NEUTRINO OSCILLATIONS IN THE CASE OF GENERAL INTERACTION*

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The process of the neutrino production, oscillation in the vacuum or in matter, and detection in the case of interactions which are beyond the Standard Model is considered. Neutrino states are described by the density matrix. The final neutrino production rate does not factorize. The known Maki–Nakagawa–Sakata neutrino states and the factorized production rate are recovered in the νSM regime.

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1. Introduction: Lagrangian of the extension of the νSM

The aim of this short presentation is to check how neutrino production, oscillation and detection processes are modified if a new physics (NP) describes neutrino interaction. Let us begin with the process of the production (P) of the massive neutrino ν_i ($i = 1, 2, 3$) accompanied by the lepton l_α ($\alpha = e, \mu, \tau$):

$$l_\alpha + P_1 \to \nu_i + P_2 \,, \tag{1}$$

followed, after traveling along the baseline L, by its detection (D):

$$\nu_i + D_1 \to l_\beta + D_2 \,, \tag{2}$$

where P_1, P_2 and D_1, D_2 are the accompanied particles.

Below we specify the NP extension of the interaction Lagrangian. Its charged current (CC) part describes the neutrino production and detection events, whereas the neutral current (NC) part modifies the neutrino interaction in a medium.

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1.1. Charged and neutral current Lagrangian for NP

The CC Lagrangian includes the νSM and the new LH and RH currents:

$$\begin{aligned}\mathcal{L}_{\rm CC} = \quad & \frac{-e}{2\sqrt{2}\sin\theta_W}\Big\{\sum_{\alpha,i}\bar{\nu}_i\,[\,\gamma^\mu(1-\gamma_5)\varepsilon_L U^{L*}_{\alpha i}+\gamma^\mu(1+\gamma_5)\varepsilon_R U^{R*}_{\alpha i}]\,l_\alpha\,W^+_\mu \\ & +\sum_{\alpha,i}\bar{\nu}_i\,[\,(1-\gamma_5)\eta_L V^{L*}_{\alpha i}+(1+\gamma_5)\eta_R V^{R*}_{\alpha i}]\,l_\alpha\,H^+ \\ & +\sum_{u,d}\bar{u}\,[\,\gamma^\mu(1-\gamma_5)\epsilon^q_L U^*_{ud}+\gamma^\mu(1+\gamma_5)\epsilon^q_R U^*_{ud}]\,d\,W^+_\mu \\ & +\sum_{u,d}\bar{u}\,[\,(1-\gamma_5)\tau_L W^{L*}_{ud}+(1+\gamma_5)\tau_R W^{R*}_{ud}]\,d\,H^+\Big\}+\text{h.c.}\,, \qquad (3)\end{aligned}$$

where U^L is the A$-$V νSM unitary neutrino mixing matrix [1]. The U^R and $V^{L,R}$ are the NP neutrino mixing matrices in the CC sector, the A+V and (pseudo)scalar (P, S) one, respectively. The coupling constants ε_L, ϵ^q_L are taken to be the global factors deviating slightly from their νSM values. For relativistic neutrinos the appropriate NC Lagrangian is:

$$\begin{aligned}\mathcal{L}_{\rm NC} = \quad & \frac{-e}{4\sin\theta_W\cos\theta_W}\left\{\sum_{i,j}\bar{\nu}_i\left[\gamma^\mu(1-\gamma_5)\varepsilon^{N_\nu}_L\delta_{ij}+\gamma^\mu(1+\gamma_5)\varepsilon^{N_\nu}_R\,\Omega^R_{ij}\right]\nu_j\right. \\ & \left.+\sum_{f=e,u,d}\bar{f}\left[\gamma^\mu(1-\gamma_5)\varepsilon^{N_f}_L+\gamma^\mu(1+\gamma_5)\varepsilon^{N_f}_R\right]f\right\}Z_\mu\,. \qquad (4)\end{aligned}$$

The νSM mixing matrix is diagonal, whereas Ω^R_{ij} depends on the NP parameters [2]. The couplings $\varepsilon^{N_\nu}_L$, $\varepsilon^{N_f}_L$ and $\varepsilon^{N_f}_R$ are the global factors very close to their νSM values. The NP factors in $\mathcal{L}_{\rm CC}$ and $\mathcal{L}_{\rm NC}$ are the small deviations from zero.

