PROGRESS IN THE EVALUATION OF THE $\overline{B} \to X_S\gamma$ DECAY RATE AT NNLO* **

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The theoretical status of NNLO QCD corrections to the inclusive radiative $\overline{B} \to X_s\gamma$ decay in the standard model is briefly overviewed. Emphasis is put on recent results for three-loop fermionic corrections to matrix elements of the most relevant four-quark operators.

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1. Introduction

The inclusive $\overline{B} \to X_s\gamma$ decay mode, a flavor-changing-neutral-current process and therefore loop-suppressed in the standard model (SM), is known to be a sensitive probe of new physics. Obviously, deriving constraints on the parameter space of physics beyond the SM relies strongly on both accurate measurements and precise theory predictions within the SM.

Combining measurements of BaBar, Belle and CLEO [1], the current world average for the branching ratio with a cut $E_{\gamma,0} > 1.6\,\mathrm{GeV}$ on the photon energy in the $\overline{B}$-meson rest frame reads [2]

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\mathrm{exp}}_{E_\gamma > 1.6\,\mathrm{GeV}} = \left(3.55 \pm 0.24 \,{}^{+0.09}_{-0.10} \pm 0.03\right) \times 10^{-4}\,, \qquad (1)$$

where the first uncertainty corresponds to a combined statistical and systematic error, the second one is due to the theory input in the extrapolation of the measured branching ratio to the reference value $E_{\gamma,0}$, whereas the third one is connected to the subtraction of $b \to d\gamma$ events. The overall error of the world average amounts to about 7% which is comparable with

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the expected size of next-to-next-to-leading order (NNLO) QCD effects to the perturbative transition $b \to X_s^{\rm parton}\gamma$. Thus, a complete SM calculation at this accuracy level is desired.

To a large extent, the NNLO program has been finished and the latest theoretical estimate based on the results [3]

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm theor}_{E_\gamma > 1.6\,{\rm GeV}} = (3.15 \pm 0.23) \times 10^{-4} \tag{2}$$

is in good agreement with the experimental value Eq. (1). Here, the error consists of four types of uncertainties added in quadrature: non-perturbative (5%), parametric (3%), higher-order (3%) and m_c-interpolation ambiguity (3%).

2. The effective theory framework

The partonic decay width $\Gamma(b \to s\gamma)$ receives large contributions of logarithms $\log M_{\rm W}^2/m_b^2$. Resumming them at each order of α_s is most suitably done in the framework of an effective low-energy theory with five active quarks by integrating out the top and heavy electroweak fields. The relevant effective Lagrangian is given by

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD\times QED}(u,d,s,c,b) + \frac{4G_{\rm F}}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu)\, Q_i(\mu)\,. \tag{3}$$

The usual QED-QCD Lagrangian for the light SM fields is stated in the first term whereas the second term gives the local operator product expansion (OPE) with Wilson coefficients $C_i(\mu)$ and operators $Q_i(\mu)$ up to dimension six built out of the light fields. V_{ij} denotes elements of the Cabibbo–Kobayashi–Maskawa matrix and $G_{\rm F}$ the Fermi coupling constant.

The operator basis reads

$$\begin{aligned}
Q_{1,2} &= (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b)\,, \\
Q_{3,4,5,6} &= (\bar{s}\Gamma_i b)\sum\nolimits_q (\bar{q}\Gamma_i' q)\,, \\
Q_7 &= \frac{e}{16\pi^2}\,\overline{m}_b(\mu)\,(\bar{s}_L\sigma^{\mu\nu} b_R)\,F_{\mu\nu}\,, \\
Q_8 &= \frac{g}{16\pi^2}\,\overline{m}_b(\mu)\,(\bar{s}_L\sigma^{\mu\nu} T^a b_R)\,G^a_{\mu\nu}\,,
\end{aligned} \tag{4}$$

where Γ and Γ' represent various products of Dirac and color matrices. $\overline{m}_b(\mu)$ is the bottom mass in the $\overline{MS}$ scheme and the sum runs over all light quark flavours q.

Consistent calculations of $\Gamma(b \to s\gamma)$ in the effective framework are performed in three steps. The Wilson coefficients $C_i(\mu_0)$ $\mu_0 \approx M_{\rm W}$ are first determined at the electroweak scale by requiring equality of Green's functions in the effective and full theory at leading order in (external momenta)/$M_{\rm W}$. Subsequently, the operator mixing under renormalization is computed by evolving $C_i(\mu)$ from μ_0 down to the low-energy scale μ_b with help of effective theory Renormalization Group Equations (RGE). Finally, the matrix elements with single insertions of effective operators are computed. Non-perturbative effects appear only as small corrections to the last step, which is connected to the heaviness of the bottom quark and the inclusiveness of the $\overline{B} \to X_s\gamma$ decay mode.

As far as the next-to-leading order precision is concerned, this program has been completed already a few years ago, thanks to the joint effort of many groups (see for Eg. [4, 5] and references therein). The NNLO calculation, which involves hundreds of three-loop on-shell vertex-diagrams and thousands of four-loop tadpole-diagrams, is a very complicated task and, as already mentioned in the introduction, large parts have already been finished.

