STUDYING OFF MASS SHELL AND OFF RESONANCE BACKGROUND EFFECTS IN $e^+e^- \to t\bar{t}H$*

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We discuss the reaction of associated production of the top quark pair and Higgs boson at the future International Linear Collider. We assume that the Higgs boson is light and hence, due to its very small decay width, it is produced on mass shell while the top quarks are produced off shell and they decay, each into 3 fermions. This causes a necessity of studying reactions with seven particles in the final state. Concentrating on one semileptonic channel of such reaction, we illustrate the off mass shell effects and calculate the off resonance background contributions. The latter are calculated with the use of a program `carlomat` for automatic computation of multiparticle cross sections.

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1. Introduction

If the Higgs boson has mass below the $t\bar{t}$ threshold, $m_H < 2m_t$, then its Yukawa coupling to the top quark

$$g_{ttH} = \frac{m_t}{v}, \qquad \text{with} \qquad v = (\sqrt{2}G_{\rm F})^{-1/2} \simeq 246\text{GeV},$$

which is by far the largest Yukawa coupling of the Standard Model (SM), can be best determined at the future International Linear Collider (ILC) [1] through measurement of the process [2]

$$e^+e^- \to t\bar{t}H\,. \tag{1}$$

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Particles on the right hand side of reaction (1) are unstable: the top and antitop decay, even before they hadronize, predominantly into bW^+ and $\bar{b}W^-$, respectively, the Higgs boson, dependent on its mass, decays either into a fermion–antifermion or an electroweak (EW) gauge boson-pair and the EW bosons subsequently decay, each into a fermion–antifermion-pair. This leads to reactions with either 8 or 10 fermions in the final state. To study such reactions is a highly complicated problem, as they typically involve thousands of the Feynman diagrams, *e.g.* reaction $e^+e^- \to bu\bar{d}\bar{b}\mu^-\bar{\nu}_\mu b\bar{b}$ receives contributions from 56550 diagrams already at the lowest order of SM in the unitary gauge.

The problem can be simplified by taking into account the fact that current constraints on the Higgs boson mass, based on the virtual effects it has on precision EW observables combined with a value of the top quark mass measured at Tevatron and the W-boson mass [3], indicate a law value of m_H, close to the limit of 114.4 GeV from the direct searches at LEP. Such a light Higgs boson is very narrow, with the width of a few MeV, almost three orders of magnitude smaller than the width of the top quark or EW gauge bosons. Therefore, it seems natural to assume that the Higgs boson of reaction (1) is on mass shell and take into account decays of the top and antitop quark, which lead to the reactions of the form

$$e^+e^- \to bf_1f_1'\bar{b}f_2f_2'H\,, \tag{2}$$

where $f_1f_2' = \nu_e, \nu_\mu, \nu_\tau, u, c$ and $f_1'f_2 = e^-, \mu^-, \tau^-, d, s$.

In the present paper, we will use this approximation in order to discuss the off shell and off resonance background effects in the associated top quark pair and Higgs boson production at ILC. To illustrate the effects we will concentrate on one selected channel of reaction (2). The associated production of the top quark pair and Higgs boson has been already studied in this context before. Processes $e^+e^- \to b\bar{b}b\bar{b}W^+W^- \to b\bar{b}b\bar{b}l^\pm\nu_l q\bar{q}'$ accounting for the signal of associated Higgs boson and top quark pair production, as well as several irreducible background reactions, were studied in [4], EW contributions to the leptonic and semileptonic reactions channels of $e^+e^- \to bf_1f_1'\bar{b}f_2f_2'b\bar{b}$ have been computed in [5] and the off shell effects in such reactions have been looked at in [6].

2. Automatic calculation with a program `carlomat`

`carlomat` [7] is a program that allows to generate the matrix element for a user specified process, with up to 12 external particles, together with phase space parameterizations which are later used for the multichannel Monte Carlo integration of the lowest order cross sections and event generation. The program is written in Fortran 90/95. It takes into account both the EW and quantum chromodynamics (QCD) lowest order contributions. Fermion masses are not neglected in the program, hence the cross sections can be usually calculated without any kinematical cuts. At the moment only the SM particles and Feynman rules are coded in the program, but extensions of SM can be implemented in a similar way.

When the process has been entered, topologies for a given number of external particles are generated and checked against Feynman rules. While doing that `carlomat` joins typically a few external particles, by means of the vertices and propagators of the implemented model, into an object that is actually an off shell particle. It is represented in the program by the derived data type object and can be later used as a building block of other Feynman diagrams. Once the diagram is created, the corresponding particles are used to construct the helicity amplitude, color factor (matrix) and phase space parametrization which are stored on the disk. When all the topologies have been already checked, `carlomat` creates subroutines for the matrix element, color matrix and phase space integration.

