# STATUS OF PRECISION MONTE CARLO TOOLS FOR LUMINOSITY MONITORING AT MESON FACTORIES*

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We review recent progress in precision calculations and the development of Monte Carlo generators for luminosity monitoring at meson factories. It is shown how the theoretical accuracy reached by presently used large-angle Bhabha tools at meson factories is at the 0.1% level and, therefore, comparable with that reached about a decade ago for luminosity monitoring through small-angle Bhabha scattering at LEP.

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## 1. Introduction

It is well known [1] that two important parameters for precision tests of the Standard Model, *i.e.* the anomalous magnetic moment of the muon $a_\mu \equiv (g-2)_\mu/2$ and the running electromagnetic coupling constant $\alpha(q^2)$, are affected by uncertainties that are totally dominated by hadronic contributions. The latter are not calculable with perturbative QCD at low virtualities and rely, therefore, on dispersion relations containing experimental data of the process $e^+e^- \to$ hadrons at low energies as input. It follows that more and more precise determinations of the hadronic contribution to $a_\mu$ and $\alpha(q^2)$ continuously require more and more accurate measurements of the hadronic cross section in $e^+e^-$ annihilation at meson factories operating in the energy region of hadronic resonances, such as DAΦNE, VEPP-2M, BEPC, CESR and the $B$-factories KEK-B and PEP-II.

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The precision measurement of the hadronic cross section requires, in turn, a detailed knowledge of the collider luminosity [2], which can be obtained at $e^+e^-$ accelerators by counting the number of events of a given reference process and normalizing this number to the corresponding theoretical cross section, as done, for example, with small-angle Bhabha scattering at LEP in the past. Because of the luminosity relation $\int \mathcal{L}\, dt = N_{\rm obs}/\sigma_{\rm th}$, the reference process must be a reaction with high statistics and calculable with high accuracy, to maintain small the total luminosity error given by the sum in quadrature of the relative experimental and theoretical uncertainty. It is important to emphasize that the comparison with the experimental data requires the development of Monte Carlo (MC) event generators, including radiative corrections at a high precision level of about 0.1%.

As a consequence of the features required for the reference process, it follows that the luminosity monitoring processes used at meson factories are the QED processes of $e^+e^-$ production (Bhabha scattering), two photon and muon pair production. At all the meson factories, the final-state products of the above reactions are detected at large scattering angles, because of the absence of dedicated luminosity counters, for example, at small scattering angles. In particular, at DAΦNE, VEPP-2M and PEP-II the large-angle Bhabha scattering is primarily used as luminosity monitoring process and the other reactions are employed as cross-checks, while at CESR all the three QED processes are considered and the luminosity is derived as an appropriate average of the measurements of the three channels. However, the large-angle Bhabha process is of major interest, because of its large cross section and particularly clean experimental signature.

## 2. Status of luminosity generators

The present status of Bhabha MC programs used for luminosity monitoring at meson factories is summarized in Table I. Considering the most precise and most widely used codes, the main theoretical features of the different generators are summarized in the following.

TABLE I

Status of MC generators for luminosity monitoring at meson factories.

| Generator | Processes | Theory | Accuracy |
|---|---|---|---|
| Bagenf [3] | $e^+e^-$ | $\mathcal{O}(\alpha)$ | 0.5% |
| BabaYaga v3.5 [4, 5] | $e^+e^-, \gamma\gamma, \mu^+\mu^-$ | Parton Shower | $0.5 \div 1\%$ |
| BabaYaga@NLO [6] | $e^+e^-, \gamma\gamma, \mu^+\mu^-$ | $\mathcal{O}(\alpha) + \mathrm{PS}$ | $\sim 0.1\%$ |
| MCGPJ [7] | $e^+e^-, \mu^+\mu^- ...$ | $\mathcal{O}(\alpha) + \mathrm{SF}$ | $< 0.2\%$ |
| BHWIDE [9] | $e^+e^-$ | $\mathcal{O}(\alpha)$ YFS | $\sim 0.5\%_{\rm (LEP1)}$ |

1. BabaYaga v3.5 — It is a MC generator developed by our group at the starting of DAΦNE operation [4] using a QED Parton Shower (PS) approach for the treatment of leading log QED corrections to luminosity processes and later improved according to Ref. [5] to account for the interference of radiation emitted by different charged legs in the generation of the momenta of the final-state particles. The main drawback of BabaYaga v3.5 is the absence of $\mathcal{O}(\alpha)$ non-log contributions, resulting in a theoretical precision of 0.5% for large-angle Bhabha scattering and of about 1% for $\gamma\gamma$ and $\mu^+\mu^-$ final states [4].

