Vol. 38 (2007) *ACTA PHYSICA POLONICA B* No 10

"INTRINSIC INTERPRETATION" OF FERMION GENERATIONS AND THE TAUON MASS*

Wojciech Królikowski

Institute of Theoretical Physics, Warsaw University
Hoża 69, 00–681 Warszawa, Poland

(Received March 5, 2007)

To the memory of Leszek Łukaszuk

First, in order to support the empirical mass formula discussed in this paper for fundamental fermions, our previous work on an "intrinsic interpretation" of lepton and quark generations is summarized. In this framework, some intrinsic dynamical factors are proposed as responsible for the structure of fundamental-fermion empirical mass formula. Then, a satisfactory mass sum rule for charged leptons is derived, predicting perfectly the (actual) experimental central value 1776.99 MeV of m_τ, when the input of experimental values of m_e and m_μ is applied. The derivation goes through identifying in the structure of three-parameter empirical mass formula for charged leptons a tiny combination of parameters which, if postulated to be exactly zero, gives a parameter constraint leading to the satisfactory mass sum rule.

PACS numbers: 12.15.Ff, 12.90.+b

1. Introduction

The successful development of quantum field theory through several recent decades (QED, QCD, Standard Model, ...), closely related to some experimental discoveries of fundamental significance, has formed a theoretical paradigm which now is critically confronted with the problem of mass that we feel to be lying at the very roots of our concept of matter. It seems that, in contrast to the notion of energy, the notion of dynamical mass creating a nontrivial spectrum is not embraced by the paradigm of quantum field theory, where the mass gets essentially the status of a free parameter, similarly as it got always in the classical dynamics since Newton's time (this is true, even if the Higgs mechanism is invoked: in this case, instead of the problem of mass, one has to confront the corresponding problem of Yukawa coupling).

* Work supported in part by the Polish Ministry of Science and Higher Education, grant 1 PO3B 099 29 (2005–2007).

For composite systems, the problem of mass spectrum is successfully replaced by the problem of observed rest-energy spectrum, when for considered systems efficient quantum theories exist (operating with a few different constituent masses), as it is the case in the atomic and (to some extent) nuclear and hadron physics. Explicitly, the genuine problem of observed mass spectrum appears in the particle physics, if elementary particles — in agreement with their postulated nature — are not composite systems (at least, in the conventional spacetime meaning).

For some time the author's idea is to treat the popular candidates for elementary constituents of matter, leptons and quarks, as point-like objects in spacetime, but composite in a new algebraic sense expressed by the multicomponent structure of their local wave function,

$$\psi_{\alpha_1 \alpha_2 \ldots \alpha_N}(x) \quad (N = 1, 3, 5, \ldots)\,, \tag{1}$$

satisfying the generalized Dirac equation found by the author several years ago [1] (this equation follows from Dirac's generalized square-root procedure)[1]. Here, $\alpha_n = 1, 2, 3, 4$ $(n = 1, 2, \ldots, N)$ are Dirac bispinor indices in the chiral representation, where all Dirac matrices γ^5_n and σ^3_n $(n = 1, 2, \ldots, N)$ are diagonal, though $[\gamma^\mu_n, \gamma^\nu_m] \neq 0$ for $n \neq m$ since in the generalized Dirac equation the Clifford algebra $\{\gamma^\mu_n, \gamma^\nu_m\} = 2g^{\mu\nu}\delta_{nm}$ $(\mu, \nu = 0, 1, 2, 3,\ n, m = 1, 2, \ldots, N)$ holds [1]. In Eq. (1), the Standard Model SU(3)×SU(2)×U(1) labels are suppressed.

