of which could be expected in certain approximation for any sources, *i.e.*, also for the sources of gravitational quanta. We will consider a point, static and monoenergetic source of particles and the corresponding charge, absolute values of the quantities and some other simplifications. We will treat the derived results as the first approximation only.

2. Similarity or "equivalency" of some quantities and equations

A more general form of the Poisson's equation for the gravitational potential φ can be written under certain assumptions in the form, see [20]:

$$\varDelta\varphi - \varLambda_{\mathrm{S}}\varphi + c^2\varLambda_{\mathrm{E}} - 4\pi\rho G = 0\,,$$

$\varLambda_{\mathrm{S}}$ — "Seeliger's" constant; $\varLambda_{\mathrm{E}}$ — Einstein's cosmological constant; c — the velocity of the light; $\rho = M\delta(r)$ — for a point gravitational charge M. Particular cases of this equation are for $\varLambda_{\mathrm{S}} = 0$ or $\varLambda_{\mathrm{E}} = 0$.

The quantity $c^2\varLambda_{\mathrm{E}}(k\pi G)^{-1}$ can be understood as "preexisting" density of matter, see [20], or density of the "physical" vacuum, see [5, 6, 22, 24, 25]; $k = 4$ or 8 on the dependence if we consider effective density of matter without pressure or density of energy with pressure, see [23, 25].

Let us assume that this matter creates some kind of "physical vacuum" and we also assume that this vacuum has the quantum (particle) structure with the mass M_1 of these quanta. Then the number of these quanta in a volume unit is obviously:

$$N = c^2\varLambda_{\mathrm{E}}(k\pi G M_1)^{-1}\,.$$

If some interaction takes place on this vacuum matter (for example the scattering of the gravitational quanta) with a cross section σ, then the macroscopic cross section for this interaction is

$$\varSigma = N\sigma = \sigma c^2\varLambda_{\mathrm{E}}(k\pi G M_1)^{-1}\,.$$

The "equivalent" diffusion equation for the flux density of the gravitational quanta has the form

$$\varDelta\phi - \varSigma_{\mathrm{a}}D^{-1}\phi + B + SD^{-1} = 0\,.$$

The term B, which is "equivalent" to $c^2\varLambda_{\mathrm{E}}$, means that all scattering points are also sources or absorbers of the scattered quanta and they create "homogeneous" background. But practically in "most" cases, the term B or $c^2\varLambda_{\mathrm{E}}$ can be neglected in these equations, however, the factor $c^2\varLambda_{\mathrm{E}}$ contents information about the density of scattering points, which is important for the value of the macroscopic cross section. D — the diffusion coefficient, which depends on $\varSigma_{\mathrm{s}}$ or $\varSigma_{\mathrm{tr}}$ (scattering or transport cross section); usually $D = C/\varSigma_{\mathrm{s}}$, where C is coefficient of the proportionality; ϕ — scalar flux (flux density); $\varSigma_{\mathrm{a}}$ — macroscopic absorption cross section; S — emissivity (strength) of the source; S is the number of emitted particles (quanta) in a time unit; $S = S\delta(\boldsymbol{r})$ for a point source.

We may write:

$$\phi(\boldsymbol{r}) = -k_1(\boldsymbol{r})\varphi(\boldsymbol{r})$$

and

$$S = k_2 M\delta(\boldsymbol{r})\,.$$

Then it holds:

$$\Delta\varphi(k_1-1)+\varphi\left(\Lambda_{\mathrm{S}}-\Sigma_{\mathrm{a}}D^{-1}k_1\right)-\left(B+c^2\Lambda_{\mathrm{E}}\right)-M\left(k_2D^{-1}-4\pi G\right)\delta(\boldsymbol{r})=0\,.$$

If we "normalise" in suitable units $\phi = -\varphi$, then for $k = 4$

$$\varphi\left(\Sigma_{\mathrm{a}}D^{-1}-\Lambda_{\mathrm{S}}\right)+M\left(k_2D^{-1}-4G\pi\right)\delta(\boldsymbol{r})+\left(c^2\Lambda_{\mathrm{E}}+B\right)=0\,.$$