From $\mathcal{L}_{\rm CC}$ and $\mathcal{L}_{\rm NC}$ we could construct [3] the general low energy four-fermion effective interaction Hamiltonian $\mathcal{H}$ which describes the coherent neutrino scattering inside matter. As we consider three massive neutrinos in two possible helicity states ($\lambda=\pm1$) hence the NP effective Hamiltonian $\mathcal{H}_{\lambda,i;\,\eta,k}$ in the mass — helicity base $|\lambda,i\rangle$ is the 6×6 dimensional matrix:

$$\mathcal{H}=\mathcal{M}+\begin{pmatrix}\mathcal{H}_{--} & \mathcal{H}_{-+}\\ \mathcal{H}_{+-} & \mathcal{H}_{++}\end{pmatrix}, \qquad (5)$$

where the mass term is equal to:

$$\mathcal{M}=\text{diag}(E^0_1,E^0_2,E^0_3,E^0_1,E^0_2,E^0_3)\quad\text{with}\quad E^0_i=E_\nu+\frac{m^2_i}{2E_\nu}\,,\quad i=1,2,3\,. \qquad (6)$$

Here E_ν is the energy for the massless neutrino [4]. The antidiagonal terms in Eq. (5) originate in the tensorial T interaction which appears as a result of the Fiertz rearrangement applied to the scalar part of $\mathcal{L}_{\rm CC}$. The diagonal terms are connected with the A$\pm$V interactions, both the νSM and NP ones. The more detailed description of the νSM and NP decomposition of $\mathcal{H}$ could be found in [5] and its full analysis is postpone to the forthcoming paper.

2. Implementation of the density matrix formalism

The density matrix approach to the neutrino oscillation phenomenon is analyzed *e.g.* in [6, 7]. Such formalism is here more than merely the elegant form of the analysis. Indeed, for relativistic neutrinos produced and detected by the left-handed (LH) mechanism of the νSM, the oscillation rates factorize ([8], see also below) and the analysis is based on the QM evolution of the pure neutrino state $|\vec{p}, \lambda, i>$ of momentum $\vec{p}$, helicity λ and mass i [8]. Yet, if the NP of the right-handed (RH), scalar, or pseudoscalar interactions is involved, then the states are mixed and the density matrix formalism becomes the necessity. Below we present the way it works.

The neutrino statistical operator of the production process (1) written in the mass-helicity basis is equal to [7]:

$$\varrho_P^\alpha(\vec{p}, L=0) = \sum_{\lambda,\lambda'=\pm1}\sum_{i,i'=1}^{3} |\vec{p}, \lambda, i\rangle\ \varrho_P^\alpha(\vec{p}; \lambda, i; \lambda', i')\ \langle \vec{p}, \lambda', i'|\,, \tag{7}$$

undergoing in the homogeneous medium the following quantum evolution (with $L = T$):

$$\rho_P^\alpha(\vec{x}=\vec{0}, t=0) \rightarrow \rho_P^\alpha(\vec{x}=\vec{L}, t=T) = e^{-i(\mathcal{H}T)}\rho_P^\alpha(\vec{x}=\vec{0}, t=0)\ e^{i(\mathcal{H}T)}\,, \tag{8}$$

where $\mathcal{H}$ is given by Eq. (5). Suppose that in the production process (1) the momentum of the initial lepton l_α lies along the z axis. In the case of the lack of the polarization in the process (1), the reduced density matrix for the neutrino in the production place has the following form:

$$\varrho_P^\alpha(\lambda, i; \lambda', i') = \frac{1}{N_\alpha}\sum_{\lambda_{P_2}\lambda_{P_1}\lambda_\alpha} A_{i\ \lambda_{P_1},\lambda_\alpha}^{\alpha\ \lambda;\lambda_{P_2}}(\vec{p})\ \left(A_{i'\ \lambda_{P_1},\lambda_\alpha}^{\alpha\ \lambda';\lambda_{P_2}}(\vec{p})\right)^*\,, \tag{9}$$

where $A_{i\ \lambda_{P_1},\lambda_\alpha}^{\alpha\ \lambda;\lambda_{P_2}}(\vec{p})$ are the amplitudes for the production process and to preserve $\mathrm{Tr}(\varrho_P^\alpha) = 1$ the normalization constant N_α has to be equal to:

$$N_\alpha = \sum_{\lambda=\pm1}\sum_{i=1}^{3}\sum_{\lambda_{P_2}\lambda_{P_1}\lambda_\alpha} A_{i\ \lambda_{P_1},\lambda_\alpha}^{\alpha\lambda;\lambda_{P_2}}(\vec{p})\ \left(A_{i\ \lambda_{P_1},\lambda_\alpha}^{\alpha\lambda;\lambda_{P_2}}(\vec{p})\right)^*\,. \tag{10}$$

Here λ and $\lambda_{P_2}, \lambda_{P_1}, \lambda_\alpha$ are the helicities for the neutrino and the other particles.

