THE NON-SINGLET QCD ANALYSIS OF PARTON DISTRIBUTIONS UP TO NNLO BASED ON JACOBI POLYNOMIALS*

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In this paper a non–singlet QCD analysis of the structure function F_2 in LO, NLO and NNLO is performed based on the Jacobi polynomials. For parameterization we used world data for charged lepton scattering. We determine the valence quark densities in a wide range of x and Q^2.

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1. Introduction

Presently the next-to leading order is the standard approximation for most of the important processes in QCD. The corresponding one- and two-loop splitting functions have been known for a long time. The next–to–next–to–leading order (NNLO) corrections should be included in order to arrive at quantitatively reliable predictions for hard processes occurring at present and future high-energy colliders.

In two recent publications [1, 2] a NNLO QCD analysis of $F_2^{e,p}(x, Q^2)$ and $F_2^{e,d}(x, Q^2)$ in the flavor non–singlet sector was presented. Here our purpose is to determine the flavor non–singlet parton distribution functions

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$xu_v(x,Q^2)$ and $xd_v(x,Q^2)$ in a wide range of x and Q^2 by using the Jacobi polynomial expansion and using the available $e(\mu)p$ and $e(\mu)d$ world data [3–7] up to the next-to-next-to leading order.

The plan of the paper is to give a brief review of the theoretical QCD formalism required for our QCD non-singlet analysis up to three loops in Section 2. In Section 3 we present the procedure of the QCD fits. Our results are summarized in this section.

2. Theoretical QCD formalism

The non–singlet (NS) parts of the structure functions $F_2^{p,d}(x,Q^2)$ for $x > 0.3$ are related to valence quarks combinations. Since in this region valence quarks dominate, the structure functions $F_2^{p,d}(x,Q^2)$ at LO given by

$$F_2^p(x,Q^2) \;=\; \frac{4}{9}\,xu_v(x,Q^2) + \frac{1}{9}\,xd_v(x,Q^2)\,, \tag{1}$$

$$F_2^d(x,Q^2) \;=\; \frac{5}{18}\,x(u_v+d_v)(x,Q^2)\,. \tag{2}$$

In the region $x \leq 0.3$ for the difference of the proton and deuteron data we use

$$\begin{aligned} F_2^{\rm NS}(x,Q^2) \;&\equiv\; 2\left[F_2^p(x,Q^2) - F_2^d(x,Q^2)\right] \\ &=\; \frac{1}{3}\,x(u_v-d_v)(x,Q^2) + \frac{2}{3}\,x(\bar{u}-\bar{d})(x,Q^2)\,, \end{aligned} \tag{3}$$

here sea quarks cannot be neglected for x smaller than about 0.3. We use the $x(\bar{d}-\bar{u})$ distribution at $Q_0^2 = 4\,{\rm GeV}^2$ as follows

$$x(\bar{d}-\bar{u})(x,Q_0^2) = 1.195\,x^{1.24}(1-x)^{9.1}(1+14.05x-45.52\,x^2)\,, \tag{4}$$

as applied in Ref. [1,2], which plays a marginal role in our analysis.

The evolution equations are solved in Mellin-N space and the Mellin transforms of the above distributions are denoted by $F_2^{p,d}(N,Q^2)$ and by $F_2^{\rm NS}(N,Q^2)$, respectively. The non–singlet structure functions are given by

$$\mathcal{F}_2^k(N,Q^2) = \Big[1 + a_s(Q^2)C_1(N) + a_s^2(Q^2)C_2(N)\Big]F_2^k(N,Q^2)\,, \tag{5}$$

for the three cases above. Here $a_s(Q^2) = \alpha_s(Q^2)/(4\pi)$ denotes the strong coupling constant and $C_i(N)(Q^2)$ are the non-singlet Wilson coefficients. The solution of the non-singlet evolution equation for the parton densities

up to 3-loop order is presented in Ref. [2]. In this work we choose the following parametrization for the valence quark densities

$$\begin{aligned} xu_v(x,Q^2) &= \mathcal{N}_u\, x^{\alpha_u}(1-x)^{\beta_u}(1+\eta_u\sqrt{x}+\gamma_u x)\,, \\ xd_v(x,Q^2) &= \mathcal{N}_d\; x^{\alpha_d}(1-x)^{\beta_d}(1+\eta_d\sqrt{x}+\gamma_d x)\,. \end{aligned} \tag{6}$$

The normalizations $\mathcal{N}_u$ and $\mathcal{N}_d$ being fixed by $\int_0^1 u_v dx = 2$ and $\int_0^1 d_v dx = 1$, respectively.