From this equation follows, under certain assumptions that

$$\Lambda_{\mathrm{S}} = \Sigma_{\mathrm{a}}D^{-1}, \quad 4\pi GM = SD^{-1}, \quad -c^2\Lambda_{\mathrm{E}} = B\,,$$

and also

$$\Lambda_{\mathrm{S}} = M\sigma_{\mathrm{a}}\Lambda_{\mathrm{E}}c^2\left(SM_1\right)^{-1}\,,$$

$$G = (c/4\pi)\left(S\sigma_{\mathrm{s}}\Lambda_{\mathrm{E}}/MM_1C\right)^{1/2}\,.$$

If we put $M = S$ also in suitable units then

$$G = (c/4\pi)\left(\sigma_{\mathrm{s}}\Lambda_{\mathrm{E}}/M_1C\right)^{1/2} = (1/4\pi)\left(\sigma_{\mathrm{s}}\Lambda_{\mathrm{S}}/C\sigma_{\mathrm{a}}\right)^{1/2} = 1/4\pi D$$

$$\Lambda_{\mathrm{S}} = \sigma_{\mathrm{a}}\Lambda_{\mathrm{E}}c^2/M_1\,.$$

If the scattering is isotropic, then it is possible to assume $C \cong 1/3$ and

$$\Sigma_{\mathrm{S}} = (4\pi/3)G, \quad \Sigma_{\mathrm{a}} = \Lambda_{\mathrm{S}}/4\pi G, \quad \sigma_{\mathrm{s}}/\sigma_{\mathrm{a}} = \left(16\pi^2/3\right)\left(G^2/\Lambda_{\mathrm{S}}\right)\,,$$

and also

$$G = 3\Sigma_{\mathrm{S}}\left(4\pi\right)^{-1} = \left(4\pi D\right)^{-1}\,, \quad \Lambda_{\mathrm{S}} = 3\Sigma_{\mathrm{S}}\Sigma_{\mathrm{a}} = 4\pi G\Sigma_{\mathrm{a}}\,,$$

which are relations derived in [1].

If the scattering is anisotropic then we must transform $\Sigma_{\mathrm{S}} \to \Sigma_{\mathrm{tr}}$ and $\sigma_{\mathrm{s}} \to \sigma_{\mathrm{tr}}$. Σ_{tr} and σ_{tr} are transport cross sections. The above mentioned relationships link the gravitational constant, cosmological constant and "Seeliger's" constant in this model in a simple way and thus "specify" the relations in [1]. From the introduced equations we can see that from mathematical point of view, there is similarity and "equivalency" among some quantities and equations. We may expect that also "the physical content" of these concepts and formulae is the same, the differences are only in definitions of individual quantities, in domains of applications (gravitation, sources of particles, diffusion, electromagnetism and so on), in the units in which we measure and state these quantities and also in the historic evolution (time and circumstances of their origin). We may ask, what is the mathematical form of the operators and which mappings transform these quantities each into the other, *i.e.*, for example, what are the operators $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ defined by relations:

$$\phi = \boldsymbol{A}\varphi, \quad \phi = \boldsymbol{B}E, \quad G = \boldsymbol{C}\Sigma, \quad \boldsymbol{j} = \boldsymbol{D}\boldsymbol{E}\,.$$

The quantities $G, \Lambda_{\mathrm{E}}, \Lambda_{\mathrm{S}}$ expressed by means of cross sections can generally be considered as a function of space and time variables, eventually of other parameters. Under certain assumptions some quantities can be constant functions. There is an open question, which quantities join each other. We can associate these quantities in "a natural" way and introduce relations which follow from this. We see, that for identity of the potential and flux density, the mass M has the negative sign in comparing with usual sources that is, gravitational charge is negative source and rather absorber than emitter.

3. Some special cases

A. The source in "ideal" vacuum without shielding, absorption and scattering of emitted quanta and the "pure" Newton's field of the point charge, *i.e.* $\Lambda_{\mathrm{S}} = 0, \Lambda_{\mathrm{E}} = 0, \Sigma_{\mathrm{S}} = 0$.