As we are interested in the evolution of the density matrix in the LAB frame hence the density matrix ϱ_P^α should be calculated in this frame also, which is more arduous, than in the CM one. However, for the large oscillation baseline L, only neutrinos which are produced in forward direction in the CM frame will reach the detector. Then for the relativistic limit, it could be proved [7] that the Wigner rotation is negligible, that means

$$\varrho_P^{\mathrm{LAB}}(\vec{p}_{\mathrm{LAB}}) = \varrho_P^{\mathrm{CM}}(\vec{p}_{\mathrm{CM}} \to \vec{p}_{\mathrm{LAB}})\,. \tag{11}$$

Therefore we perform the calculation of ϱ_P^α in the CM frame and (for the Lagrangian (3) and (4)) from Eq. (9) we obtain the 3×3 density submatrices:

$$\begin{aligned}\varrho_P^\alpha(-1,i;\,-1,i') = {} & \frac{1}{N_\alpha}\left[A_{\varepsilon_L^2}\varepsilon_L^2 U_{\alpha i}^{L*}U_{\alpha i'}^{L} + A_{\eta_R^2}\eta_R^2 V_{\alpha i}^{R*}V_{\alpha i'}^{R}\right.\\ & \left. + A_{\varepsilon_L\,\eta_R}\,\varepsilon_L\eta_R\,(U_{\alpha i}^{L*}V_{\alpha i'}^{R} + V_{\alpha i}^{R*}U_{\alpha i'}^{L})\right], \end{aligned}\tag{12}$$

$$\begin{aligned}\varrho_P^\alpha(+1,i;\,+1,i') = {} & \frac{1}{N_\alpha}\left[A_{\varepsilon_R^2}\varepsilon_R^2 U_{\alpha i}^{R*}U_{\alpha i'}^{R} + A_{\eta_L^2}\eta_L^2 V_{\alpha i}^{L*}V_{\alpha i'}^{L}\right.\\ & \left. + A_{\varepsilon_R\eta_L}\,\varepsilon_R\eta_L\,(U_{\alpha i}^{R*}V_{\alpha i'}^{L} + V_{\alpha i}^{L*}U_{\alpha i'}^{R})\right], \end{aligned}\tag{13}$$

and $\varrho_P^\alpha(\mp 1,i;\,\pm 1,i') = 0$. The normalization constant N_α is equal to

$$\begin{aligned}N_\alpha = {} & A_{\varepsilon_R^2}\varepsilon_R^2 + A_{\varepsilon_L^2}\varepsilon_L^2 + A_{\eta_L^2}\eta_L^2 + A_{\eta_R^2}\eta_R^2\\ & + A_{\varepsilon_R\eta_L}\varepsilon_R\eta_L\sum_{i=1}^{3}\left(U_{\alpha i}^{R*}V_{\alpha i}^{L} + V_{\alpha i}^{L*}U_{\alpha i}^{R}\right)\\ & + A_{\varepsilon_L\,\eta_R}\,\varepsilon_L\eta_R\sum_{i=1}^{3}\left(U_{\alpha i}^{L*}V_{\alpha i}^{R} + V_{\alpha i}^{R*}U_{\alpha i}^{L}\right). \end{aligned}\tag{14}$$

The A-coefficients are the functions of the energies and momenta of the particles in the production process (1) of the neutrino. *E.g.* $A_{\varepsilon_L^2}$ and $A_{\varepsilon_R^2}$ are the amplitudes for the CC processes, A$-$V in SM and A+V in NP, respectively. Similarly, $A_{\eta_L^2}$ and $A_{\eta_R^2}$ are the amplitudes for the CC processes, S$-$P and S+P, respectively. Next, $A_{\varepsilon_L\eta_R}$ and $A_{\varepsilon_R\eta_L}$ are the functions of the amplitudes that mix different helicities of the neutrino. Finally, from Eqs. (12)–(14) it could be easily noticed that for the νSM interaction the standard setup for the neutrino, *i.e.*:

$$\varrho_P^\alpha(-1,i;\,-1,i') = U_{\alpha i}^{L*}U_{\alpha i'}^{L} \qquad \text{and} \qquad \varrho_P^\alpha(+1,i;\,+1,i') = 0\,, \tag{15}$$

is regained.

