

CHIRAL LOW-ENERGY CONSTANTS: STATUS AND PROSPECTS*

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Chiral low-energy constants (LECs) carry the information of short-distance dynamics involving heavier degrees of freedom not present in the chiral Lagrangian. Our knowledge of the LECs is all-important at phenomenological level because their relevance in the prediction of hadronic observables at low-energies and, on the other side, because they provide hints on the construction of a dual theory of QCD in the low-energy regime. I review briefly the status of these important couplings.

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1. Introduction

Chiral symmetry of massless Quantum Chromodynamics has turned to be a key tool in order to deal with the low-energy domain of strong interactions (typically $E \sim M_\pi$) where hadron dynamics is not appropriately described by partonic QCD. It is indeed the guiding principle in the construction of Chiral Perturbation Theory (χ PT) [1–3] that intends to be the dual effective theory of QCD, *i.e.* it describes a perturbative quantum field theory of strong interactions at low-energies.

The χ PT Lagrangian involves a perturbative expansion guided by powers of external momenta (p) and light quark masses (m), with $p \sim m$, over a hadronic scale Λ_χ , driven by the loop expansion required by unitarity, p^2/Λ_χ^2 or the one that rules the lightest heavier degrees of freedom omitted in the Lagrangian, as the $\rho(770)$ mass, p^2/M_ρ^2 . It includes the hadronic fields that live in this energy region, the multiplet of Goldstone bosons (π , K , η), and classical auxiliary fields that help to determine Green functions satisfying the appropriate Ward identities. The theory can be systematically constructed

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by looking for all the operators that, with the ingredients above and up to a fixed order in the $\mathcal{O}(p^n)$ expansion, satisfy the chiral symmetry constraint [4]:

$$\mathcal{L}_{\chi\text{PT}} = \mathcal{L}_2^{\chi\text{PT}} + \mathcal{L}_4^{\chi\text{PT}} + \mathcal{L}_6^{\chi\text{PT}} + \dots \quad (1)$$

$\mathcal{L}_2^{\chi\text{PT}}$ embodies the spontaneous breaking of chiral symmetry and depends only on two parameters: $F \sim 92.4$ MeV, related with the decay constant of the pion, and $B_0 F^2 = -\langle 0 | \bar{\psi} \psi | 0 \rangle$, the vacuum expectation value of the light quarks¹.

Higher orders in the expansion bring in the information of the dynamics of short-distance contributions arising from heavier degrees of freedom that have been integrated out, for instance, resonance states. As in any effective field theory (EFT) this information is incorporated into the couplings of the operators:

$$\mathcal{L}_4^{\chi\text{PT}} = \sum_{i=1}^{10} L_i \mathcal{O}_i^{(4)}, \quad \mathcal{L}_6^{\chi\text{PT}} = \sum_{i=1}^{90} C_i \mathcal{O}_i^{(6)}, \quad (2)$$

for SU(3). Explicit expressions for the operators can be read from Refs. [3,5]. In Eq. (2) L_i and C_i are the chiral LECs at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ respectively; they are not provided by chiral symmetry. At present we do not know how to construct this Lagrangian directly from partonic QCD and, accordingly, we do not know how to determine the LECs from that framework. Hence we have to use our knowledge on the foundations of EFTs. LECs in χPT should receive contributions from the energy regime at or above the scale that rules the chiral perturbative expansion [6]. It is reasonable to infer that the dynamics of the lightest meson resonances in the hadronic spectrum, that are not included explicitly in the Chiral Lagrangian, would provide the larger contribution. Following this assumption the determination of the input of the resonance spectrum to the L_i LECs in $\mathcal{L}_4^{\chi\text{PT}}$ [7] showed that they indeed saturate the values extracted from phenomenological analyses. As a consequence, it is reasonable to think that the most important contribution to the LECs is provided by the energy region immediately above the integrated scale ($E \sim A_\chi \sim M_\rho$).

The determination of LECs is crucial for the predictability of low-energy hadronic observables determined using χPT . In the last ten years many of these processes have been evaluated up to $\mathcal{O}(p^6)$ ([8] and references therein) but our ignorance of the values of the involved LECs lessens the practical value of that enormous task. Therefore it is mandatory to explore procedures that allow us to determine or, at least, provide reliable estimates for the values of chiral LECs up to $\mathcal{O}(p^6)$.

¹ It happens that the loop expansion of the theory tells us that $A_\chi \sim 4\pi F$ that is of the same order than M_ρ . Hence there is only one scale in the perturbative expansion of the theory.