Then $\phi = S\left(4\pi r^2\right)^{-1}$ is the flux density of emitted particles; $|\boldsymbol{j}| = \phi$ is the current of particles in direction of the radius and $n = \phi c^{-1} = S\left(4\pi c r^2\right)^{-1}$ is the space density of monoenergetic quanta with the velocity c.

For the Newton's field is:

$$\Delta\varphi = 4\pi G M \delta(\boldsymbol{r})\,,$$
$$\varphi = -GMr^{-1}, \quad \boldsymbol{E} = -\mathrm{grad}\varphi, \quad E = -GMr^{-2}$$
$$F = -GMmr^{-2}\,,$$

where R — distance; m — "testing" mass in this field; G — Newton's gravitational constant; φ — potential; E — intensity, $E = |\boldsymbol{E}|$; $\boldsymbol{F}$ — force acting on the mass m, $F = |\boldsymbol{F}|$; M — point gravitational charge. In this case is flux density $\phi = \boldsymbol{K}_1 E$, where $\boldsymbol{K}_1 = \phi E^{-1} = -S\,(4\pi GM)^{-1}$. If we demand "normalisation" $\phi = -E$ then $S = 4\pi GM$ or $G = S\,(4\pi M)^{-1}$. In units in which $M = S$ we have $G = (4\pi)^{-1}$. If we make the same consideration for the space density n then from $n = -E$ follows formula $S = 4\pi cGM$ and for $M = S$ is $G = (4\pi c)^{-1}$. These relations may be "more natural" and "more suitable" than the relation between ϕ and E.

B. Shielded point source without diffusion in an infinite homogeneous environment with absorption and "Seeliger's" or "Yukawa's" field:

The flux density:

$$\phi = S\left(4\pi r^2\right)^{-1}\exp(-\Sigma_{\rm a} r), \quad \Sigma_{\rm a} = N\sigma_{\rm a}\,,$$

N — number of "absorbing" points in a volume unit; $\sigma_{\rm a}$ — microscopic cross section for absorption.

The current of the quanta in radial direction is $|\boldsymbol{j}| = \phi$ and the space density of monoenergetic quanta with the velocity c is

$$n = \phi c^{-1} = S\left(4\pi c r^2\right)^{-1}\exp(-\Sigma_{\rm a} r)\,.$$

Then relation between the flux density and intensity is $\phi = \boldsymbol{K}_2 E$, where operator

$$\boldsymbol{K}_2 = \phi E^{-1} = -S\,(4\pi GM)^{-1}\exp\left[\left(\Lambda_{\rm S}^{1/2} - \Sigma_{\rm a}\right) r\right]\,.$$

We use the Seeliger's form for E

$$E = -GMr^{-2}\exp\left(-\Lambda_{\rm S}^{1/2} r\right)\,.$$

The case where $\varphi = -GMr^{-1}\exp\left(-\Lambda_{\rm S}^{1/2} r\right)$ will be introduced later. If we "normalise" $\phi = -E$ then $G = S\,(4\pi M)^{-1}$ and $\Lambda_{\rm S} = \Sigma_{\rm a}^2$. Analogous formulae are also given for the particle density n.

C. We suppose diffusion and "small" absorption *i.e.* the source is in an environment in which the elastic and inelastic scattering and absorption occur.

The flux density is

$$\phi = S\,(4\pi r D)^{-1}\exp(-\kappa r)\,,$$

D — diffusion coefficient in this environment

$$\kappa^2 = \Sigma_{\rm a} D^{-1}, \quad \Sigma_{\rm a} = N\sigma_{\rm a}\,.$$

Under certain assumptions we get $D = C\Sigma_{\rm S}^{-1}$ or $C\Sigma_{\rm tr}^{-1}$ and for isotropic or weak anisotropic scattering $D = (3\Sigma_{\rm S})^{-1}$ or $D = (3\Sigma_{\rm tr})^{-1}$. In this case corresponding diffusion equation has the form

$$\Delta\phi - \Sigma_{\rm a} D^{-1}\phi + SD^{-1} = 0\,.$$

Fick's law

$$|\boldsymbol{j}| = |-D\,\mathrm{grad}\,\phi| = S(1+\kappa r)\left(4\pi r^2\right)^{-1}\exp(-\kappa r)\,,$$