Now, let us discuss the detection process (2) of the neutrino after it propagates to $T = L \neq 0$. If the particles D_1 are not polarized and the final particles polarizations are not measured, the differential cross section for the flavor β neutrino detection in the LAB frame is given by:

$$\frac{d\sigma_{\beta\,\alpha}}{d\Omega_\beta} = f_D \sum_{\substack{\lambda, i;\, \lambda', i' \\ \lambda_{D_1}, \lambda_{D_2}, \lambda_\beta}} A^{\beta\ \lambda, \lambda_{D_1}}_{i\ \lambda_\beta, \lambda_{D_2}}(\vec{p}_\beta)\ \varrho^\alpha_P(\lambda, i; \lambda', i'; L = T)\,(A^{\beta\ \lambda', \lambda_{D_1}}_{i'\ \lambda_\beta, \lambda_{D_2}}(\vec{p}_\beta)\,)^*\,, \quad (16)$$

where $\varrho^\alpha_P(\,\lambda, i; \lambda', i'; L = T \neq 0)$ is the density matrix of the produced neutrino after it evolves to the detection point and $\lambda, \lambda_{D_1}, \lambda_\beta, \lambda_{D_2}$ are the particles helicities. f_D is the kinematical factor:

$$f_D = \frac{1}{64\,\pi^2\,(2\,s_{D_1} + 1)\ E_\nu\ m_{D_1}}\,\frac{p^3_\beta}{(E_\nu + m_{D_1})\,p^2_\beta - E_\beta\ (\vec{p}\cdot\vec{p}_\beta)}\,. \quad (17)$$

Here m_{D_1} and s_{D_1} are the mass and spin of the detector particle. The neutrino momentum and energy are noted as $\vec{p}$ and E_ν, respectively. Similarly $\vec{p}_\beta$ and E_β are the momentum and energy of the β lepton. The amplitudes $A^{\beta\ \lambda=\pm, \lambda_{D_1}}_{i\ \lambda_\beta, \lambda_{D_2}}(\vec{p}_\beta)$ describe the detection process (2) and are calculated in the rest frame of the detector (LAB). The formula (16) could be, after summing over all helicities of the particles, rewritten as follows:

$$\begin{aligned} \frac{d\sigma_{\beta\,\alpha}}{d\Omega_\beta} = \ & f_D \sum_{i;\, i'} \Big[a^{--}_{\beta;ii'}\ \varrho^\alpha_P(-1, i;\ -1, i'; L) + 2\ \cos\varphi\,\mathrm{Re}(a^{+-}_{\beta;ii'}\ \varrho^\alpha_P(1, i; -1, i'; L)) \\ & -2\ \sin\varphi\,\mathrm{Im}(a^{+-}_{\beta;ii'}\ \varrho^\alpha_P(1, i; -1, i'; L)) + a^{++}_{\beta;ii'}\ \varrho^\alpha_P(1, i; 1, i'; L) \Big]\,. \end{aligned} \quad (18)$$

The a-coefficients (*e.g.* $a^{--}_{\beta;ii'}$) are the functions of the energies and momenta of the particles in the detection process. They depend on the polar angle θ of the momentum of the β lepton but the dependence on the azimuthal angle φ is openly factorized out. Further investigations of the shape of the differential cross sections enables us to write the a-coefficients (see Eqs.(3) and (4)) as follows:

$$\begin{aligned} a^{--}_{\beta;ii'} &= A^L_{\varepsilon\varepsilon}|\varepsilon_L|^2 U^{L*}_{\beta i} U^L_{\beta i'} + A^L_{\varepsilon\eta}(\eta_L \varepsilon^*_L V^L_{\beta i} U^{L*}_{\beta i'} + h.c.) + A^L_{\eta\eta}|\eta_L|^2 V^L_{\beta i} V^{L*}_{\beta i'}\,, \\ a^{++}_{\beta;ii'} &= A^R_{\varepsilon\varepsilon}|\varepsilon_R|^2 U^{R*}_{\beta i} U^R_{\beta i'} + A^R_{\varepsilon\eta}(\eta_R \varepsilon^*_R V^R_{\beta i} U^{R*}_{\beta i'} + h.c.) + A^R_{\eta\eta}|\eta_R|^2 V^R_{\beta i} V^{R*}_{\beta i'}\,, \\ a^{+-}_{\beta;ii'} &= A^{RL}_{\varepsilon\eta}(\eta^*_L \varepsilon_R U^R_{\beta i} V^{L*}_{\beta i'} - \varepsilon^*_L \eta_R V^R_{\beta i} U^{L*}_{\beta i'}) + A^{RL}_{\varepsilon\varepsilon}\,\varepsilon^*_L \varepsilon_R U^R_{\beta i} U^{L*}_{\beta i'} = (a^{-+}_{\beta;i'i})^*\,. \end{aligned} \quad (19)$$

