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REDUCTION OF ONE-LOOP AMPLITUDES AT THE INTEGRAND LEVEL: CURRENT STATUS AND FUTURE OUTLOOK*

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We review the main features of the recently presented method for the evaluation of one-loop amplitudes of arbitrary scattering processes, in which the reduction to scalar integrals is performed at the integrand level. The coefficients of the scalar integrals are extracted by means of simple algebraic equations constructed by numerically evaluating the numerator of the integrand for specific choices of the integration momentum. The method is very well suited for an automatized and efficient implementation.

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1. Introduction and general motivations

The experimental programs at future colliders require high precision predictions for multi-particle processes. At the tree level, the introduction of efficient recursive algorithms [1] improved the theoretical description of such processes. However, the current need for precision goes beyond tree order. Starting with LHC, all analyses will require at least next-to-leading order calculations (NLO).

Many of the ingredients needed to accomplish such calculations were already developed in the fundamental work of Refs. [2, 3]. However, after almost three decades, only few one-loop calculations involving more than five particles have been completed [4].

The difficulties arising in this kind of calculation are well known: on the one hand the presence of a very large number of Feynman diagrams, on the other the appearance of numerical instabilities, that should be cured or avoided.

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In the last few years, several groups have been working on the problem of constructing efficient and automatized methods for the computation of one-loop corrections for multi-particle processes. Many different interesting techniques have been proposed: these contain numerical and semi-numerical methods [5], as well as analytic approaches [6] that make use of unitarity cuts to build NLO amplitudes by gluing on-shell tree amplitudes [7]. Some of these techniques require additional rational terms to be computed separately [8]. For a recent review of existing methods, see [9].

The main purpose of this talk is to illustrate a different approach to NLO calculations [10, 11]. The method benefits from previous work of Pittau and del Aguila [12]. We will review the main features of the method including a simple recipe for the computation of rational terms. We will then illustrate the status of the numerical implementation and the applications already performed. Finally, we will briefly comment on possible future developments.

2. General method

Any m-point one-loop amplitude can be written, before integration, as

$$A(\bar{q}) = \frac{N(q)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}\,, \tag{1}$$

where $\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2$. The bar denotes objects living in $n = \ 4 + \varepsilon$ dimensions and a tilde objects of dimension ε. Physical external momenta p_i are 4-dimensional objects, while the integration momentum q is in general n-dimensional. Following this notation, we have $\bar{q}^2 = q^2 + \tilde{q}^2$ and $\bar{D}_i = D_i + \tilde{q}^2$.

The integrated amplitude can be expressed in a basis of known integrals, such as 4-, 3-, 2- and 1-point scalar integrals. The task of the reduction is therefore to determine the coefficients in front of each one of the integrals.

In order to perform the calculation, we need three main building blocks: the evaluation of the numerator function $N(q)$, the determination of the coefficients via reduction method, and finally the evaluation of the scalar functions. This talk will describe in some detail how the second step can be accomplished. Concerning the other two steps, $N(q)$ is evaluated numerically using a separate routine, and only for the values of q needed in the reduction, while scalar integrals are provided by FF [13] for massive internal particles, and OneLOop [14] in the massless case.

Let us now focus on the reduction. The procedure described below is the main idea on which the method is based. When performing a calculation, it can be considered as a black-box, since it is completely process-independent: for any given numerator function $N(q)$, the reduction can be performed in an efficient automatized way. We provide an implementation of the general algorithm in the package `CutTools` [15], that will soon be available upon request from the authors.

2.1. Reduction

Assuming for the moment that the numerator $N(q)$ is fully 4-dimensional, we can rewrite it at the integrand level in terms of D_i as

$$\begin{aligned} N(q) \;=\; & \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0i_1i_2i_3) + \tilde{d}(q;i_0i_1i_2i_3)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i \\ & + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0i_1i_2) + \tilde{c}(q;i_0i_1i_2)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i \\ & + \sum_{i_0<i_1}^{m-1} \left[b(i_0i_1) + \tilde{b}(q;i_0i_1)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i \\ & + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q;i_0)\right] \prod_{i\neq i_0}^{m-1} D_i \,. \end{aligned} \tag{2}$$

The quantities $d(i_0i_1i_2i_3)$ are the coefficients of 4-point scalar functions with denominators labeled by i_0, i_1, i_2, and i_3. In the same way, we call $c(i_0i_1i_2)$, $b(i_0i_1)$, $a(i_0)$ the coefficients of all possible 3-point, 2-point and 1-point scalar functions, respectively. The quantities $\tilde{d}$, $\tilde{c}$, $\tilde{b}$, $\tilde{a}$ are what we define as "spurious" terms, *i.e.* terms that are present in the decomposition at the integrand level, but will vanish upon integration. These terms still depend on the integration momentum q.

After fixing the form of all the spurious terms, our calculation is reduced to the algebraic problem of extracting all the coefficients. This is achieved simply by evaluating $N(q)$ at different values of the integration momentum q. As a further simplification, there is a very good set of such points: we can use values of q for which a subset of denominators D_i vanish. Operating in this manner, the system becomes "triangular"; we can first solve for 4-point functions, then 3-point functions and so on.