$\boldsymbol{j}$ — the current of quanta. Then $\phi = \boldsymbol{K}_3\varphi$ (φ is potential), where operator

$$\boldsymbol{K}_3 = \phi\varphi^{-1} = -S\,(4\pi MG)^{-1}\exp\left[\left(\Lambda_{\mathrm{S}}^{1/2}-\kappa\right)r\right]\,.$$

"The Seeliger's" form was used for φ. For "normalisation" $\phi = -\varphi$ is $G = S\,(4\pi DM)^{-1}$ and $\Lambda_{\mathrm{S}}^{1/2} = \kappa$, which are again relations derived in the paper [1]. Analogous considerations may be done also for the space density n that differs by the factor c.

Let us consider, as an example, a simple model where there is an elastic scattering of the gravitons in "physical" vacuum with the space density of target scattering points

$$N = c\Lambda_{\mathrm{E}}\,(k\pi GM_1)^{-1} \quad (k=4)\,,$$

and with the cross section σ_{s} (*i.e.* $\Sigma_{\mathrm{s}} = N\sigma_{\mathrm{s}}$).

Gravitational potential φ:

$$\Delta\varphi - \Lambda_{\mathrm{S}}\varphi = 4\pi GM\delta(\boldsymbol{r})\,.$$

We admit a small absorption with $\Sigma_{\mathrm{a}} = N\sigma_{\mathrm{a}}$ then for the "normalisation" $\varphi = -\phi$ and for $M = S$ in suitable units, we obtain the same relationships as in Section 2.

The other relations that differ by the factor c, can be obtained when we set higher priority on the space density n and if we make the same considerations as were given above.

D. The Coulomb's field.

Absolute values of the potential and intensity are:

$$\Delta\varphi + Q\delta(\boldsymbol{r})\varepsilon^{-1} = 0,\ \ \boldsymbol{E} = -\mathrm{grad}\varphi,$$
$$\varphi = Q\,(4\varepsilon\pi r)^{-1}\,, \quad E = Q\left(4\pi\varepsilon r^2\right)^{-1}\,.$$

The flux density of an "equivalent" source is $\phi = S\left(4\pi r^2\right)^{-1}$. After "normalisation" $\phi = E$ we get $S = Q\varepsilon^{-1}$ (ideal vacuum without diffusion and absorption).

If we suppose a modification from the Proca's equation, which leads to an electrostatic potential of Yukawa's form (physical vacuum with diffusion and small absorption)

$$\varphi = Q\,(4\pi\varepsilon r)^{-1}\exp(-br)\,,$$

then also

$$\phi = S\,(4\pi r D)^{-1}\exp(-\kappa r)$$

(for the scattering and a small absorption of corresponding quanta) and operator

$$\boldsymbol{A} = \varphi\phi^{-1} = QD\,(S\varepsilon)^{-1}\exp[(\kappa - b)r]\,.$$

If we demand (at suitable normalisation) $\boldsymbol{A} = 1$ then $Q = S\varepsilon D^{-1} = C^{-1}S\varepsilon N\sigma_{\rm s}^{\rm el}$ and

$$b = \kappa = \left(\Sigma_{\rm a}^{\rm el}D^{-1}\right)^{1/2} = N\left(C^{-1}\sigma_{\rm a}^{\rm el}\sigma_{\rm s}^{\rm el}\right)^{1/2}\,.$$

If also $Q = S$ in suitable units then $\varepsilon = D = C\left(N\sigma_{\rm s}^{\rm el}\right)^{-1}$, where $C \cong 1/3$ for isotropic scattering and $\Sigma_{\rm s}^{\rm el} = (3\varepsilon)^{-1}$.

If we suppose that, in this case, the same "vacuum" plays role as for the gravitation, *i.e.* $N_{\rm g} = N_{\rm el} = N$, then

$$\varepsilon = \left(4/3c^2\right)\left(\pi G M_1/\Lambda_{\rm E}\sigma_{\rm s}^{\rm el}\right)\,.$$

It is a relation associating some electromagnetic and gravitation quantities and cross sections.