To be more detailed, we correlate in our approach all Standard Model charges carried by leptons and quarks (suppressed in the notation) with the first bispinor index α_1, and then conjecture that the additional bispinor indices $\alpha_2, \ldots, \alpha_N$ are fully antisymmetrized *i.e.*, they describe indistinguishable physical objects obeying Fermi statistics along with Pauli principle (realized in an intrinsic way) [1,3]. It is convenient to introduce for all objects $n = 1, 2, \ldots, N$ defined by the bispinor indices $\alpha_1, \alpha_2, \ldots, \alpha_N$ the term of *algebraic partons* (then, we can say that all Standard Model charges are "carried" by the distinguished algebraic parton $n = 1$ characterized by the bispinor index α_1).

[1] The generalized Dirac equation [1] works also for boson states with $N = 2, 4, 6, \ldots$; in particular for $N = 2$ it is known as the Kähler equation [2].

It is exciting enough to observe that the above simple conjecture on the full antisymmetry of the wave function (1) in the additional $\alpha_2, \ldots, \alpha_N$ indices implies the existence in Nature of *exactly* three generations of fundamental fermions (leptons and quarks): the index N in Eq. (1) can take *only* three values $N = 1, 3, 5$, since each bispinor index assumes four different values [1, 3][2].

The author would like to point out that this simple picture of algebraic partons "within" leptons and quarks can be obtained from the picture of conventional composite system of spatial partons through an act of logical abstraction that may be compared to Dirac's famous act of logical abstraction which led from the conventional spatial rotator to the spinning electron. In both cases, this is an act of algebraic abstraction from the notion of spatial extension. This analogy would be spoilt, if our hypothetic algebraic partons were the visible top of an iceberg of conventional spin-1/2 spatial partons, bound by some new forces within composite leptons and quarks, mainly in S states.

Thus, three odd integers,

$$N_i = 1, 3, 5 \quad (i = 1, 2, 3)\,, \tag{2}$$

equal to the number of bispinor indices involved in $\psi_{\alpha_1\alpha_2\ldots\alpha_{N_i}}(x)$, numerate in our picture three generations $i = 1, 2, 3$ of fundamental fermions (leptons and quarks) whose relativistic wave functions can be reduced to the forms [3]

$$\begin{aligned} N_1 = 1 : \psi^{(1)}_{\alpha_1}(x) &= \psi_{\alpha_1}(x)\,, \\ N_2 = 3 : \psi^{(2)}_{\alpha_1}(x) &= \frac{1}{4}\sum_{\alpha_2\,\alpha_3} \left(C^{-1}\gamma^5\right)_{\alpha_2\alpha_3} \psi_{\alpha_1\alpha_2\alpha_3}(x) = \psi_{\alpha_1 12}(x) = \psi_{\alpha_1 34}(x)\,, \\ N_3 = 5 : \psi^{(3)}_{\alpha_1}(x) &= \frac{1}{24}\sum_{\alpha_2\,\alpha_3\,\alpha_4\,\alpha_5} \varepsilon_{\alpha_2\alpha_3\alpha_4\alpha_5}\psi_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}(x) = \psi_{\alpha_1 1234}(x) \end{aligned} \tag{3}$$

[2] Similarly, for boson states, *only* two values $N = 2, 4$ are possible, implying two generations of some fundamental scalars, both with leptonlike or quarklike Standard Model charges (the first scalars might play the role of two Higgs doublets leading then to the 2H scheme, while the second could not spontaneously break the electroweak symmetry). Note that if in the wave function (1) the index α_1 were not correlated with the Standard Model charges and all indices $\alpha_1, \alpha_2, \ldots, \alpha_N$ were antisymmetrized, then for N odd, *only* two values $N = 1, 3$ would be possible, giving two sterile spin-1/2 fundamental fermions (they might be two light sterile neutrinos or two sterile fermions in the cold dark matter or, eventually, one of the first and one of the second sort of these sterile particles). Similarly, for N even, *only* two values $N = 2, 4$ would be possible, leading to two sterile fundamental scalars (they might play the role of quintessence or be sterile bosons in the cold dark matter). Thus, in our scheme there is a place also for such sterile fermion and boson fundamental states.