Matching the four-quark operators $Q_1, ..., Q_6$ and the dipole operators Q_7 and Q_8 at the two- and three-loop level, respectively, has been performed in [6, 7]. The three-loop renormalization in the $\{Q_1, \ldots, Q_6\}$ and $\{Q_7, Q_8\}$ sectors was found in [8,9], and results for the four-loop mixing of $Q_1, \ldots, Q_6$ into Q_7 and Q_8 were lately provided in [10] completing the anomalous-dimension matrix. The two-loop matrix element of the photonic dipole operator Q_7 was found, together with the corresponding bremsstrahlung, in [11, 12] and confirmed in [13]. Moreover, contributions of the dominant operators in the so-called large-β_0 approximation ($\mathcal{O}\left(\alpha_s^2\beta_0\right)$) to the photon energy spectrum have been computed in [14]. Three-loop matrix elements of the operators Q_1 and Q_2 at $\mathcal{O}\left(\alpha_s^2\beta_0\right)$ and two-loop matrix elements of Q_7 and Q_8 were found in [15] as expansions in the quark mass ratio m_c^2/m_b^2. Recently, we confirmed the findings of [15] on the matrix elements of $Q_{1,2}$ and were able to extend the calculation beyond the large-β_0 approximation by evaluating the full fermionic contributions [16]. This calculation is briefly reviewed below. Furthermore, in [17], the full matrix elements of Q_1 and Q_2 have been computed in the large m_c limit, $m_c \gg m_b$, and subsequently used to perform an interpolation to the physical region assuming that the large-β_0 part is a good approximation at $m_c = 0$. This is the source of the interpolation ambiguity mentioned beneath Eq. (2).

3. NNLO fermionic corrections to the matrix elements of $Q_{1,2}$

Matrix elements of Q_1 and Q_2 constitute a crucial input for the accuracy of the current NNLO estimate Eq. (2). The intention of our recent work [16] was the determination of full fermionic corrections to these matrix elements to cross-check the results of [15] and, at the same time, to test the validity of the massless approximation used in the large-β_0 approximation. Our calculation is based on two different techniques, that were applied to the master integrals obtained from IBP reduction. In the case of massless quark loop insertions into the gluon propagator of the relevant NLO diagrams, all integrals have been performed using the Mellin–Barnes (MB) method. The MB representations were derived using an automated package [18] and analytically continued with help of the `MB` package [19]. After expanding in $z = m_c^2/m_b^2$ the resulting coefficients represented as series over residues could be resummed with `XSummer` [20]. In addition, an exact solution through direct numerical integration keeping the full z-dependence was obtained. This procedure was not applicable in the case of massive quark loop insertions due to poor convergence. Instead, the method of differential equations as a second approach was utilized. Using the fact that the master integrals $V_i(z,\epsilon)$ (after rescaling by a trivial factor) are functions of ϵ and the mass ratio z^{-1} a system of differential equations has been generated where the right-hand side was again expressed through master integrals with the help of relations obtained from the reduction. The solution of this system for arbitrary values of y proceeded in two steps. First, an expansion in ε and y for ϵ, $y \to 0$ was performed and the coefficients were calculated recursively up to high powers of y [21]. Using the resulting high precision values at a starting point $y \ll 1$, the unexpanded system was integrated numerically up to physical values of y with help of the Fortran package ODEPACK [22]. The path was shifted into the complex plane to avoid special points. Figure 1 shows the resulting data points together with fits for the renormalized

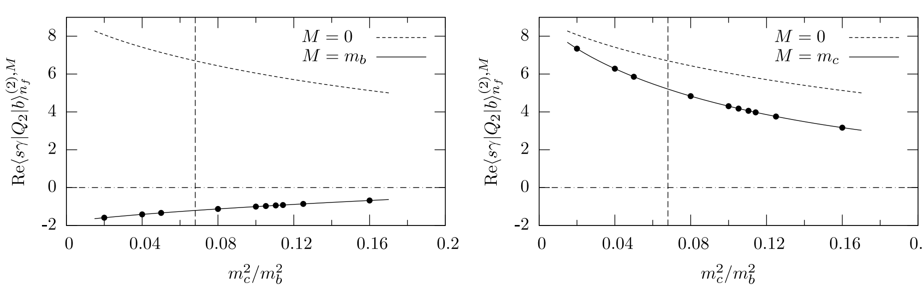

Fig. 1. Plots of the $\mathcal{O}(\alpha_s^2 n_f)$ corrections to matrix elements of Q_1 as function of $z = m_c^2/m_b^2$ with fermionic loops of mass M and $\mu = m_b$. (a): $M = m_b$, (b): $M = m_c$. The $M = 0$ case is also shown for comparison.

matrix elements of Q_1 with an internal quark of mass $M = 0, m_c, m_b$. In the case $M = m_b$ it is evident that the massless approximation overestimates the massive result by a large factor. For $M = m_c$ this difference is not that pronounced but still not negligible.

4. Conclusions

Taking new results for the full fermionic corrections at NNLO into account, the branching ratio is enhanced by about one to two percent in comparison to [17]. Moreover, an evaluation of bosonic corrections at this order, thereby completing three-loop matrix elements, is essential to resolve the interpolation ambiguity and to further improve the SM prediction for the $\overline{B} \to X_s\gamma$ decay.

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