E. Some further relations.

In literature [10] we sometimes find the form

$$\phi(r) = S\beta\,(4\pi D r)^{-1}\exp(-\kappa r) + S\left(4\pi r^2\right)^{-1}\exp(-\Sigma' r)$$

as a good approximation for both small and large r. D, κ, Σ' and β are again quantities which depend on $\Sigma, \Sigma_{\rm a}$ and on anisotropy of the scattering. It is also possible to write the basic equation for ϕ in the form:

$$\phi = S\,(kr^a)^{-1}\exp(-br)\,.$$

Some modifications of Newton's law are for example:

$E = -GMr^{-2}\,(1 + \varepsilon_{\rm C} r^{-m})$	A.C. Clairaut
$E = -GMr^{-2}\exp(-hr)$	H. Seeliger (also Laplace, Neumann, Yukawa)
$E = -GMr^{-(2+\alpha_{\rm H})}$	A.H. Hall

and many other modifications. The quantities $\varepsilon_{\rm C}, h, \alpha_{\rm H}$ are certain parameters.

"New and present" modifications of Newton's law are expressed by means of potentials of the type Fujii, Scherk, Long, Fishbach, see *e.g.* [9, 17–19], *i.e.*, for example, in the form:

$$\varphi = -GMmr^{-1}\left[1 + \alpha\exp\left(-r\lambda^{-1}\right)\right] .$$

Let us consider $\varphi = -GMr^{-1}\exp\left(-\Lambda_{\rm S}^{1/2}r\right)$, then

$$E = -|\mathrm{grad}\,\varphi| = -GMr^{-1}\exp\left(-\Lambda_{\rm S}^{1/2}r\right)\left(\Lambda_{\rm S}^{1/2} + r^{-1}\right) .$$

Expression for E corresponds to the relation derived in case (C) for Fick's laws $\boldsymbol{j} = -D\,\mathrm{grad}\,\phi$. The quantity for intensity E, can be compared in this case, with quantity for flux density

$$\phi(r) = \underset{(\text{large } r)}{S\beta\left(4\pi Dr\right)^{-1}\exp(-\kappa r)} + \underset{(\text{small } r)}{S\left(4\pi r^2\right)^{-1}\exp - \Sigma' r} .$$

In more general cases, it is necessary to include also the so-called built-up factor and a change of energy spectrum (slow-down moderation) in these relations or to solve transport Boltzmann equation.

4. Conclusion

From all these relations we can see that there is a connection between "classical" quantities and quantities which describe the particle structures of these fields and sources. It seems that it is suitable to consider equivalency $|\varphi| \Leftrightarrow |\phi|$ (respectively $|n| \Leftrightarrow |\varphi|$) and $|\boldsymbol{j}|\Leftrightarrow|\boldsymbol{E}|$ but it requires to somewhat change the "classical" definitions of quantities φ (potential) and E (intensity), which have arisen historically before, and on the basis of "phenomenological" associations. Relation $\boldsymbol{j} = \gamma\boldsymbol{E}$ is analogy of the Ohm law applied for gravitation for current of gravitons, which can be expressed as $\boldsymbol{j} = \rho\boldsymbol{v} = \boldsymbol{j}(\psi)$. $\psi, \rho, \boldsymbol{v}$ are wave function, density and velocity of the gravitons and $\boldsymbol{j}\boldsymbol{E}$ is power of gravitational forces.

There is an open question which quantity is to be associated with the force respectively with the fluency and with further quantities. The force in the Newton's (or Coulomb's) field is given by relation

$$F = mE = GMmr^{-2} = \left(G^{1/2}Mr^{-1}\right)\left(G^{1/2}mr^{-1}\right) = G^{-1}\varphi_1\varphi_2$$

(for absolute values).

