Vol. 38 (2007) *ACTA PHYSICA POLONICA B* No 11

RESUMMATION OF MASS DISTRIBUTIONS IN b DECAYS* **

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(Received October 23, 2007)

Results obtained by resumming threshold logarithms in the QCD form factor are presented for the $B \to X_c + l + \nu_l$ decays to next-to-leading logarithmic accuracy. An interpolation formula is presented by including soft and collinear effects for a non-vanishing charm mass.

PACS numbers: 11.15.Bt, 12.38.Gc, 13.20.He

1. Introduction

Distributions in semileptonic B decays

$$B \,\to\, X_c \,+\, l \,+\, \nu_l \,, \tag{1}$$

such as for example the hadron-mass event fraction, often receive logarithmic contributions of infrared origin of the form

$$\alpha_S^n \, \log^k \left(\frac{m_{X_c}^2 - m_c^2}{m_b^2 - m_c^2} \right) \log^l \left(\frac{m_{X_c}^2}{m_b^2} \right) \,, \tag{2}$$

where m_{X_c} is the final hadron mass and α_S is the QCD coupling evaluated at the hard scale. Even in weak coupling regime $\alpha_S \ll 1$, the terms above tend to spoil the convergence of the perturbative series in the threshold region, the latter being defined as the one having parametrically

$$m_{X_c} \,\ll\, m_b \,. \tag{3}$$

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* Presented at the XXXI International Conference of Theoretical Physics, "Matter to the Deepest", Ustroń, Poland, September 5–11, 2007.

** Work done in collaboration with U. Aglietti, L. Di Giustino, G. Ferrera, A. Renzaglia and G. Ricciardi.

The large infrared logarithms are a result of an incomplete cancellation between real corrections to the decay distributions, integrated on the border of the phase space, and virtual ones. An all-order resummation is needed in order to have a reliable description of the spectra in the whole kinematical range. The infrared logarithms coming from soft-gluon emission are of the form $\log\left(\frac{m_{X_c}^2 - m_c^2}{m_b^2 - m_c^2}\right)$ and formally diverge for $m_{X_c} \to m_c^+$. Since there is at most one soft logarithm for each gluon emission, *i.e.* for each power of α_S, it follows that $k \leq n$ in Eq. (2). The infrared logarithms of collinear origin $\log\left(\frac{m_{X_c}^2}{m_b^2}\right)$ never become infinite but may become large in the threshold region (3). Since there are at most two infrared logarithms for each power of α_S, $k + l \leq 2\,n$ in Eq. (2).

Our main result is the following expression for the resummed QCD form factor in N or Mellin space [1]:

$$
\begin{aligned}
\sigma_N(\rho, Q^2) = \; & \exp \int_0^1 dy \Big[(1-y)^{N-1} - 1\Big] \Bigg\{ \frac{1}{y} \int_{\frac{Q^2\,y^2}{1+\rho}}^{\frac{Q^2\,y^2}{y+\rho}} A\left[\rho, \alpha_S(k^2)\right] \frac{dk^2}{k^2} \\
& + \frac{1}{y} D\left[\alpha_S\left(\frac{Q^2\,y^2}{1+\rho}\right)\right] + \left(\frac{1}{y} - \frac{1}{y+\rho}\right) \Delta\left[\alpha_S\left(\frac{Q^2 y^2}{y+\rho}\right)\right] \\
& + \frac{1}{y+\rho} B\left[\alpha_S\left(\frac{Q^2 y^2}{y+\rho}\right)\right] \Bigg\} . \qquad (4)
\end{aligned}
$$