(the Standard Model labels accompanying the bispinor index α_1 are here suppressed). In Eq. (3), other relativistic $\psi_{\alpha_1\alpha_2\alpha_3}$ and $\psi_{\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5}$ vanish. It can be seen that, due to the full antisymmetry in $\alpha_2, \ldots, \alpha_N$ indices, the wave functions (3) can appear in various calculations with multiplicities 1, 4, 24, respectively (up to the $\pm$ sign). So, there are defined three *generation-weighting factors*,

$$\rho_i = \frac{1}{29}, \frac{4}{29}, \frac{24}{29} \quad (i = 1, 2, 3) \tag{4}$$

($\sum_i \rho_i = 1$), such that the following reduction formulae hold for bilinear forms of $\psi_{\alpha_1\alpha_2\ldots\alpha_{N_i}}(x)$:

$$\sum_{\alpha_2\ldots\alpha_{N_i}} \psi^\dagger_{\alpha_1\alpha_2\ldots\alpha_{N_i}}(x)\psi_{\beta_1\alpha_2\ldots\alpha_{N_i}}(x) = 29\rho_i\psi^{(i)\dagger}_{\alpha_1}(x)\psi^{(i)}_{\beta_1}(x) \ (i = 1, 2, 3)\,. \tag{5}$$

2. Formal intrinsic interactions and the mass formula

Thus, we will look for the mass formula for fundamental fermions of three generations in the form

$$m_i = \rho_i \, h_i \quad (i = 1, 2, 3)\,, \tag{6}$$

where ρ_i are three generation-weighting factors (4), while the other three factors

$$h_i = a u_i + b v_i + c w_i \quad (i = 1, 2, 3) \tag{7}$$

involve three free parameters a, b, c dependent on the kind $f = \nu, l, u, d$ of fundamental fermions (suppressed in our notation), but independent of their generations $i = 1, 2, 3$.

In order to build up nine terms u_i, v_i, w_i in Eqs. (7), we have to our disposal only three numbers N_i (three other numbers ρ_i available in our approach are already used in Eqs. (6) as generation-weighting factors). We will do it introducing the following three types of terms which — when multiplied by the constants a, b, c, respectively — may be technically called *formal intrinsic interactions* "within" fundamental fermions:

(a) Two-body intrinsic interaction between all N_i algebraic partons $n = 1, 2, ..., N_i$ treated on an equal footing:

$$a u_i = a \sum_{n,m=1}^{N_i} 1 = a N_i^2 = a N_i + a N_i(N_i - 1) \quad (i = 1, 2, 3)\,, \tag{8}$$

where the first term aN_i on the r.h.s. describes the sum of intrinsic self-interactions of all N_i algebraic partons, while the second term $aN_i(N_i-1)$ presents the sum of their intrinsic mutual interactions.

(b) A correction to the intrinsic self-interaction of the algebraic parton $n=1$ distinguished from all other N_i-1 algebraic partons $n\neq 1$ which, in turn, are indistinguishable from each other:

$$bv_i = b\left(P^{(N_i)}_{n=1}\right)^2 = b\left[\frac{N_i!}{(N_i-1)!}\right]^{-2} = b\frac{1}{N_i^2} \quad (i=1,2,3)\,, \qquad (9)$$

where $P^{(N_i)}_{n=1} = 1/N_i$ describes the probability of finding such a distinguished algebraic parton (among N_i algebraic partons of which N_i-1 are indistinguishable).

(c) A collective "binding" correction to the intrinsic self-interactions of all N_i algebraic partons $n=1,2,\ldots,N_i$ treated on an equal footing:

$$cw_i = -c\left(P^{(N_i)}_{n=1} + P^{(N_i)}_{n\neq 1}\right) = -c \quad (i=1,2,3)\,, \qquad (10)$$

where $P^{(N_i)}_{n\neq 1} = 1 - P^{(N_i)}_{n=1}$, is the probability of finding any of the indistinguishable algebraic partons $n=2$ or $3\ldots$ or N_i.