2. Tools: Large- N_C and QCD asymptotic constraints

As illustrated in the $\mathcal{O}(p^4)$ case [7, 9], a procedure to systematically disclose the structure of the resonance contributions to the LECs in χ PT is available. The key point is to construct a Lagrangian theory that includes resonances, Goldstone bosons and auxiliary fields respecting the underlying chiral symmetry. There are several tools that allow us to grab important properties of QCD and to implement them in an EFT-like Lagrangian model. Two relevant features to consider are:

- (i) Weinberg [1] and Leutwyler [10] state that if one writes down the most general Lagrangian, including all terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will be the most general possible S -matrix amplitude consistent with analyticity, perturbative unitarity, cluster decomposition and the principles of symmetry that have been required.
- (ii) The $1/N_C$ coupling (N_C is the number of colours in QCD) can be taken as a perturbative expansion parameter [11]. Indeed large- N_C QCD shows features that resemble, both qualitatively and quantitatively, the $N_C = 3$ case [12]. In practice the consequences of this approach are that meson dynamics in the large- N_C limit is described by tree diagrams of an effective local Lagrangian involving an infinite spectrum of zero-width mesons.

Both statements can be combined by constructing a Lagrangian in terms of SU(3) (Goldstone mesons) and U(3) (heavier resonances) flavour multiplets as explicit degrees of freedom respecting the underlying chiral symmetry. Then upon integration of the heavier states, the χ PT Lagrangian is to be recovered. This procedure has been systematically established [7, 9, 13] and devises what is known as Resonance Chiral Theory (R χ T). Its content is schematically given by:

$$\mathcal{L}_{\text{R}\chi\text{T}} = \mathcal{L}_2^{\chi\text{PT}} + \sum_n \mathcal{L}_{n>2}^{\text{GB}} + \mathcal{L}_{\text{R}}, \quad (3)$$

where $\mathcal{L}_{n>2}^{\text{GB}}$ has the same structure than $\mathcal{L}_4^{\chi\text{PT}}$, $\mathcal{L}_6^{\chi\text{PT}}$, ... in Eq. (2) though with different coupling constants, and $\mathcal{L}_{\text{R}}$ involves terms with resonances and their couplings to Goldstone modes:

- (1) The construction of the operators in the $\mathcal{L}_{\text{R}}$ is guided by chiral symmetry for the Goldstone mesons and by unitary symmetry for the resonances. The general structure of these couplings is:

$$\mathcal{O} = \langle R_1 R_2 \dots R_m \chi(p^n) \rangle \in \mathcal{L}_{(n)}^{\overbrace{RR\dots R}^m}, \quad (4)$$

where R_j indicates a resonance field and $\chi(p^n)$ is a chiral structured tensor, involving the pseudoscalar mesons and auxiliary fields only. With these settings chiral symmetry is preserved upon integration of the resonance fields and, at the same time, the low-energy behaviour of the amplitudes is guaranteed.

- (2) Symmetries do not provide information on the couplings in $\mathcal{L}_R$ as these incorporate short-distance dynamics not included explicitly in the Lagrangian. The latter is supposed to bridge between the energy region below resonances ($E \ll M_V$) and the parton regime ($E \gg M_V$). This hypothesis indicates that it should match both regions and it satisfies, by construction, the chiral constraints. To suit the high-energy behaviour one can match, for instance, the OPE of Green functions (that are order parameters of chiral symmetry) with the corresponding expressions evaluated within our theory. In addition, the asymptotic trend of form factors of QCD currents is estimated from the spectral structure of two-point functions and it is enforced on the observables. This heuristic strategy is well supported by the phenomenology [12–16] and provides information on the resonance couplings.

R χ T does not have an expansion parameter, hence it does not admit a conventional perturbative treatment. There is, of course, the guide provided by $1/N_C$ that translates into the loop expansion, however, there is no counting that limits the number of operators with resonances that have to be included in the initial Lagrangian. The number of resonance fields to be kept relies essentially in the physical system that we are interested in. Moreover, the maximum order of the chiral tensor $\chi(p^n)$ in Eq. (4) is constrained by the high-energy behaviour.

As commented above, large- N_C requires, already at $N_C \rightarrow \infty$, an infinite spectrum in order to match the leading QCD logarithms, though we do not know how to implement this in a model-independent way. The usual approach in R χ T is to include the lightest resonances because their phenomenological relevance, though there is no conceptual problem that prevents the addition of a finite number of multiplets. This cut in the spectrum may produce inconsistencies in the matching procedure outlined above [17]. To deal with this, one can include more multiplets that may delay the appearance of that problem.

3. Status and prospects

I comment briefly the present status and several developments on our knowledge of chiral LECs.