We have defined the hadron variable with unitary range $y \equiv \frac{m_{X_c}^2 - m_c^2}{Q^2 - m_c^2}$. which equals zero in the Born kinematics, and the mass-correction parameter

$$
\rho \equiv \frac{m_c^2}{Q^2 - m_c^2}\,, \qquad (5)
$$

where Q, the hard scale in the heavy flavor decay, is given by $Q \equiv E_{X_c} + |\vec{p}_{X_c}|$ with E_{X_c} and $\vec{p}_{X_c}$ the total energy and 3-momentum of the final hadron state X_c. The function $A(\rho, \alpha_S)$ has an expansion in power of α_S,

$$
A\left(\rho, \alpha_S\right) = \sum_{n=1}^{\infty} A^{(n)}(\rho) \alpha_S^n \,, \qquad (6)
$$

and describes soft radiation collinearly enhanced off the charm quark; it reduces to the standard double-logarithmic function $A(\alpha_S)$ in the massless limit:

$$
A(\rho, \alpha_S) \to A(\alpha_S) \qquad \text{for} \qquad \rho \to 0^+ \,. \qquad (7)
$$

The first-order coefficient is:

$$A^{(1)}(\rho) \equiv \frac{C_{\rm F}}{\pi}\,(1+2\rho)\ , \tag{8}$$

where $C_{\rm F} = (N_c^2-1)/(2N_c) = 4/3$ for $N_c = 3$. The second-order coefficient $A^{(2)}(\rho)$ (as well as the third-order one $A^{(3)}(\rho)$) is only known in the massless limit. The function $B(\alpha_S)$ is the standard massless jet-function, describing hard collinear emission off the charm quark, while $D(\alpha_S)$ and $\Delta(\alpha_S)$ describe soft radiation not collinearly enhanced off the beauty and charm quarks respectively. All these functions have a perturbative expansion in α_S. For a compilation of the massless coefficients see [3], while for the heavy flavor case [4].

$$B(\alpha_S) = \sum_{n=1}^{\infty} B^{(n)}\alpha_S^n\,, \quad D\,(\alpha_S) = \sum_{n=1}^{\infty} D^{(n)}\alpha_{\rm S}^n\,, \quad \Delta\,(\alpha_S) = \sum_{n=1}^{\infty} \Delta^{(n)}\alpha_S^n\,. \tag{9}$$

The $\mathcal{O}(\alpha_S)$ coefficients read:

$$B^{(1)} = -\frac{3}{4}\frac{C_{\rm F}}{\pi}\,, \qquad D^{(1)} = -\frac{C_{\rm F}}{\pi}\,, \qquad \Delta^{(1)} = -\frac{C_{\rm F}}{\pi}\,. \tag{10}$$

The form factor (4) aims at describing the three different dynamical regions in the semileptonic decay (1), which are identified by the value of the (ordinary) charm velocity $u_c = p_c/E_c$. Without loosing in generality we can work in the beauty rest frame. It holds $\rho \simeq \frac{1-u_c}{2u_c}$.
These regions are:

(i) **Very slow charm quark**: $u_c \gtrsim 0$ or, equivalently, $\rho \gg 1$. For $u_c = 0$ there is no *soft-gluon* emission to any order in perturbation theory [5]. By "soft" we mean a gluon with energy $E_g < m_c, m_b$. That is a coherence effect, namely destructive interference between emission off the initial and the final state. The physical picture is the following. The b decays into the c at rest and with the same color state of the b. Soft gluons "view" the b as well as the c as static color charges, and therefore do not see any acceleration or color-spin flip, *i.e.* any change occurring in the decay. As a consequence, there is no radiation. Furthermore, for small u_c radiation is proportional to u_c^2, *i.e.* first order terms $\mathcal{O}(u_c)$ vanish [6]. Radiative corrections to our form factor vanish in the no-recoil point: $\sigma_N(\rho, Q^2) \to 1$ for $\rho \to +\infty$, implying that this region is correctly described by the form factor.

(ii) **Non-relativistic charm quark**: $u_c \approx \frac{1}{3}$ or $\rho \approx 1$. The final state X_c contains the final charm quark together with soft gluons emitted at any angle with respect to it, because there is a significant soft emission.

