

## MEASUREMENTS OF CKM PARAMETERS AT THE $B$ FACTORIES\*

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We present selected constraints on the CKM quark mixing matrix elements from the  $B$  factories BaBar and Belle. In particular, we discuss the latest developments on  $|V_{ub}|$  from inclusive and exclusive  $b \rightarrow u$  decays and the constraints on the three angles of the unitarity triangle from various CP violation measurements. We conclude with a discussion of prospects for the future.

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### 1. Introduction

Aside from the neutrino sector, the CKM quark mixing matrix [1] is the only source of CP-violating phases in the Standard Model. The primary mission of the current  $B$  factories, BaBar [2] and Belle [3], is to test the Kobayashi–Maskawa mechanism for CP violation by measuring the magnitudes and relative phases of the elements of the CKM matrix. This is done with as many complementary measurements as possible. Close attention is paid to the reliability of theoretical calculations relating experimental observables to Standard Model parameters. Uncertainties from the strong interaction must be contained in this endeavor. This involves a great deal of interaction between experimentalists and theorists in devising strategies for finding the optimal balance between experimental statistical and systematic uncertainties and theoretical uncertainties.

We are now in the seventh year of data accumulation for both BaBar and Belle. Both experiments have logged more than  $500 \text{ fb}^{-1}$  of integrated luminosity<sup>1</sup>, which is an increase of two orders of magnitude over previous

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<sup>1</sup> The cross section for  $b\bar{b}$  production in  $e^+e^-$  collisions with  $\sqrt{s} = m_{\Upsilon(4S)}$  is about 1.1 nb. Every  $\text{fb}^{-1}$  of integrated luminosity yields about 1.1 million  $B\bar{B}$  pairs, half of which are  $B^+B^-$  and half of which are  $B^0\bar{B}^0$ .

$B$  factory experiments. The center-of-mass reference frame in PEP-II [4] and KEKB [5], the  $e^+e^-$  storage rings for the BaBar and Belle experiments, respectively, is boosted by using asymmetric beam energies in order to make the flight length difference of the two  $B$  mesons measurable, enabling proper-time-dependent CP violation measurements. This feature, combined with the huge increase in the size of the datasets, has allowed us to make significant progress on our primary mission.

The CKM matrix is often given in the useful Wolfenstein parametrization [6], which is an expansion in powers of the sign of the Cabibbo angle  $\lambda \approx 0.22$

$$V \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (1)$$

An interesting constraint from the unitarity of the CKM matrix comes from the first and third columns, namely

$$V_{ub}^*V_{ud} + V_{cb}^*V_{cd} + V_{tb}^*V_{td} = 0. \quad (2)$$

Dividing by  $|V_{cb}^*V_{cd}|$  and using the phase convention of Wolfenstein, this gives the so-called Unitarity Triangle (UT), which has a unit-length base along the real axis and an apex described by the point  $\bar{\rho} \equiv \rho(1 - \lambda^2/2) \approx \rho$ ,  $\bar{\eta} \equiv \eta(1 - \lambda^2/2) \approx \eta$ . The current experimental constraints on the point  $\bar{\rho}, \bar{\eta}$ , as computed by the CKMfitter collaboration [7], are shown in Fig. 1. From the impressive agreement of all of the constraints in Fig. 1, we can safely say that the CKM matrix is indeed the dominant source of CP violation in the Standard Model, excluding the neutrino sector. All measurements

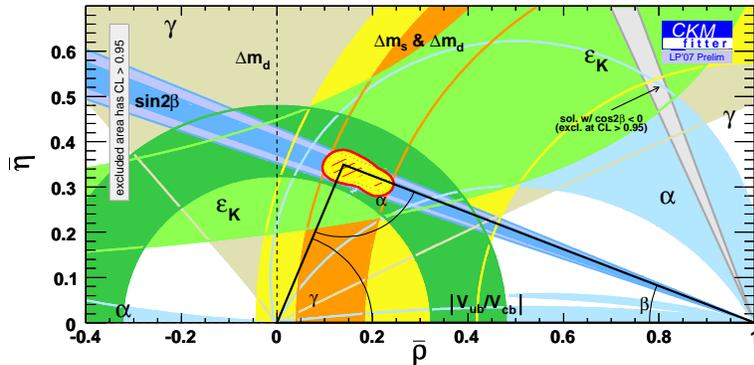


Fig. 1. Constraints on the apex of the Unitarity Triangle ( $\bar{\rho}, \bar{\eta}$ ) as of the Lepton-Photon '07 conference from the CKMfitter group [7].

to date are consistent with the CKM model, though there is still room for improvement on both the experimental and the theory sides. In the remainder of this note, I will summarize the current constraints on  $|V_{ub}|$  and the three angles of the Unitarity Triangle ( $\alpha$ ,  $\beta$ , and  $\gamma$  or  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$ , depending on which side of the pacific you are on). Much of the pedagogical discussion has been left out, due to the length restriction on the paper. Most of this discussion can be found in the PDG review articles on CP violation [8] and  $|V_{ub}|$  [9].