2. BabaYaga@NLO — It is the presently released version of BabaYaga, based on the matching of exact $\mathcal{O}(\alpha)$ corrections with QED PS, as described in detail in Ref. [6]. The accuracy of the current version is estimated to be at 0.1% level, as detailed in the following, for large-angle Bhabha scattering, two-photon and $\mu^+\mu^-$ [1] production.

3. MCGPJ — It is the generator developed by a Dubna–Novosibirsk collaboration [7] and used at VEPP collider. This program includes exact $\mathcal{O}(\alpha)$ corrections supplemented with higher-order leading logarithmic contributions related to the emission of collinear photon jets and taken into account through collinear QED Structure Functions (SF) [8]. The theoretical precision is estimated to be better than 0.2%.

4. BHWIDE — It is a MC code realized in Ref. [9] at the time of LEP operation. In this generator, exact $\mathcal{O}(\alpha)$ corrections are matched with the resummation of soft and collinear logarithms through the Yennie–Frautschi–Suura (YFS) exponentiation approach. According to the authors, the precision is estimated about 0.5% for LEP1. However, since the theoretical ingredients of BHWIDE are very similar to the formulation of both BabaYaga@NLO and MCGPJ, it is reasonable to assume that its theoretical accuracy for physics at meson factories is at the level of 0.1%.

Concerning the theoretical precision of the above generators, it is important to emphasize that the bulk of the most important sub-leading $\mathcal{O}(\alpha^2)$ corrections, namely $\alpha^2 L$ photonic contributions enhanced by infrared logarithms, where $L = \ln(Q^2/m^2)$ is the large collinear logarithm, is effectively incorporated in the tools BabaYaga@NLO, MCGPJ and BHWIDE by means of factorization of $\mathcal{O}(\alpha)$ non-log terms with the leading $\mathcal{O}(\alpha)$ contributions taken into account in the PS, collinear SF and YFS exponentiation approaches, as argued and demonstrated in Ref. [10].

[1] At present, finite mass effects in the virtual corrections to $e^+e^- \to \mu^+\mu^-$, which should be taken into for precision simulations at the $\Phi$-factories, are not included in BabaYaga@NLO.

## 3. Large-angle Bhabha scattering: numerical results

### *3.1. Size of radiative corrections*

To get an idea of which corrections are relevant to achieve a 0.1% theoretical precision in luminosity measurements at meson factories, we show in Table II the relative effect of various contributions to the large-angle Bhabha scattering cross section, when considering typical selection criteria at $\Phi$- (set up (a) and (b)) and $B$-factories (set up (c) and (d)) (see Ref. [6] for details).

TABLE II

Relative size of different sources of correction (in per cent) to the large-angle Bhabha scattering cross section at meson factories.

| Set up | (a) | (b) | (c) | (d) |
|---|---|---|---|---|
| $\delta_\alpha$ | $-13.06$ | $-17.16$ | $-19.10$ | $-24.35$ |
| $\delta_\alpha^{\text{non-log}}$ | $-0.39$ | $-0.66$ | $-0.41$ | $-0.70$ |
| $\delta_{\text{HO}}$ | 0.43 | 0.93 | 0.87 | 1.76 |
| $\delta_{\alpha^2 L}$ | 0.04 | 0.09 | 0.06 | 0.11 |
| $\delta_{\text{VP}}$ | 1.73 | 2.43 | 4.59 | 6.03 |

From Table II, it can be seen that $\mathcal{O}(\alpha)$ corrections decrease the Bhabha cross section of about 15% at the $\Phi$-factories and of about 20–25% at the $B$-factories. Within the full set of $\mathcal{O}(\alpha)$ corrections, non-log terms are of the order of 0.5%, almost independently of the centre-of-mass (c.m.) energy, as expected, and with a mild dependence from the angular acceptance cuts, as due to box/interference contributions. The effect of higher-order corrections due to multiple photon emission is about 0.5–1% at the $\Phi$-factories and reaches 1–2% at the $B$-factories. The contribution of (approximate) $\mathcal{O}(\alpha^2 L)$ corrections is not exceeding the 0.1% level, while the vacuum polarization increases the cross section of about 2% around 1 GeV and of about 5–6% around 10 GeV. Concerning the latter correction, the non-perturbative hadronic contribution to the running of $\alpha$ is included in BabaYaga@NLO both in the lowest-order and one-loop diagrams through the HADR5N routine [11], that returns a data-driven error, thus affecting the accuracy of the theoretical calculation, as discussed in the following.