In this listing of three types of formal intrinsic interactions it was assumed that the Standard Model charges "carried" by the distinguished algebraic parton $n=1$ exert their influence on its formal intrinsic self-interaction corrected as at point (b) and on free parameters a,b,c dependent on the label $f=\nu,e,u,d$ (suppressed in our notation). Here, the convenient term of "formal intrinsic interactions" for the expressions au_i, bv_i, cw_i has a technical character and might be not applied at all (though it is suggestive for model building). At any rate, these expressions (together with the generation-weighting factors ρ_i) are responsible in our approach for the structure of fundamental-fermion mass formula.

In fact, with the use of notation

$$\begin{aligned} a &\equiv \mu > 0\,,\\ b &\equiv \mu(\varepsilon-1)\,,\\ c &\equiv \mu\xi > 0\,, \end{aligned} \qquad (11)$$

where μ,ε,ξ are new free parameters dependent on the kind $f=\nu,l,u,d$ of fundamental fermions (suppressed in our notation), the forms (6) and (7) together with the proposals (8), (9) and (10) imply the following fundamental-fermion mass formula:

$$m_i = \mu\rho_i \left(N_i^2 + \frac{\varepsilon - 1}{N_i^2} - \xi \right) \quad (i = 1, 2, 3) \tag{12}$$

or explicitly:

$$\begin{aligned} m_1 &= \frac{\mu}{29}(\varepsilon - \xi)\,, \\ m_2 &= \frac{\mu}{29}\, 4 \left(\frac{80 + \varepsilon}{9} - \xi \right)\,, \\ m_3 &= \frac{\mu}{29}\, 24 \left(\frac{624 + \varepsilon}{25} - \xi \right)\,. \end{aligned} \tag{13}$$

This mass formula was proposed previously as an empirical mass formula for fundamental fermions [4].

Strictly speaking, the mass formula (12) or (13) is a specific transformation of three masses m_1, m_2, m_3 into three free parameters μ, ε, ξ or *vice versa*, giving no predictions for the masses, *unless* the parameters are constrained. To fit the free parameters to the experimental masses, and so to make the mass formula (12) or (13) empirical, we can use the formulae inverse to Eqs. (13) [4]:

$$\begin{aligned} \mu &= 29 \frac{25}{9216} \left[m_3 - \frac{6}{25}(27m_2 - 8m_1) \right]\,, \\ \varepsilon &= 10 \frac{m_3 - \frac{6}{125}(351m_2 - 904m_1)}{m_3 - \frac{6}{25}(27m_2 - 8m_1)}\,, \\ \xi &= 10 \frac{m_3 - \frac{6}{125}(351m_2 - 136m_1)}{m_3 - \frac{6}{25}(27m_2 - 8m_1)}\,. \end{aligned} \tag{14}$$

Notice the relation following from Eqs. (14) that will play an important role for charged leptons:

$$\frac{\varepsilon - 97\xi}{960} = -\frac{m_3 - \frac{6}{125}(351m_2 - 128m_1)}{m_3 - \frac{6}{25}(27m_2 - 8m_1)}\,. \tag{15}$$

From the values of parameters μ, ε, ξ experimentally fitted to the masses m_1, m_2, m_3, we may read off some parameter constraints (*e.g.* $\xi = 0$ in the case of charged leptons) and then, try to postulate them and so get some predictions for the masses. Later on in this paper, we will concentrate on the case of charged leptons $l_i = e, \mu, \tau \quad (i = 1, 2, 3)$, where the experimental mass values are [5]

$$\begin{aligned} m_1 \equiv m_e = 0.5109989 \text{ MeV}, \quad m_2 \equiv m_\mu = 105.65837 \text{ MeV}\,, \\ m_3 \equiv m_\tau = 1776.99^{+0.29}_{-0.26} \text{ MeV}\,. \end{aligned} \tag{16}$$

3. Charged leptons and a satisfactory mass sum rule

From the inverse formulae (14) we obtain for charged leptons the parameter values

$$\mu = 86.0076 \text{ MeV}, \quad \varepsilon = 0.174069, \quad \xi = 0.0017706\,, \tag{17}$$

if the experimental masses (16) are inserted (for m_τ the experimental central value 1776.99 MeV is used).