The second mission of the  $B$  factories is to search for physics beyond the Standard Model. This is done primarily with measurements involving rare decays or with searches for decays that are forbidden within the Standard Model but allowed in extension of the Standard Model. This topic is covered in the contribution from Rosenberg [10].

## 2. Measurements related to $|V_{ub}|$

To measure the magnitude of  $V_{ub}$ , one would like to isolate the  $b \rightarrow u$  weak charged-current transition. Unfortunately, the  $b$  and  $u$  quarks are confined within hadrons by the strong interaction, which is non-perturbative at low energies. This leads to “hadronic uncertainties” in relating our measured quantities to fundamental parameters, such as  $|V_{ub}|$ . Devising optimal methods for minimizing these hadronic uncertainties and the overall uncertainty on  $|V_{ub}|$  has been and still is an active area of research both on the experimental and theoretical sides.

In the inclusive approach, one attempts to measure the branching fraction for  $b \rightarrow u\ell\nu$ , summing over all hadronic final states. This is attractive from the theoretical side, since the *total* inclusive  $b \rightarrow u$  rate is straightforward to calculate. The main experimental challenge is to reject the mountain of background from the CKM-favored  $b \rightarrow c\ell\nu$  decays. This is accomplished by making cuts to select regions of phase space where the  $b \rightarrow u$  transition is strongly favored or the  $b \rightarrow c$  transition is kinematically forbidden. Extrapolating from the region of phase space where the measurement is made to the full spectrum is where the hadronic uncertainties enter. This extrapolation depends on understanding the Fermi motion of the  $b$  quark within the  $B$  meson. Heavy quark effective theory can be applied to this problem and the parameters of the shape function describing the Fermi motion can be experimentally determined [11]. An alternative is to use the photon energy spectrum in  $b \rightarrow s\gamma$  decays to determine the Fermi motion directly from data with minimal theoretical modeling [12].

In the exclusive approach, one measures the branching fraction for a specific final state, such as  $B \rightarrow \pi\ell\nu$ . Experimentally, better signal-to-background ratios are achieved, compared to the inclusive approach.

Extracting  $|V_{ub}|$  from an exclusive branching fraction requires knowing the shape and normalization of the  $B$  to  $f$  form factor. There has been some recent progress in this area from the BaBar collaboration. Using a novel loose neutrino reconstruction technique [13], they were able to measure the shape of the  $q^2$  spectrum for  $B \rightarrow \pi \ell \nu$  with enough precision to rule out one form factor model.

Fig. 2 shows various calculations of  $|V_{ub}|$  from inclusive and exclusive decays as performed by the Heavy Flavor Averaging Group [14]. In all cases, the theoretical uncertainties are greater than the experimental uncertainties. The differing theoretical frameworks for dealing with the shape function describing the Fermi motion of the  $b$  quark (top three points in left plot of Fig. 2) agree within errors. The more model independent approaches utilizing the  $b \rightarrow s\gamma$  photon energy spectrum (bottom four points in left plot of Fig. 2) are in agreement with the other techniques, though the uncertainties are still large at this point. The uncertainties in the  $|V_{ub}|$  determinations from  $B \rightarrow \pi \ell \nu$  (right plot of Fig. 2) are dominated by the uncertainties in the form factor calculations, though the different calculations yield very consistent results. The  $|V_{ub}|$  value from inclusive decays is somewhat higher than the value from  $B \rightarrow \pi \ell \nu$ , though the difference is less than two standard deviations.