In the next section we can determine the unknown parameters by using the available world data [3–7] and the Jacobi polynomials method.

3. The procedure of the QCD fit

The evolution equations allow one to calculate the Q^2-dependence of the PD's provided at a certain reference point Q_0^2. These distributions are usually parameterized on the basis of plausible theoretical assumptions concerning their behavior near the end points $x = 0, 1$.

In the phenomenological investigations of the unpolarized and polarized structure functions, for example xg_1, xF_3 or F_2, one of the simplest and fastest possibilities in the structure function reconstruction from the QCD predictions for its Mellin moments is Jacobi polynomials expansion. The Jacobi polynomials are especially suited for this purpose since they allow one to factor out an essential part of the x-dependence of the structure function into the weight function [8]. Thus, given the Jacobi moments $a_n(Q^2)$, a structure function $f(x,Q^2)$ may be reconstructed in a form of the series [9]

$$xf(x,Q^2) = x^\beta(1-x)^\alpha \sum_{n=0}^{N_{\max}} a_n(Q^2)\Theta_n^{\alpha,\beta}(x)\,, \tag{7}$$

where $N_{\max}$ is the number of polynomials and $\Theta_n^{\alpha,\beta}(x)$ are the Jacobi polynomials of order n. For the moments, we note that the Q^2 dependence is entirely contained in the Jacobi moments

$$\begin{aligned} a_n(Q^2) &= \int_0^1 dx\; xf(x,Q^2)\Theta_k^{\alpha,\beta}(x) \\ &= \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta) f(j+2,Q^2)\;. \end{aligned} \tag{8}$$

Using Eqs. (7)–(8), one can relate the structure function with its Mellin moments

$$\mathcal{F}_2^k(x,Q^2) \;=\; x^\beta(1-x)^\alpha \sum_{n=0}^{N_{\max}} \Theta_n^{\alpha,\beta}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha,\beta)\mathcal{F}_2^k(j+2,Q^2)\,, \quad (9)$$

where $\mathcal{F}_2^k(j+2,Q^2)$ are the moments of the structure function. $N_{\max}$, α and β have to be chosen so as to achieve the fastest convergence of the series on the r.h.s. of Eq. (9) and to reconstruct $\mathcal{F}_2^k$ with the required accuracy. In our analysis we use $N_{\max}=9$, $\alpha=3.0$ and $\beta=0.5$. Obviously the Q^2-dependence of the structure function is defined by the Q^2-dependence of the moments. The same method has been applied to calculate the polarized structure function xg_1 from their moments [10].

Now we can use the structure function data measured in charged lepton proton and deuteron deep inelastic scattering. We used all of the world data for $\mathcal{F}_2^{p,d,\mathrm{NS}}$. Using the CERN subroutine MINUIT, we defined a global χ^2 for all the experimental data points in the NNLO case. In Fig. 1 the proton and deuteron data for $F_2(x,Q^2)$ are shown in the valence quark region $x \geq 0.3$. The solid lines correspond to the NNLO QCD fit. Also this figure shows the results of NNLO fit for the kinematic region $x \leq 0.35$. Fig. 2 illustrates the evolution of the valence quark densities $xu_v(x,Q^2)$ and $xd_v(x,Q^2)$ for $Q^2 = 1\,\mathrm{GeV}^2$ and $Q^2 = 10\,\mathrm{GeV}^2$ at NNLO.