As a whole, these results indicate that both exact $\mathcal{O}(\alpha)$ and higher-order corrections (including vacuum polarization) are necessary for 0.1% theoretical precision.

### *3.2. Tuned comparisons*

A typical procedure followed in the literature for establishing the technical precision of a given generator is to perform tuned comparisons between independent predictions, using the same set of experimental cuts. An example of such tuned comparisons is given in Ref. [6], where it is shown that the agreement between the predictions of BabaYaga@NLO and BHWIDE at the $\Phi$-factories is well below 0.1%, and that also the agreement between BabaYaga@NLO and LABSPV, which is a benchmark code by our group with a formulation based on collinear SF very similar to MCGPJ, is very good, below the 0.1% level. This level of agreement, together with further considerations about two-loop corrections discussed in the next section, is the reason why in the latest publication by KLOE Collaboration about the measurement of the hadronic cross section at DAΦNE [12] the relative uncertainty assigned to theory in the luminosity measurement is now 0.1%, resulting in a total luminosity error of 0.3%.

Similar comparisons have been recently performed between the results of BabaYaga@NLO and BHWIDE by Denig and Hafner [13] of BABAR collaboration, in the presence of realistic selection cuts for luminosity at PEP-II. Their studies show that the two generators agree at the 0.1% level, both at the level of integrated cross sections and differential distributions. On the other hand, the work presented in Ref. [7] shows that also the agreement between BHWIDE and MCGPJ is quite good, with relative differences at 0.1–0.2% level for the integrated Bhabha cross section as a function of the c.m. energy at VEPP-2M collider.

## 4. Estimate of the theoretical accuracy

In order to assess the physical precision of existing generators, the methods typically used are *(i)* to compare with $\mathcal{O}(\alpha^2)$ calculations, if the latter — as in the case of large-angle Bhabha scattering — are available in the literature [14–18], *(ii)* to estimate the size of unaccounted higher-order contributions.

Concerning point *(i)* and considering, for definiteness, the MC generator BabaYaga@NLO, the strategy consists in deriving from the general formulation the cross section expansion up to $\mathcal{O}(\alpha^2)$, which can be, in general, cast in the following form

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}\,, \tag{1}$$

where, in principle, each of the above $\mathcal{O}(\alpha^2)$ contributions is affected by an uncertainty, to be properly estimated. In Eq. (1), the first contribution is the cross section including $\mathcal{O}(\alpha^2)$ soft plus virtual corrections, whose uncertainty can be evaluated by comparison with the available next-to-next-to-leading order (NNLO) calculations. The $\sigma^{\alpha^2}_{\rm SV}$ of BabaYaga@NLO has

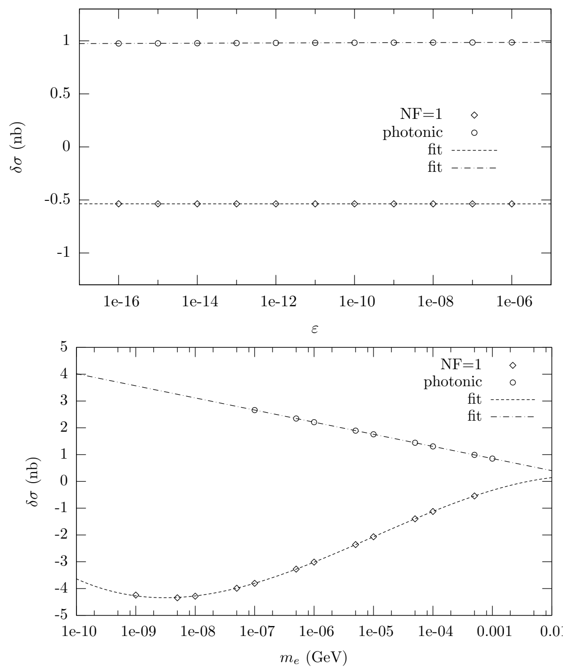


Fig. 1. Differences between the $\sigma_{\rm SV}^{\alpha^2}$ of BabaYaga@NLO and the $\mathcal{O}(\alpha^2)$ calculations of Ref. [16] (photonic) and of Ref. [17] ($N_F = 1$), as a function of the infrared regulator (top) and of a fictitious electron mass (bottom).