To conclude this section, let us summarize once more the recipe for the calculation. We first calculate all the coefficients in Eq. (2), by evaluating the numerator of the integrand $N(q)$ for a set of values of the integration momentum q. Note, that we do not need to repeat this for all Feynman diagrams: we can group them and expand for (sub)amplitudes directly. We just need to specify external momenta, polarization vectors and masses, and proceed with the reduction. Concerning $N(q)$ we can choose how to proceed according to the specific calculation: as an interesting development, $N(q)$ could be determined numerically via recursion relations. Once all the coefficients have been determined, we can multiply them by the corresponding scalar integrals.

2.2. Rational terms

As mentioned in the previous section, the reduction has been performed assuming a purely four dimensional numerator (this singles out the so called cut-constructable part of the amplitude). In this section we describe one possible method to calculate the rational parts of the amplitude [11, 16]. In our approach, these contributions originate from the fact that up to now we expressed the numerator in terms of D_i, while the functions appearing in the denominator are n-dimensional $\bar{D}_i$. Let us go back to the integrand $A(\bar{q})$ of Eq. (1) and insert the expression for $N(q)$ of Eq. (2), that we obtained after determining all the coefficients for both regular and spurious terms. Now, we rewrite D_i by means of

$$\frac{D_i}{\bar{D}_i} = \bar{Z}_i\,, \qquad \text{with} \qquad \bar{Z}_i \equiv \left(1 - \frac{\tilde{q}^2}{\bar{D}_i}\right).$$

The rational part is produced, after integrating over $d^n q$, by the $\tilde{q}^2$ dependence in $\bar{Z}_i$. The expressions for all relevant integrals are reported in the Appendix of Ref. [11].

In addition to the one just described, there might be other sources of rational terms coming from objects of dimension ε in the numerator $N(q)$, according to the specific calculation. These, for example, could originate from the contraction of Dirac matrices or from powers of $\tilde{q}^2$ [17].

3. Numerical tests

3.1. Four-photon and six-photon amplitudes

As a first example of application of the method, we calculated 4-photon and 6-photon amplitudes, via fermionic loop of mass $m_{\rm f}$ [11]. Those processes do not have an immediate physical interest, however they provide a playground for testing and comparing different methods of calculation.

For the 4-photon amplitudes, we performed a comparison with the analytic result presented by Gounaris *et al.* [18], finding perfect agreement both in the massive and massless cases.

Coming to the 6-photons, there are a few previous results available in the literature for the massless case. Some time ago, Mahlon presented an exact analytic result for the helicity configuration $[++----]$ [19]. More recently, Nagy and Soper, with a fully numerical approach, obtained the results for the configurations $[++----]$ and $[++--+-]$ [20]. The same results were also recently presented by Binoth *et al.* [21], that also provide analytic expressions. Our results are in full agreement with all previous calculations. We also checked that the cut-constructable part for configurations $[+-----]$ and $[------]$ and the rational terms for all helicity configurations are identically zero. Finally, we calculated the 6-photon amplitudes with massive internal fermions. The results have been presented in Ref. [11].

3.2. ZZZ production

We recently completed the evaluation of the virtual QCD corrections to the process $q\bar{q} \to ZZZ$. These results have been recently presented by Lazopoulos *at al.* [22], together with the evaluation of the contributions from real emission. The virtual part of the calculation involves eight different diagrams, that have been depicted in Fig. 1. Each diagram should be evaluated for six permutations of the final particles. It is interesting to notice that, using our technique, the eight diagrams can be combined in a single numerator $N(q)$, allowing for a one-shot evaluation of the resulting scalar coefficients.

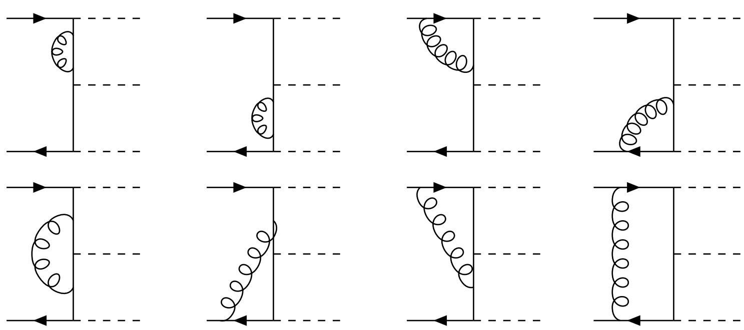

Fig. 1. Diagrams contributing to virtual QCD corrections to $q\bar{q} \to ZZZ$.

The results that we obtain, both for poles and finite parts, agree with the results obtained by the authors of Ref. [22]. We will report the details of our calculation in a forthcoming publication.

4. Summary and future outlook

The discovery potential of LHC requires NLO calculations. At present, there is a variety of interesting options available to perform one-loop multileg calculations, however not a universal method. Moreover, the need for automatized and efficient implementations puts additional constraints in the construction of tools to accomplish such tasks.

We recently proposed a method for the numerical evaluation of one-loop amplitudes in which the reduction is performed at the integrand level. With respect to other existing methods, it presents several advantages. First of all, the information required in order to perform the reduction is minimal, simply the numerical value of the numerator of the integrand for a set of values of the integration momentum (no analytical expression is necessary). Moreover, the method does not require any computer algebra, which is usually very time consuming, and it incorporates a solid way to compute rational terms. The first tests performed gave good results, both in terms of precision and efficiency. Moreover, there is still plenty of room for improvement and optimization. For example, a very appealing possibility is the automatized evaluation of the numerator function $N(q)$, by means of recursion relations.

An implementation of the general algorithm, together with the main routines needed to perform the reduction, can be found in the package `CutTools`, that will soon be available upon request from the authors.

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