3.1. $\mathcal{O}(p^4)$

Contributions from the lightest multiplets of vector resonances ($J^{\text{PC}} = 1^{--}$) to SU(2) [2] and vector, axial-vector (1^{++}), scalar (0^{++}) and pseudoscalar (0^{-+}) resonances to SU(3) [7] LECs are in general good agreement with those values extracted from observables [18], showing that those resonances saturate the phenomenological values. This has been lately confirmed by the inclusion of one multiplet of resonances with 2^{++} and 1^{+-} quantum numbers [19] that are shown to play, quantitatively, a minor role.

A study of the strange quark mass dependence of the $\mathcal{O}(p^4)$ SU(2) LECs (ℓ_i^r) up to $\mathcal{O}(p^6)$ has also been recently performed [20]. The authors essentially obtain:

$$\ell_i^r = \delta_{i7} \frac{F^2}{8B_0 m_s} + a_i + b_i x + \mathcal{O}(x^2), \quad i = 1, \dots, 7, \quad (5)$$

where $x = M_K^2/(16\pi^2 F^2)$ and m_s is the mass of the strange quark. Here $a_i = a_i(L_j, l_K)$ arise at $\mathcal{O}(p^4)$ and $b_i = b_i(L_j, C_k, l_K)$ at $\mathcal{O}(p^6)$, with $l_K = \log(M_K^2/\mu^2)$. The dependence on the strange quark or kaon mass is explicitly stated above. In addition, and for later discussion, the b_i parameters can be written as $b_i = p_{0,i} + p_{1,i} l_K + p_{2,i} l_K^2$ with an explicit dependence $p_{0,i} = p_{0,i}(L_j L_k, C_l)$, $p_{1,i} = p_{1,i}(L_j)$.

3.2. $\mathcal{O}(p^6)$

In Ref. [13] we have constructed the R χ T Lagrangian needed to evaluate the resonance contributions to the $\mathcal{O}(p^6)$ LECs in Eq. (2). It can be shown [9] that, at $\mathcal{O}(p^4)$, all local terms in $\mathcal{L}_4^{\text{GB}}$ (see Eq. (3)) have to vanish in order not to spoil the asymptotic behaviour of QCD correlators. A corresponding result at $\mathcal{O}(p^6)$ is still lacking but we have also assumed that all the couplings in $\mathcal{L}_6^{\text{GB}}$ are set to zero.

$\mathcal{L}_{\text{R}\chi\text{T}}$ in Eq. (3) involves 124 *a priori* unknown couplings. Some additional work provides an enormous simplification:

- (i) Upon integration of resonances not all couplings appear independently in the LECs. In general only several combination of couplings intervene and, to take into account this case, one can perform suitable redefinitions of the fields. This procedure may upset the high-energy behaviour of the theory but it is correct for the evaluation of the LECs. Indeed, the unknown couplings are reduced to 77.

- (ii) The next step is to enforce short-distance information, *i.e.* the leading behaviour at large momenta, for two and three-point functions and form factors. This procedure, set in Ref. [9], relies on the well-known properties of partonic scattering or asymptotic QCD [21]. Two-current correlators and associated form-factors provide 19 new constraints on couplings, while the three-point Green functions studied till now: $\langle \text{VAP} \rangle$ [15] and $\langle \text{SPP} \rangle$ [13, 16], give 6 and 5 independent restrictions, respectively. We are left with 47 couplings. Further studies along this line can diminish even more number of unknown constants.

Notwithstanding, we can already determine fully, at this point, the resonance contribution to the $\mathcal{O}(p^6)$ couplings C_{78} and C_{89} (that appear in $\pi \rightarrow \ell \nu_\ell \gamma$ and $\pi \rightarrow \ell \nu_\ell \gamma^*$, respectively), C_{87} (in $\langle A_\mu A_\nu \rangle$), C_{88} and C_{90} (in $F_V^\pi(q^2)$) and the q^2 dependence of the form factors in $K_{\ell 3}$, C_{38} (in $\langle SS \rangle$) and C_{12} and C_{34} (in $F_V^{\pi, K}(q^2)$ and $f_+^{K^0 \pi^-}(0)$).

The evaluation of resonance contributions to C_j carried out in Ref. [13] can be termed as an $N_C \rightarrow \infty$ evaluation (with a cut spectrum). It is interesting to notice that when the values obtained for C_j (namely C_j^R) are substituted in the expressions obtained in Ref. [20] for $p_{0,i}$, the different combinations of C_j couplings vanish (except for $i = 7$) [22]. Something similar happens with the combinations of $L_i L_j$ products under additional conditions. The reason of these unexpected cancellations is still not understood and it would imply that, in the large- N_C limit, resonance saturation of $\mathcal{O}(p^4)$ SU(2) chiral LECs is already achieved at leading order. This would be very much satisfactory both for the $1/N_C$ expansion and for the determination of resonance contributions to chiral LECs. It is nice to have something left to understand for the future.

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