been compared, in particular, with the calculation by Penin [16], who computed the complete set of two-loop virtual photonic corrections in the limit $Q^2 \gg m_e^2$ supplemented by real soft-photon radiation up to non-logarithmic accuracy, and the calculations by Bonciani *et al.* [17], who computed two-loop fermionic corrections (in the one-family approximation) with finite mass terms and the addition of soft bremsstrahlung and real pair contributions. The results of such comparisons are shown in Fig. 1, for the set up (a) at the $\Phi$-factories. In the upper panel, $\delta\sigma$ is the difference between $\sigma_{\rm SV}^{\alpha^2}$ of BabaYaga@NLO and the cross sections of the two $\mathcal{O}(\alpha^2)$ calculations, denoted as photonic (Penin) and $N_F = 1$ (Bonciani *et al.*), as a function of the logarithm of the infrared regulator $\varepsilon$. It can be seen that the differences are given by flat functions, demonstrating that such differences are infrared-safe, as expected, as a consequence of the universality and factorization properties of the infrared divergences. In the lower panel, $\delta\sigma$ is shown as a function of the logarithm of a fictitious electron mass and for a fixed value of $\varepsilon = 10^{-5}$. Since the difference with the calculation by Penin is given by a straight line, this indicates that the soft plus virtual two-loop photonic corrections missing in BabaYaga@NLO are $\mathcal{O}(\alpha^2 L)$ not infrared-enhanced contributions. On the other hand, the difference with the calculation by

Bonciani *et al.* is fitted by a quadratic function, showing that the fermionic two-loop effects missing in BabaYaga@NLO are of the order of $\alpha^2 L^2$. It is important to emphasize that, as shown in detail in Ref. [6], the sum of the differences with the two $\mathcal{O}(\alpha^2)$ calculations does not exceed the $1.5 \times 10^{-4}$ level, for all the considered set up at $\Phi$- and $B$-factories. The second term in Eq. (1) is the cross section containing the one-loop corrections to single hard bremsstrahlung and its uncertainty can be estimated by relying on partial results existing in the literature. Actually, the exact perturbative expression of $\sigma^{\alpha^2}_{\mathrm{SV,H}}$ is not available yet for full $s+t$ Bhabha scattering, but, using the results valid for small-angle Bhabha scattering [19] and large-angle $s$-channel processes [20], the relative uncertainty of BabaYaga@NLO in the calculation of $\sigma^{\alpha^2}_{\mathrm{SV,H}}$ can be safely estimated at the level of 0.05% [6]. The third contribution in Eq. (1) is the double hard bremsstrahlung cross section, whose uncertainty can be evaluated by comparison with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section. As shown in Ref. [6], the differences registered between $\sigma^{\alpha^2}_{\mathrm{HH}}$ as in BabaYaga@NLO and the exact calculation are really negligible, at the $10^{-5}$ level.

Concerning point *(ii)*, the most important higher-order corrections still missing in BabaYaga@NLO are those due to light pairs radiation. Their effect has been evaluated by considering the leading contribution due to real pair radiation in soft approximation [21] in combination with virtual electron pair corrections [22] and found to be at the level of a few 0.01%, in agreement with previous studies about such a correction in small-angle Bhabha scattering at LEP.

Summing all the results for the various sources of uncertainty, it turns out that the total theoretical error in BabaYaga@NLO is at the 0.1% level, when also considering the uncertainty due to the running of $\alpha_{\mathrm{QED}}$ as returned by the HADR5N routine.

## 5. Conclusions

During the last few years, there has been a significant progress in reducing the theoretical uncertainty in the luminosity measurement at meson factories down to 0.1%. Exact $\mathcal{O}(\alpha)$ and multiple photon corrections are necessary ingredients to achieve such a precision. These corrections are implemented in three generators (BabaYaga@NLO, BHWIDE and MCGPJ) for large-angle Bhabha scattering, which agree within $\sim 0.1\%$ for integrated cross sections and $\sim 1\%$ (or better) for differential distributions.

NNLO QED calculations are important to establish the theoretical accuracy of existing generators and, if necessary, to improve it below 0.1%. In particular, the one-loop corrections to single hard bremsstrahlung should be calculated for full Bhabha scattering, to get a better control of the theoretical precision.

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