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# INTERPOLATING LAGRANGIAN AND BOUNDARY CONDITION FOR THE SUPERSTRING

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We are going to construct the interpolating action for the free superstring. We start from Nambu–Goto action and construct interpolating Lagrangian. We generate a first class algebra with primary constraint. Then this leads us to obtain the Lagrangian density in Polyakov form. Also we calculate interpolating boundary condition.

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## 1. Introduction

For the last few decades string theory has been regarded as the most promising step toward the fundamental theory uniting all the basic interactions at the Plank scale [1]. The evolution of a string is described either by Nambu–Goto (NG) or Polyakov action. The Polyakov action is classically equivalent to another action which does not have square root. This action was found by Brink, Deser, Divecchia, Howe and Zumino [2,3] but is usually known as the Polyakov action. The NG formalism is inconvenient for path integral quantisation whereas Polyakov action involves more degrees of freedom. However another formulation is interpolating between these two versions of string action. The interpolating Lagrangian is good candidate to description of the string theory. In Ref. [4,5], they derived a master action for free bosonic strings which is interpolating between the NG and Polyakov formalism. In Ref. [6] the interpolating action for interacting bosonic string is constructed and also the essential modification in the Poisson bracket structure of this interpolating theory that generates non-

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commutativity coordinates is obtained. Also they concluded that a gauge fixing is necessary to give an exact non-commutative solution between the string coordinates [7,8]. In the present paper we give supersymmetric version to NG and Polyakov strings [9] and find the interpolating action between them in the free case. Then we construct the interpolating boundary condition from the corresponding Lagrangian.

## 2. Free superstring in the Nambu–Goto formalism

For the construction of interpolating superstring action, we first start with NG action. So in this section, we analyse the NG formulation of the free superstring. The Green–Schwarz space-time supersymmetric version of the Nambu–Goto string can be written as

$$S_{\rm NG} = -T\int d\tau d\sigma(\sqrt{-g} + L_{\rm WZ})\,, \tag{1}$$

where $g$ is the determinant of the induced two-dimensional metric $g_{ab} = \delta_{\mu\nu} Z^\mu_a Z^\nu_b$ with $Z^\mu_a = \partial_a X^\mu - i\bar\theta\Gamma^\mu\partial_a\theta$. $L_{\rm WZ}$ is the Wess–Zumino term

$$L_{\rm WZ} = -i\,\varepsilon^{ab} Z_a \bar\theta\Gamma\partial_b\theta\,, \tag{2}$$

which is necessary for the presence of the local fermionic symmetry [10]. Obviously if we discard the fermions in action (1) we recover the usual Nambu–Goto action for the bosonic string.
The canonical momenta can be obtained from action (1)

$$\begin{aligned} P_\mu &= -T(\sqrt{-g}g^{0a}Z_{a\mu} + i\bar\theta\Gamma_\mu\dot\theta)\,,\\ \Pi_\alpha &= i(P - TZ_1)\,(\Gamma\theta)_\alpha\,. \end{aligned} \tag{3}$$

The nontrivial Poisson brackets of the theory are given by

$$\begin{aligned} \left\{X_\mu(\tau,\sigma), P_\nu(\tau,\sigma')\right\} &= \eta_{\mu\nu}\delta(\sigma-\sigma')\,,\\ \left\{\Pi_\alpha(\tau,\sigma), \theta_\beta(\tau,\sigma')\right\} &= \delta_{\alpha\beta}\delta(\sigma-\sigma')\,. \end{aligned} \tag{4}$$

From Eq. (3) one can obtain three primary constraints

$$\begin{aligned} \phi_0 &= \frac{1}{2}\left(Q^2 + T^2 Z_1^2\right) = 0\,,\\ \phi_1 &= Q\,Z_1 = 0\,,\\ \psi_\alpha &= \Pi_\alpha - i(P - TZ_1)(\Gamma\theta)_\alpha\,, \end{aligned} \tag{5}$$

where the mechanical momentum is $Q_\mu = P_\mu + iT\bar\theta\Gamma_\mu\dot\theta$. From Eqs. (4) it is straightforward to generate a first class algebra:

$$
\begin{aligned}
\{\phi_0(\sigma),\phi_0(\sigma')\} &= 4\left[\phi_1(\sigma)+\phi_1(\sigma')\right]\partial_\sigma\delta(\sigma-\sigma')\,,\\
\{\phi_0(\sigma),\phi_1(\sigma')\} &= \left[\phi_0(\sigma)+\phi_0(\sigma')\right]\partial_\sigma\delta(\sigma-\sigma')\,,\\
\{\phi_1(\sigma),\phi_1(\sigma')\} &= \left[\phi_1(\sigma)+\phi_1(\sigma')\right]\partial_\sigma\delta(\sigma-\sigma')\,.
\end{aligned}
\tag{6}
$$

The canonical Hamiltonian density corresponding to action (1) is

$$
\mathcal{H}_c = P\dot{X} + \bar{\Pi}\dot{\theta} - \mathcal{L}\,, \tag{7}
$$

which identically vanishes. This is a characteristic feature of reparametrization invariant systems. This can be easily seen by substituting (3) in (7). The total Hamiltonian density is given by a linear combination of the first class constraints

$$
\mathcal{H}_T = \lambda^a\phi_a + \xi\psi_\alpha\,, \tag{8}
$$

where $\lambda^a$ are two bosonic Lagrange multipliers and $\xi$ is a fermionic one.