In the approximation, where the small parameter ξ is put exactly zero *i.e.*, the parameter constraint $\xi = 0$ is imposed, we get from the third Eq. (14) the following charged-lepton mass sum rule [4]:

$$m_\tau = \frac{6}{125}\,(351 m_\mu - 136 m_e) = 1776.80 \text{ MeV}\,. \tag{18}$$

In this way, we receive an approximate *prediction* for m_τ that is in a good agreement with the experimental value $m_\tau = 1776.99^{+0.29}_{-0.26}$ MeV. We can also evaluate the parameters μ and ε that in this approximation become

$$\begin{aligned} \mu &= \frac{29(9m_\mu - 4m_e)}{320} = 85.9924 \text{ MeV}\,, \\ \varepsilon &= \frac{320 m_e}{9m_\mu - 4m_e} = 0.172329\,. \end{aligned} \tag{19}$$

In Eqs. (18) and (19), only the experimental values of m_e and m_μ are used as an input. If it turned out that the experimental central value of m_τ were close to 1776.80 MeV rather than to 1776.99 MeV, then the postulated constraint $\xi = 0$ would be perfect, telling us that for charged leptons there should be no formal intrinsic interaction of the type (c).

However, with the actually accepted value of m_τ [5], it turns out in the case of charged leptons that in the structure of our mass formula (12) or (13) there is another really tiny parameter which, if put exactly zero as an imposed parameter constraint, leads to a perhaps more satisfactory mass sum rule than the relation (18). In fact, the parameter standing on the l.h.s. of relation (15), when calculated with the use of the experimentally fitted parameter values (17), is very small:

$$\frac{\varepsilon - 97\xi}{960} = 2.4175 \times 10^{-6} \tag{20}$$

(note that $\xi/10 = 1.7706 \times 10^{-4}$ in the third Eq. (14) is also very small, but larger by two orders; it gives the mass sum rule (18), when it is put exactly zero). Thus, putting exactly $(\varepsilon - 97\xi)/960 = 0$ or

$$\varepsilon = 97\xi \tag{21}$$

as an imposed parameter constraint for charged leptons, we obtain from the r.h.s. of relation (15) the following satisfactory charged-lepton mass sum rule:

$$m_\tau = \frac{6}{125}(351 m_\mu - 128 m_e) = 1776.9926 \text{ MeV}\,, \tag{22}$$

being in a perfect agreement with the (actually accepted) experimental value $m_\tau = 1776.99^{+0.29}_{-0.26}$ MeV. Here, only the experimental values of m_e and m_μ are applied as an input. Now, making use of the *prediction* (22) and of the experimental values of m_e and m_μ as an input, we can calculate from Eqs. (14) the following parameter values for charged leptons:

$$\mu = 86.007807 \text{ MeV}, \qquad \varepsilon = 0.1740927, \qquad \xi = 0.0017948\,. \tag{23}$$

Our predicted value (22) of m_τ is even closer to the (actual) experimental central value 1776.99 MeV of m_τ than the prediction $m_\tau = 1776.9689$ MeV following (as one of two solutions [6]) from the wonderful nonlinear Koide equation [7]. Note that our mass sum rule (22) is linear and involves integers as its coefficients (when multiplied by 125).