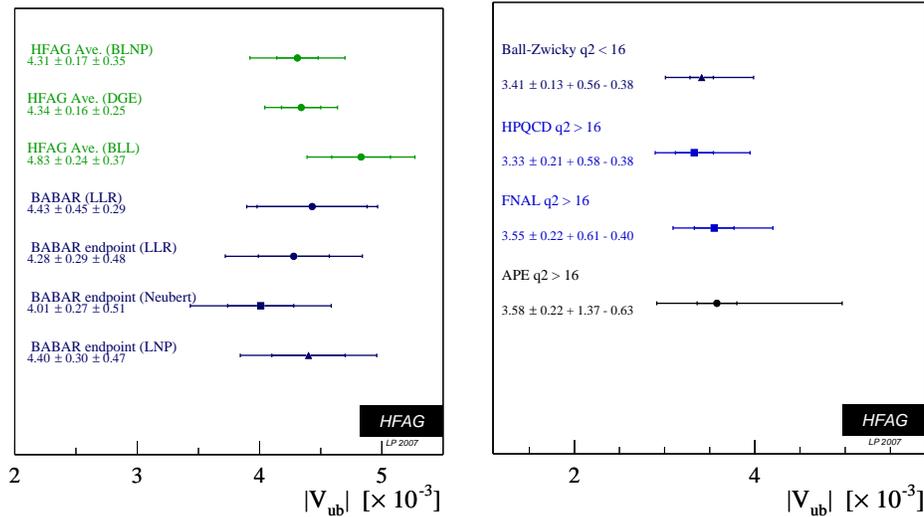


Fig. 2. Calculations of  $|V_{ub}|$  from inclusive  $b \rightarrow u \ell \nu$  decays (left) and exclusive  $B \rightarrow \pi \ell \nu$  as performed by the Heavy Flavor Averaging Group [14]. The theoretical frameworks for the inclusive determinations are described in references [12, 15–19]. The references for the form factor calculations are described in [20–23]. The inner error bars show the experimental uncertainty, while the full error bars show the experimental and theoretical uncertainties added in quadrature.

### 3. Measurements of $\sin 2\beta$ and $\beta$

The measurement of the angle  $\beta$  of the Unitarity Triangle is the single most powerful constraint on the apex  $(\bar{\rho}, \bar{\eta})$ . Relatively speaking, it is easy both experimentally (given the large  $B$  factory datasets) and theoretically. Since hadronic uncertainties mostly cancel in the CP asymmetries, the relationship between the measured asymmetry and  $\beta$  is very precise. The measurement of  $\sin 2\beta$  from  $B^0 \rightarrow (c\bar{c})K^0$  is the benchmark for time-dependent CP violation analysis at BaBar and Belle.

For  $B^0$  decays to final states that are CP eigenstates, the amplitudes for direct decay and decay after a  $B^0 \rightarrow \bar{B}^0$  flavor oscillation interfere. The proper-time dependent CP asymmetry for a CP eigenstate  $f$  is defined as

$$A_{\text{CP}}(f; t) \equiv \frac{N(\bar{B}^0(t) \rightarrow f) - N(B^0(t) \rightarrow f)}{N(\bar{B}^0(t) \rightarrow f) + N(B^0(t) \rightarrow f)}, \quad (3)$$

where notation  $\bar{B}^0(t) \rightarrow f$  indicates that the flavor of the  $B$  meson that decayed to  $f$  was known to be  $\bar{B}^0$  at a reference proper time  $t = 0$ . This can be written as

$$A_{\text{CP}}(f; \Delta t) = S_f \sin \Delta m_d \Delta t - C_f \cos \Delta m_d \Delta t, \quad (4)$$

with

$$S_f \equiv \frac{2 \text{Im} \lambda_f}{(1 + |\lambda_f|^2)}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{(1 + |\lambda_f|^2)}. \quad (5)$$

The parameter  $\lambda_f$ , in the Standard Model and the Wolfenstein phase convention, is given by  $\lambda_f \equiv e^{-i2\beta} \bar{A}_f/A_f$ , where  $A_f(\bar{A}_f)$  is the amplitude for the  $B^0$  ( $\bar{B}^0$ ) to decay to  $f$ . For  $B^0 \rightarrow J/\psi K_S^0$ , the color-suppressed, tree-level decay amplitude, proportional to  $V_{cb}^* V_{cs}$ , is dominant. Other decay amplitudes are both loop and CKM suppressed (by more than  $\approx 1/20$ ). If only a single decay amplitude is relevant for the final state  $f$ , the hadronic matrix element cancels in  $\lambda_f$  and the ratio  $\bar{A}_f/A_f$  is a pure phase. The final state  $J/\psi K_S^0$  is a CP-odd eigenstate and the CKM factors  $V_{cb}^* V_{cs}$  are real, so we have  $\lambda_{J/\psi K_S^0} = -1 e^{-i2\beta}$ , giving  $S_{J/\psi K_S^0} = \sin 2\beta$  and  $C_{J/\psi K_S^0} = 0$ , assuming a single dominant decay amplitude.