Since the QCD matrix elements do not have any collinear enhancement, X_c does not have a jet structure. In the threshold region $y \ll 1$, so that $y \ll \rho$ and the form factor (4) simplifies into:

$$\sigma_{S,N}(\rho, Q^2) = \exp \int_0^1 \frac{dy}{y}\Big[(1-y)^{N-1} - 1\Big]\Bigg\{ \int_{\frac{Q^2y^2}{1+\rho}}^{\frac{Q^2y^2}{\rho}} A\left[\rho, \alpha_S(k^2)\right] \frac{dk^2}{k^2} + D\left[\alpha_S\left(\frac{Q^2y^2}{1+\rho}\right)\right] + \Delta\left[\alpha_S\left(\frac{Q^2\,y^2}{\rho}\right)\right]\Bigg\}. \tag{11}$$

(iii) **Fast charm quark**: $u_c \lesssim 1$ or $\rho \ll 1$. Soft gluons are mostly radiated at small angle with respect to the charm quark and there is a jet structure of the final state. For $\rho \to 0^+$ the form factor (4) reduces to the standard massless expression [4, 7–9]:

$$\sigma_N(0, Q^2) = \exp \int_0^1 \frac{dy}{y}\Big[(1-y)^{N-1} - 1\Big]\Bigg\{ \int_{Q^2y^2}^{Q^2y} A\left[\alpha_S(k^2)\right] \frac{dk^2}{k^2} + D\left[\alpha_S\left(Q^2y^2\right)\right] + B\left[\alpha_S\left(Q^2y\right)\right]\Bigg\}, \tag{12}$$

where:

$$y \to u = \frac{E_X - p_X}{E_X + p_X} \simeq \frac{m_X^2}{4E_X^2}\,, \qquad Q \to E_X + p_X \simeq 2E_X\,. \tag{13}$$

On the last member we have expanded for $m_X \ll E_X$. The inclusion of first-order corrections in ρ on the r.h.s. of Eq. (12) basically amounts to take into account the dead-cone effect in gluon radiation off the charm [2]. The form factor factorizes into the product of the form factor for a massless charm times the universal mass jet correction:

$$\sigma_N(\rho, Q^2) \simeq \sigma_N(0, Q^2)\delta_N(\rho, Q^2) \qquad \text{for} \qquad \rho \ll 1\,, \tag{14}$$

where

$$\delta_N(\rho, Q^2) = \exp \int_0^1 dy \frac{(1-y)^{\rho(N-1)} - 1}{y}\Bigg\{ - \int_{\rho Q^2y^2}^{\rho Q^2 y} \frac{dk_\perp^2}{k_\perp^2} A\left[\alpha\left(k_\perp^2\right)\right] - B\left[\alpha\left(\rho Q^2 y\right)\right] + D\left[\alpha\left(\rho Q^2y^2\right)\right]\Bigg\}. \tag{15}$$

Let us note that in [2] the *universal* correction $\delta_N(\rho, Q^2)$ was found for $\rho \ll 1$, while in this work we consider a *specific* process for *any* ρ.

Since the ratio $m_c/m_b \approx 1/3 \div 1/4$ in the real world, *i.e.* it is not so small, region *(iii)* is tiny. With typical values of the on-shell masses, $m_b = 4.7$ GeV and $m_c = m_b - m_B + m_D \simeq 1.29$ GeV [10], one has a maximal charm velocity $u_{c\,\max} \simeq 0.86$ corresponding to a Lorentz factor $\gamma_{c\,\max} \simeq 2$, or $0.081 \lesssim \rho \leq \infty$. With hadron kinematics, *i.e.* with $m_b = m_B$ and $m_c = m_D$, one obtains instead: $u_{c\,\max} = 0.78$, $\gamma_{c\,\max} \simeq 1.6$ and $\rho_{\min} \simeq 0.14$. Since the y and k^2 integrals on the r.h.s. of Eq. (4) extend down to zero, one hits the infrared singularity in the QCD couplings $\alpha_S(k^2)$ and $\alpha_S(\ldots y^2)$ — the well-known Landau pole: some prescription is needed in order to render the integrands well behaved. As it is usually the case with resummation formulae, Eq. (4), as it stands, has an "algebraic" sense: upon expansion in powers of α_S, it allows to predict the logarithmic corrections which would be explicitly found in higher-order Feynman diagram computations.