If, alternatively, the coefficient 128 at the small m_e in Eq. (22) were replaced by 129, then m_τ would be nearly equally excellent: 1776.9681 MeV, but our postulated parameter constraint (21), $\varepsilon = 97\xi$, should be replaced by $7\varepsilon = 775\xi$. In this case, in place of the relation (15), another relation

$$\frac{7\varepsilon - 775\xi}{7680} = -\frac{m_3 - \frac{6}{125}(351 m_2 - 129 m_1)}{m_3 - \frac{6}{25}(27 m_2 - 8 m_1)}\,, \tag{24}$$

also following from Eqs. (14), would work. Its l.h.s., evaluated with the use of the experimentally fitted parameter values (17), is

$$\frac{7\varepsilon - 775\xi}{7680} = -2.0017 \times 10^{-5} \tag{25}$$

so, its magnitude, though still very small, becomes larger by one order than $(\varepsilon - 97\xi)/960$ (see Eq. (20)).

One may ask the question, for what fractional number lying around 128 the calculated mass m_τ is exactly equal to its (actually accepted) experimental value $1776.99^{+0.29}_{-0.26}$ MeV. Obviously, this is

$$\frac{351 m_\mu - \frac{125}{6} 1776.99^{+0.29}_{-0.26} \text{ MeV}}{m_e} = 128.108^{-11.823}_{+10.660}\,, \tag{26}$$

where, beside $m_\tau = 1776.99^{+0.29}_{-0.26}$ MeV, the experimental values of m_e and m_μ are used as an input. Then, the experimental mass identity

$$m_\tau = \frac{6}{125}\left(351 m_\mu - 128.108^{-11.823}_{+10.660} m_e\right) = 1776.99^{+0.29}_{-0.26} \text{ MeV}\,, \tag{27}$$

holds, rather than the mass sum rule (22) predicting m_τ. Here, the coefficient $128.108^{-11.823}_{+10.660}$ at the small m_e is experimentally fitted, in contrast to Eq. (22), where the coefficient 128 at the small m_e is imposed through the parameter constraint $\varepsilon = 97\xi$ which is postulated in this case.

Note, however, that — in the mass formula (12) for charged leptons — the alternative parameter constraint $\xi = 0$ in place of $\varepsilon = 97\xi$ is still possible within the present experimental limits, leading to the alternative charged-lepton mass sum rule (18) instead of (22).

4. Conclusions

In this paper, a satisfactory mass sum rule for charged leptons is derived from an empirical mass formula proposed previously for fundamental fermions. This sum rule predicts perfectly the (actual) experimental central value 1776.99 MeV of m_τ, when the input of experimental values of m_e and m_μ is applied. The derivation goes through identifying in the structure of three-parameter empirical mass formula for charged leptons a tiny combination of parameters that, if postulated to be exactly zero, gives a parameter constraint leading to the satisfactory mass sum rule. But first, in order to support the discussed empirical mass formula, we summarize our previous work on an "intrinsic interpretation" of lepton and quark generations, explaining the existence in Nature of exactly three generations. In this framework, some expressions, technically called formal intrinsic interactions "within" point-like leptons and quarks, are proposed as dynamical factors responsible for the structure of fundamental-fermion empirical mass formula.

REFERENCES

[1] W. Królikowski, *Acta Phys. Pol. B* **20**, 849 (1989); *Acta Phys. Pol. B* **21**, 871 (1990); *Phys. Rev.* **D45**, 3222 (1992).

[2] E. Kähler, *Rendiconti di Matematica*, **27**, 425 (1962); *cf.* also D. Ivanenko, L. Landau, *Z. Phys.*, **48**, 341 (1928).

[3] For more recent presentations *cf.* W. Królikowski, *Acta Phys. Pol. B* **33**, 2559 (2002) [`hep-ph/0203107`]; `hep-ph/0504256` (unpublished) and references therein.

[4] W. Królikowski, *Acta Phys. Pol. B* **37**, 2601 (2006) [`hep-ph/0602018`].

[5] W.M. Yao *et al.* (Particle Data Group), *Review of Particle Physics, J. Phys.* **G33**, 1 (2006).

[6] W. Królikowski, `hep-ph/0508039` (unpublished).

[7] For a recent discussion *cf.* Y. Koide, `hep-ph/0506247` and references therein; *cf.* also A. Rivero, A. Gsponer, `hep-ph/0505220`.