The latest results from the BaBar [24] and Belle [25] experiments for the cleanest, highest-statistics mode  $J/\psi K_S^0$  are at the 6% level and are in good agreement. The current statistical errors are more than double the systematic error, so these measurements will remain statistics limited with the ultimate  $B$  factory datasets. Averaging all measurements from charmonium  $K^0$  decays, which are all consistent, gives  $\sin 2\beta = 0.680 \pm 0.025$ .

One should keep in mind that long-distance corrections to the relation  $S = \sin 2\beta$  for charmonium  $K^0$  decays could be on the order of 0.017 and that the corrections could be mode dependent [26].

#### 4. Measurements related to $\alpha$

Determining the UT angle  $\alpha$  requires time-dependent CP asymmetry measurements in  $b \rightarrow u$  decays, such as  $B^0 \rightarrow \pi^+\pi^-$ . If the  $b \rightarrow u$  tree amplitude were the only decay amplitude for a hypothetical, CP-even final state  $f$ , we would have  $\lambda_f = e^{-i2\beta}$  ( $\bar{A}_f/A_f = e^{-i2\beta} e^{-i2\gamma} = e^{i2\alpha}$ ). This would give the expectation of  $S_f = \sin 2\alpha$ ,  $C_f = 0$ . Unfortunately, no such decay exists. The  $\pi^+\pi^-$  decay mode, for example, has both tree ( $T$ ) and 1-loop ‘‘penguin’’ ( $P$ ) amplitudes with different CKM phases that must be taken into account.

Both  $\delta_f$ , the CP-conserving phase difference between the tree and penguin amplitudes ( $T_f$  and  $P_f$ ) and the ratio of the two amplitudes  $|T_f/P_f|$  cannot be reliably calculated and are treated as unknowns that must be determined experimentally. The  $S_f$  coefficient is  $\sqrt{1 - C_f^2} \sin 2\alpha_{\text{eff}}$ , where  $\alpha_{\text{eff},f} = \alpha + \kappa_f$ . Gronau and London [27] proposed using isospin symmetry to disentangle the penguin and tree amplitudes. For the  $\pi^+\pi^-$  mode, this procedure requires measuring the decay rates of the three isospin-related decays ( $\pi^+\pi^-$ ,  $\pi^+\pi^0$ , and  $\pi^0\pi^0$ ) separately for  $B^0$  and  $\bar{B}^0$  decays.

The left plot of Fig. 3 shows the results of the most recent measurements of the time-dependent CP violation coefficients for  $B^0 \rightarrow \pi^+\pi^-$  ( $S_{\pi^+\pi^-}$  and  $C_{\pi^+\pi^-}$ ) from the BaBar [28] and Belle [29] experiments, both of which have more than 1000  $B^0 \rightarrow \pi^+\pi^-$  signal events. Both experiments see significant indirect CP violation ( $S_{\pi^+\pi^-} \neq 0$ ) indicating  $\alpha_{\text{eff}} \neq 0$ . There is some disagreement on  $C_{\pi^+\pi^-}$ , however, the disagreement has a statistical significance of less than  $3\sigma$  and is probably due to statistical fluctuations — the measurements from both experiments have been thoroughly validated. The fact that the average value of  $C_{\pi^+\pi^-}$  is inconsistent with zero is an indication of direct CP violation or CP violation in decay. Within the Standard Model, this would be from the interference of the tree and penguin decay amplitudes.