For the identity $|\varphi| = |\phi|$ and $S = M$, it corresponds to quantity

$$G^{-1}\phi_1\phi_2 = 4\pi D\phi_1\phi_2 \cong 4\pi C\Sigma_{\mathrm{S}}^{-1}\phi_1\phi_2 \cong (4/3)(\pi/\Sigma_{\mathrm{S}})\phi_1\phi_2 \quad (\text{for } C = 1/3)\,,$$

which we may interpret as a certain kind of "probability" that there are simultaneously the flux ϕ_2 in point 1 and the flux ϕ_1 in point 2. It is also possible to choose another approach. Let $\boldsymbol{j} = \boldsymbol{E}$ and $m = S$; then $\boldsymbol{F} = m\boldsymbol{E} = S\boldsymbol{j}$ or more generally $\boldsymbol{F} = \boldsymbol{K}(S\boldsymbol{j})$, where $\boldsymbol{K}$ is an operator. In this case the quantity $\boldsymbol{F}$ expresses "interaction of the source S with the current $\boldsymbol{j}$". It is possible to say that a similar relation will hold for any source or charge which is in the current $\boldsymbol{j}$ of the quanta of another source, see also consideration in [1]. (It is certain analogy of current-current interactions, current-field and field-field interactions in elementary particle physics, see for example [26]).

In the case of the fluency $F = \int \phi dt$ and equivalency $|\phi| = |\varphi|$, is $F = G\,M\,T\,/\,r$. By my opinion it will be needed, in the future, determine absolutely and independently the values $\Sigma_{\mathrm{a}}, \Sigma_{\mathrm{S}}, N, \sigma_{\mathrm{a}}, \sigma_{\mathrm{s}}$ and also S, and so on and then state the "normalisation" relationships on "classical" phenomenological quantities $M, \varphi, E, G, \Lambda_{\mathrm{S}}, \Lambda_{\mathrm{E}}$ and other ones. This approach will also allow compute reaction rates

$$R_i = \int \varphi(E)\sigma_i(E)dE$$

of various gravitational interactions. Here $\varphi(E)$ is the spectrum of gravitational quanta and $\sigma_i(E)$ is the cross section for given interaction. The symbolic form of many equations of mathematical physics, see for example [21], is the same. Consequently we could compare some quantities derived from these equations for phenomena in which we may expect certain "similarity" and then we can obtain further new relations. Some another relations will be presented in [29–31] which are prepared.

Appendix

Some basic relations for a field of particles are

Density of particles in phase space $[\boldsymbol{r}, \boldsymbol{v}, t]$:	$n = n(\boldsymbol{r}, \boldsymbol{v}, t)$
Space density of particles:	$\rho(\boldsymbol{r}, t) = \int nd^3v$
Flux density of particles:	$\phi(\boldsymbol{r}, t) = \int n(E)v(E)dE = \int \varphi(E)dE$
Energy spectrum of particles:	$\varphi(E, \boldsymbol{r}, t) = \int nvd\boldsymbol{\Omega}$
Current of particles:	$\boldsymbol{j}(\boldsymbol{r}, t) = \int n\boldsymbol{v}d^3v$
Energy of particles:	E

Solid angle:	$\boldsymbol{\Omega}$
Fluency of particles:	$F = \int \phi(\boldsymbol{r}, t) dt$
The reaction rate of given interaction:	$R_i = \int \varphi(E)\sigma_i(E) dE$
Microscopic cross section for given interaction:	σ_i
Total cross section:	$\sigma_t = \sum \sigma_i$
Macroscopic cross section:	$\Sigma_i = N\sigma_i$
Space density of target scattering points (*i.e.* the number of target points in a volume unit):	N
Decay constant:	$\lambda = \ln 2 / T_{1/2}$
Half-life of decay (we assume that it is sufficiently large):	$T_{1/2}$
Fick's law:	$\boldsymbol{j} = -D \mathrm{grad} \phi$

We can suppose that these basic relations will also play some role in the theory of gravitational interactions. Under certain assumptions these quantities may be also expressed by means of the wave function which is the solution of corresponding equations of quantum mechanics (nonrelativistic Schroedinger equation and relativistic equations Klein–Gordon, Dirac, Proca and equations for higher spins).

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