## 3. Interpolating Lagrangian and boundary conditions for free superstring

Now we will construct interpolating Lagrangian of the free superstring. The construction of the interpolating action for the free and interacting bosonic string has been discussed in [4,6]. By using Eq. (8) in the following Lagrangian

$$
\mathcal{L} = P\dot{X} + \bar{\Pi}\dot{\theta} - \mathcal{H}_T\,,
$$

one can obtain

$$
\mathcal{L} = P\dot{X} + \bar{\Pi}\dot{\theta} - \lambda^a\phi_a - \xi\psi_\alpha\,. \tag{9}
$$

If we substitute (5) in (9) and solve the classical equation of motion and obtain the $P_\mu$ and $\bar{\Pi}_\alpha$, so we rewrite the Eq. (9) as follows

$$
\mathcal{L}_{\mathrm{I}} = \frac{1}{2\lambda_0}(Z_0 - \lambda^1 Z_1)^2 - \frac{1}{2}\lambda_0 T^2 Z_1^2 + T L_{\mathrm{WZ}}\,, \tag{10}
$$

where is interpolating Lagrangian of the free superstring. We note that $\lambda_0$ and $\lambda_1$ will be treated as independent fields. In order to obtain the $\lambda_0$ and $\lambda_1$ we calculate $P^\mu$ and compare to Eq. (3). From Eq. (10) we have

$$
P^\mu = \frac{1}{\lambda_0}(Z_0 - \lambda^1 Z_1) - i\,T\bar{\theta}\Gamma_\mu\dot{\theta}\,, \tag{11}
$$

finally one can obtain:

$$\begin{aligned}\frac{1}{\lambda_0} &= -T\sqrt{-g}g^{00}\,,\\ \frac{\lambda_1}{\lambda_0} &= T\sqrt{-g}g^{01}\,.\end{aligned} \tag{12}$$

Now we are going to represent the Eq. (12) as form of metric and that metric play the important role in any action

$$g^{ab} = -\frac{1}{T\sqrt{-g}}\begin{pmatrix} \frac{1}{\lambda_0} & -\frac{\lambda_1}{\lambda_0} \\ -\frac{\lambda_1}{\lambda_0} & \frac{\lambda_1^2 - T^2\lambda_0^2}{\lambda_0} \end{pmatrix}. \tag{13}$$

If we use explicit form of $g^{ab}$ we can write $\lambda_0$ and $\lambda_1$ in terms of $Z_0$ and $Z_1$

$$\begin{aligned}\lambda_0 &= -\frac{\sqrt{(Z_0\,Z_1)^2 - Z_0^2 Z_1^2}}{Z_1^2}\,,\\ \lambda_1 &= -\frac{Z_0\,Z_1}{Z_1^2}\,.\end{aligned} \tag{14}$$

Now by using (13) in interpolating Lagrangian (10) we arrive to the Polyakov Lagrangian density

$$\mathcal{L}_{\mathrm{P}} = -\frac{T}{2}(\sqrt{-g}g^{ab}Z_aZ_b - 2L_{\mathrm{WZ}})\,, \tag{15}$$

with $a, b = 1, 2$. The interpolating Lagrangian (10) leads us to construct the interpolating boundary condition

$$K_\mu = \left[\frac{\lambda_1}{\lambda_0}(Z_0 - \lambda^1 Z_1) - \lambda^0 T^2 Z_1 - iT\bar{\theta}\Gamma\theta'\right]_{\sigma=0,\Pi} = 0\,. \tag{16}$$

Also, we put $\lambda_0$ and $\lambda_1$ from Eq. (12) to (16), so we will arrive at

$$\left[g^{1a}Z_a + \frac{i}{\sqrt{-g}}\bar{\theta}\Gamma\theta'\right]_{\sigma=0,\Pi} = 0\,, \tag{17}$$

which is the Polyakov form of boundary condition and also obtained by the action (15). Hence, it is possible to interpret (16) as an interpolating boundary condition. Now it is turn to discuss the structure of constraint in interpolating free superstring. Note, that the fields in Eq. (10) $X^\mu$, $\lambda_0$ and $\lambda_1$ correspond to momenta $P_\mu$, $\pi_{\lambda_0}$ $\pi_{\lambda_1}$ respectively and is given by

$$\begin{aligned}P_\mu &= \frac{1}{\lambda_0}(Z_0 - \lambda^1 Z_1) - iT\bar{\theta}\Gamma\dot{\theta}\,,\\ \pi_{\lambda_0} &= 0\,,\\ \pi_{\lambda_1} &= 0\,.\end{aligned} \tag{18}$$

In addition to the Poisson brackets in Eq. (6) we have,

$$\begin{aligned} \left\{\lambda_0(\tau,\sigma), \pi_{\lambda_0}(\tau,\sigma')\right\} &= \delta(\sigma-\sigma')\,, \\ \left\{\lambda_1(\tau,\sigma), \pi_{\lambda_1}(\tau,\sigma')\right\} &= \delta(\sigma-\sigma')\,. \end{aligned} \tag{19}$$

Generally, we can say that the Eqs. (6) and (19) describe the full Poisson brackets of interpolating free superstring action.

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