The same theoretical framework applies for the  $B^0 \rightarrow \rho\rho$  system. The  $\rho^+\rho^-$  mode is not a CP eigenstate, since the  $\rho$  has spin 1 and the final state can have  $L = 0, 1, 2$ , though it is effectively a CP eigenstate, since it turns out that the  $\rho\rho$  system is almost fully longitudinally polarized (thus CP even). There are two key differences in the isospin analysis of  $\pi\pi$  and  $\rho\rho$ . One is that the geometry of the isospin triangles is very different because  $\mathcal{B}(B^0 \rightarrow \pi^0\pi^0)/\mathcal{B}(B^0 \rightarrow \pi^+\pi^-) \approx 0.25$  and  $\mathcal{B}(B^0 \rightarrow \rho^0\rho^0)/\mathcal{B}(B^0 \rightarrow \rho^+\rho^-) \approx 0.036$ . The  $\rho\rho$  triangles are much more squashed, since the  $A^{00}$  and  $\bar{A}^{00}$  sides are

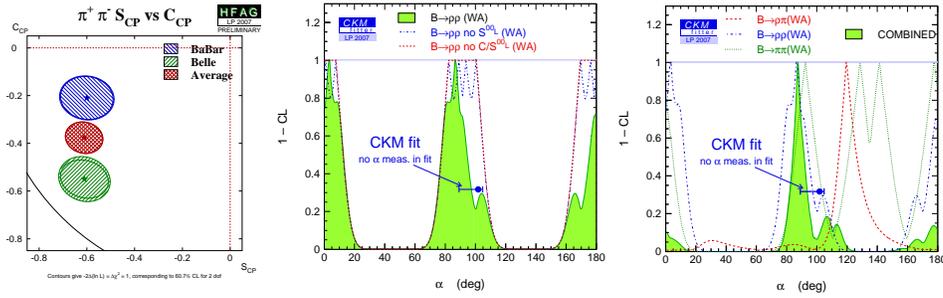


Fig. 3. The plot on the left shows the latest measurements of the time-dependent CP violation coefficients  $S$  and  $C$  for  $B^0 \rightarrow \pi^+\pi^-$ . Figure courtesy of the heavy flavor averaging group [14]. The middle and right plots show constraints on the Unitarity Triangle angle  $\alpha$  from the isospin analysis of the  $\rho\rho$  system (middle) and from  $\pi\pi$ ,  $\rho\rho$ , and  $\rho\pi$  combined. Figures courtesy of the CKMfitter group [7].

small, forcing  $\kappa = \alpha_{\text{eff}} - \alpha$  to also be small. The second key difference is that a time-dependent CP analysis of the  $\rho^0\rho^0$  mode is possible, since the  $\rho^0 \rightarrow \pi^+\pi^-$  decay vertex can be reconstructed (unlike  $\pi^0 \rightarrow \gamma\gamma$ ). A time-dependent analysis of  $\rho^0\rho^0$  measures  $S_{00} = \sqrt{1 - C_{00}^2} \sin 2\alpha_{\text{eff}}^{00}$  with  $\alpha_{\text{eff}}^{00} = \alpha + \kappa_{00}$ , where  $\kappa_{00}$  is the angle between the  $A^{00}$  and  $\bar{A}^{00}$  sides of the  $\rho\rho$  isospin triangles. This, combined with  $\alpha_{\text{eff}}^{+-} = \alpha + \kappa_{+-}$ , gives two independent measures of the relative orientation of the isospin triangles, so the four-fold discrete ambiguity can be broken.

The BaBar experiment recently released preliminary results of the first time-dependent CP analysis of  $B^0 \rightarrow \rho^+\rho^-$  [30]. This was done using only  $85 \pm 28 \pm 17$  signal events from analyzing 427 million  $B\bar{B}$  events. The large statistical and systematic errors on the signal yield are a reflection of the substantial amount of background. The analysis gives  $S_{00} = 0.5 \pm 0.9 \pm 0.2$  and  $C_{00} = 0.4 \pm 0.9 \pm 0.2$ . The interpretation of this measurement in the  $\rho\rho$  isospin analysis is shown in Fig. 3. One can see that the four solutions are close together (since  $\kappa$  must be small), and that the  $S^{00}$  breaks the four-fold ambiguity, although only weakly.

The combination of the  $\pi\pi$  and  $\rho\rho$  isospin analyses, in addition to the constraint from  $\rho\pi$  which I did not discuss, as performed by the CKMfitter group is shown in Fig. 3. There are only four  $\alpha$  solutions from  $\pi\pi$  because one of the isospin triangles is completely flat (zero area), which removes a factor of two in the number of solutions. One can see that the range favored by  $\rho\rho$  nicely overlaps with one of the  $\pi\pi$  solutions, largely resolving the discrete ambiguities. This also happens to be the solution consistent with indirect constraints on  $\alpha$ . Although not shown in Fig. 3, the areas near 0 and 180 degrees can be excluded using the branching fraction for  $B_s \rightarrow K^+K^-$  and SU(3) symmetry [31].