3. Density matrix evolution in a medium. The νSM regime

If the inclusion of the NP parameters is important it should be at first seen in the cumulative phenomenon of the long propagation in matter. Let $(W_{i\,\lambda;\,a}) \equiv (\langle\lambda,\, i|a\rangle)$ be the unitary transition matrix from the $|\lambda, i\rangle$ helicity-mass eigenbasis to $|a\rangle$ energy eigenbasis of the effective Hamiltonian (5), and E_a are the eigenvalues of $\mathcal{H}$. As the general NP structure of the effective Hamiltonian is non-diagonal hence $(W_{i\,\lambda;\,a})$ is not diagonal too, and it is clear that the evolution Eq. (8) mixes all entries of the original neutrino density matrix (9). Hence the evolution of the density matrix ϱ_P^α:

$$\begin{aligned}\varrho_P^\alpha(\lambda, i; \lambda', i'; L = T \neq 0) = &\sum_{n\sigma}\sum_{n'\sigma'}\sum_{a,b} W_{i\lambda;a}\, W^*_{n\sigma;a} \\ &\times \varrho_P^\alpha(\sigma, n; \sigma', n'; L = T = 0) e^{i(E_b - E_a)\,T}\, W_{n'\sigma';b}\, W^*_{i'\lambda';b}\,, \end{aligned} \tag{20}$$

is in this respect different in medium and vacuum, where $W_{i\,\lambda;\,a} = \delta_{\lambda,i;\,a}$. Clearly in vacuum, Eq. (20) has the form:

$$\varrho_P^\alpha(\lambda, i; \lambda', i'; L = T \neq 0) = \varrho_P^\alpha(\lambda, i; \lambda', i'; L = 0) e^{i(E_{\lambda,i} - E_{\lambda',i'})\,L}\,, \tag{21}$$

that does not mix ϱ_P^α entries. Hence for νSM in vacuum, Eq. (18) (after using Eq. (15)) reduces to the νSM, factorized, differential cross section formula:

$$\begin{aligned}\frac{d\sigma_{\beta\,\alpha}}{d\Omega_\beta} &= f_D\, A^L_{\varepsilon\varepsilon} \sum_{i;i'} U^L_{\beta i} U^{L*}_{\beta i'} U^{L*}_{\alpha i} U^L_{\alpha i'} e^{\,i\frac{m_i^2 - m_{i'}^2}{2\,E_\nu}\,L} \\ &\equiv \frac{d\sigma_\beta}{d\Omega_\beta}(m_i = 0) \times P_{\alpha\to\beta}(L)\,, \end{aligned} \tag{22}$$

and we recover the widely used factorization formula for the neutrino production rate. However, in the general case the full expression (18) for the final cross section does not factorize and it should be used in the future, more precise neutrino experiments, anywhere NP effects might be studied.

4. Summary and perspectives for the NP corrections and the νSM medium terms

It has been shown that for the NP Lagrangian (3) and (4), the neutrino production and detection states are in general mixed. We have presented the density matrix formalism which enables us to tackle with this sort of the NP phenomena. It has been shown that only for relativistic neutrino produced and detected by the LH mechanism the oscillation rate factorizes. We indicated that analyzed NP corrections may have the V$\pm$A or S$\pm$P origin and that the later one may lead to the tensorial spin-flip interactions.

The phenomenon is well known [3] although its significance has been mainly ignored as it is effective in the presence of the nonzero mean magnetization of the medium. Yet recent, systematically collected data on the magnetic field of the earth seem to support the view that part of it is connected with the correlated magnetized structures in the earth's crust [9]. If so, the spin-flip phenomenon induced by the possible existence of the S$-$P terms of the proposed NP should not be ignored. Searching on this phenomenon pushed us towards the necessity of the recalculation of the crust magnetization impact in the νSM regime also, as it appears in its V$\pm$A terms in the CC and NC interactions. The introductory calculations suggest that the effect can be meaningful [5], depending on the value of the mean magnetization. The problem is postpone to the forthcoming paper.

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