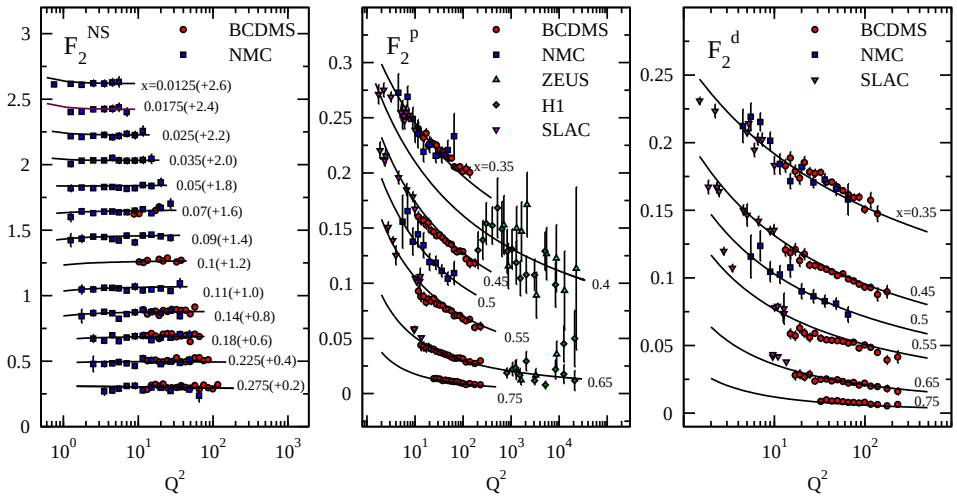

Fig. 1. The structure functions F_2^{NS}, F_2^p and F_2^d as function of Q^2 in intervals of x.

According to our QCD fits it seems that one should take into account the target mass corrections and higher twist in the region of large values of x and small values of Q^2.

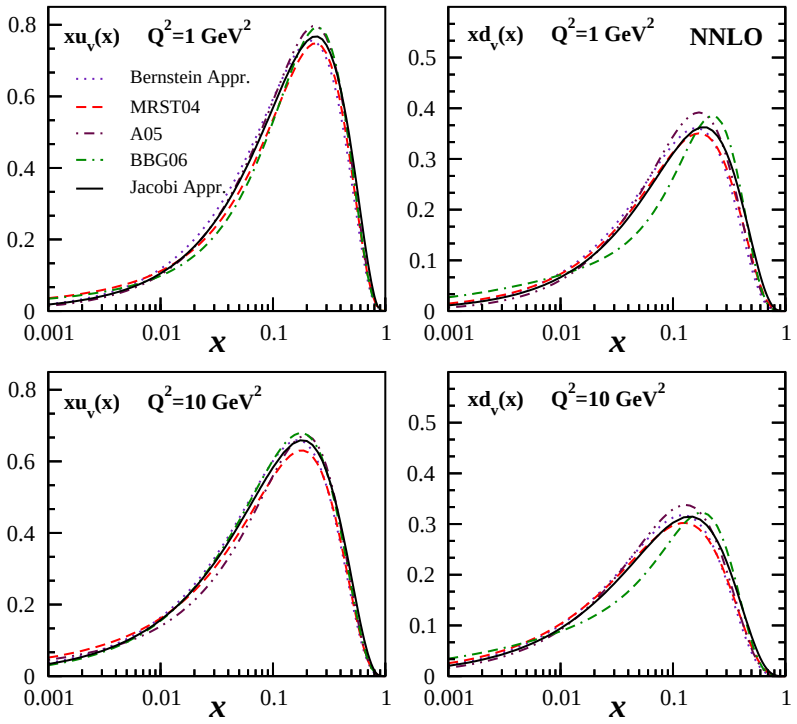

Fig. 2. The parton distribution xu_v and xd_v at some different values of Q^2. The solid (dotted) line is our model for Jacobi approach (Bernstein approach [11]), the dashed line is the MRST model [12], dashed-dotted line is the A05 model [13], dashed-dotted-dotted line is the BBG model [2].

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