### 5. Measurements related to $\gamma$

The UT angle  $\gamma$  is the most difficult to measure, though it is the cleanest from a theory point of view. The task is to measure the CP-violating phase difference between the  $b \rightarrow u$  and  $b \rightarrow c$  transitions using CP violation in decay (or direct CP violation). The most straightforward tool for this is to use  $B^- \rightarrow D^0 K^-$  and  $B^- \rightarrow \bar{D}^0 K^-$ , where the former (latter) is a  $b \rightarrow c$  ( $b \rightarrow u$ ) transition. If the neutral  $D$  meson decays to a final state that can be reached by both the  $D^0$  and the  $\bar{D}^0$ , the two  $B$  decay amplitudes will interfere. Since the relative weak phase between the two  $B$  decay amplitudes ( $\gamma$ ) is CP-violating, the  $B^-$  and  $B^+$  decay rates will in general be different (a manifestation of direct CP violation). The relative size of the  $b \rightarrow u$  and  $b \rightarrow c$   $B$  decay amplitudes ( $r_b$ ) and the relative CP-conserving strong phase  $\delta_b$  are treated as unknowns that must be determined experimentally. The  $r_b$  parameter is expected to be between 0.1 and 0.2 due to CKM and color-suppression of the  $b \rightarrow u$  amplitude.

Many  $B \rightarrow DK$  methods for determining  $\gamma$  have been proposed [32–34] over the years, all of which can be combined to help determine  $r_b$ ,  $\delta_b$ , and  $\gamma$ . The most powerful method [34] was proposed relatively recently. The technique is to use a three-body  $D$  decay, such as  $D^0 \rightarrow K_S^0 \pi^+ \pi^-$  which has many intermediate resonances. The  $D$  decay amplitude structure is determined from a dedicated Dalitz analysis using  $D$  mesons that are flavor tagged by the pion charge in  $D^{*+} \rightarrow D^0 \pi^+$ . The direct CP violation in the  $B \rightarrow DK$  analysis can be large in areas of the  $D$  Dalitz plot where the  $B$  decay is suppressed and the  $D$  decay is favored or *vice versa*.

Table I gives the measured values for  $\gamma$  from the most recent BaBar [35] and Belle [36] measurements. The statistical uncertainty for  $\gamma$  is larger for the BaBar analysis, even though the size of the  $DK$  sample is comparable, because the BaBar data favor a smaller value of  $r_b$ .

TABLE I

The latest results on constraining  $\gamma$  with direct CP violation in  $B^+ \rightarrow DK^+$ . The BaBar (Belle) analysis uses the  $D^* K^+$  ( $D^* K^+$  and  $DK^{*+}$ ) mode(s) in addition to  $DK^+$ .

	Ref.	$N_{B\bar{B}}$	$N_{\text{sig}}(DK^\pm)$	$\gamma$
BaBar	[35]	347M	$398 \pm 23$	$(92 \pm 41 \pm 11 \pm 12)^\circ$
Belle	[36]	386M	$331 \pm 23$	$(53_{-18}^{+15} \pm 3 \pm 9)^\circ$

## 6. Summary and outlook

The results from the current  $B$  factories have established the CKM mechanism for CP violation within the Standard Model. The determination of  $|V_{ub}|$  is limited by hadronic uncertainties. The uncertainty is currently around 8%. The experimentalists and theorists hope to reach the 5% level with the analysis of the ultimate  $B$  factory datasets in the next couple of years. All measurements of the Unitarity Triangle angles, even  $\beta$ , are statistics limited (*not* systematics or theory limited). You may expect better than  $1/\sqrt{N_{B\bar{B}}}$  improvements in the ultimate  $B$  factory measurements, since we have a long history of improving our analyses with each iteration as we add more data.

Within the next two years, the LHC will be the flavor physics frontier, in addition to being the energy frontier, though the focus of the LHCb experiment will likely be on the  $B_s$  system and searches for new physics in  $B$  decays.

There are efforts underway to explore the feasibility of a so-called “Super B” factory [37], which would be an asymmetric-energy  $e^+e^-$  machine on the  $\Upsilon(4S)$ , like the current  $B$  factories, but with 100 times the instantaneous luminosity of the current machines!

Such a machine would have discovery potential for New Physics, complementary to the LHC, in addition to providing an enormous dataset which we can use to pinpoint the tip of the Unitarity Triangle.

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