In general, we expect less radiation to be emitted in the decay (1) because of the rather large charm mass compared to the charmless channel $B \to X_u + l + \nu_l$. As a consequence, the typical Sudakov effects, namely suppression of non-radiative channels and broadening of sharp structures, are expected to be less pronounced for our process. In principle, for the decay (1), one has single-logarithmic corrections, which are not strong enough to shift the peak of tree-level distributions. For $y \to 0^+$ they produce indeed a form factor of the form

$$\sigma(y) \approx \frac{d}{dy} e^{-\alpha_S \log \frac{1}{y}} = \frac{\alpha_S}{y^{1-\alpha_S}}\,, \tag{16}$$

which still has an (infinite) peak in $y = 0$, like the lowest-order form factor $\sigma^{(0)}(y) = \delta(y)$. That is to be contrasted with a double-logarithmic form factor $\approx d/dy \exp(-\alpha_S \log^2 y)$. By looking at the y-integration on the r.h.s. of Eq. (4), one finds that the threshold region cannot be described within a "pure" perturbative framework when the QCD coupling comes close to the infrared Landau singularity, namely when

$$\frac{Q^2 y^2}{1+\rho} \approx \Lambda_{\rm QCD}^2\,, \tag{17}$$

where $\Lambda_{\rm QCD}$ is the QCD scale, *i.e.* when the final hadron mass becomes as small as[1]

$$m_{X_c}^2\Big|_{\rm NP} \approx m_c^2 + \Lambda_{\rm QCD}\sqrt{Q^2 - m_c^2}\,. \tag{18}$$

These effects are related to soft interactions only because hard collinear terms ($\propto B(\alpha_S)$) are controlled by a smaller coupling, evaluated at the larger scale $Q^2y^2/(y+\rho)$. These soft interactions are non-perturbative in

[1] For $Q = m_b$ and $\Lambda_{\rm QCD} = 300$ MeV one obtains for the current values of the b and c masses $m_X\Big|_{\rm NP} \simeq 1.7$ GeV.

region (18) and can be factorized into the shape function for a massive final quark [11]. Note that for $m_c \to 0$, the slice (18) reduces to the well-known one $m_X^2 \approx \Lambda_{\rm QCD} Q$. In region (18), also soft interactions between the b quark and the light degrees of freedom in the B-meson, namely the valence quark, are important. That is the well-known Fermi-motion, characterized by momentum exchanges k_μ in the B-meson of the order of the hadronic scale, $|k_\mu| \approx \Lambda_{\rm QCD}$. Physical intuition would suggest that Fermi motion is related to the initial B-meson state only, *i.e.* that it is independent on the mass of the final quark. In quantum field theory that is not true and the light quark mass enters into it. The resummation is valid for any chosen value of the ratio $m_c/m_b \in [0,1]$ and reduces to the standard next-to-leading logarithmic approximation for $m_c \to 0$. Coherence effects, occurring for a small relative velocity of the c and b quarks and leading to destructive soft-gluon interference, are correctly incorporated in the form factor, whose corrections vanish in the no-recoil point. Our form factor is basically a smooth interpolation of a *soft* resummation and a *soft + collinear* one and constitutes a consistent description of a multi-scale process like semileptonic $b \to c$ decay; such interpolation is not unique and has been constructed on the basis of simplicity's requirements. The resummation formula in Eq. (4) has been derived form "first principles", namely from the universal properties of QCD radiation.

I would like to thank the Organizers for inviting me to this lively Conference. This work has been carried out in part under the EU Contract No. MRTN-CT-2006-035482 FLAVIAnet and under the INTAS project 05-1000008-8328.

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