The matrix element is calculated with the helicity amplitude method for a given set of external momenta in a fixed reference frame. The actual method of calculation was described in [8]. `carlomat` makes use of the routines developed for a Monte Carlo program `eett6f v. 1.0`, for calculating lowest order cross sections of $e^+e^- \to 6$ fermions relevant for a $t\bar{t}$-pair production and decay [9], which have been tailored to meet needs of the automatic generation of amplitudes. A novel feature with respect to other programs, as *e.g.* MadGraph [10], is that spinors or polarization vectors representing particles, both the on shell and off shell ones, are computed only once, for all the helicities of the external particles they are made of. They are stored in arrays and can be multiply used while the program is performing sum over helicities in a calculation of the unpolarized cross section.

The dedicated phase space parameterizations taking into account mappings of the Breit–Wigner behavior caused by propagators of unstable particles and $\sim 1/s$ behavior of the photon and gluon propagators are performed, if necessary. They are incorporated into a multichannel MC integration routine.

Let us use `carlomat` in order to calculate the cross section of a semileptonic channel of reaction (2)

$$e^+(p_1)\, e^-(p_3) \;\to\; b(p_3)\, u(p_4)\, \bar{d}(p_5)\, \bar{b}(p_6)\, \mu^-(p_7)\, \bar{\nu}_\mu(p_8)\, H(p_9), \qquad (3)$$

where the particle four momenta are indicated in the parentheses. The program generates helicity amplitudes and dedicated phase space parameterizations for 5456 Feynman diagrams which contribute to reaction (3) in the lowest order of SM in the unitary gauge together with a reduced 2×2 color matrix. If we neglect the Higgs boson coupling to fermions lighter than the b quark then the number of diagrams is reduced to 1294. The "signal" diagrams of the associated production of the top quark pair and Higgs boson are depicted in Fig. 1. It is worth mentioning that the number of building blocks which are necessary to construct the full lowest order matrix element with 5456 Feynman diagrams in `carlomat` is less than 600, while MadGraph needs for that about 2600 wave functions. The reader is referred to the forthcoming article [7] for a description of the algorithm of amplitude generation used in `carlomat`.

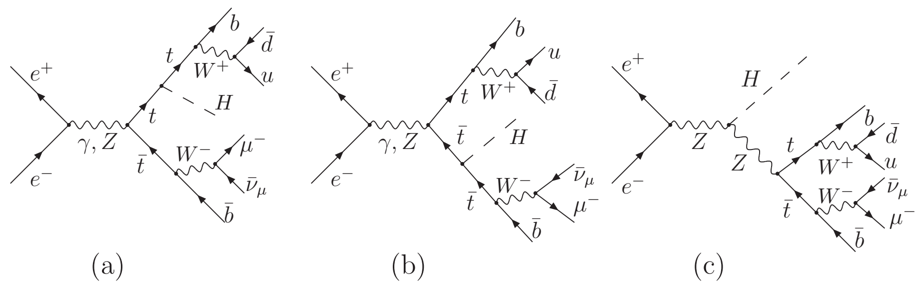

Fig. 1. "Signal" Feynman diagrams of reaction (3).

3. Results

The numerical results presented in this section have been obtained with the following set of initial physical parameters. We have chosen the Fermi coupling and fine structure constant in the Thomson limit

$$G_\mu = 1.16639 \times 10^{-5}\ \mathrm{GeV}^{-2}\,, \qquad \alpha_0 = 1/137.03599976\,, \tag{4}$$

as well as the W- and Z-boson masses

$$m_W = 80.4515\ \mathrm{GeV}\,, \qquad m_Z = 91.1867\ \mathrm{GeV}\,, \tag{5}$$

as the EW input parameters. The top quark mass and the external fermion masses of reaction (3) are the following:

$$\begin{gathered} m_t = 173.8\ \mathrm{GeV}\,, \quad m_b = 4.7\ \mathrm{GeV}\,, \quad m_u = 62\ \mathrm{MeV}\,, \quad m_d = 83\ \mathrm{MeV}\,, \\ m_e = 0.5109991\ \mathrm{MeV}\,, \quad m_\mu = 105.6584\ \mathrm{MeV}\,, \end{gathered} \tag{6}$$

where the light quark masses have been chosen in such a way that, together with $m_s = 215$ MeV and $\alpha_{\mathrm{s}} = 0.123$, they reproduce correctly virtual quark loop contributions to the photon vacuum polarization. The value of the Higgs boson mass is assumed at $m_H = 130$ GeV. Widths of unstable particles are calculated to the lowest order of SM resulting in

$$\Gamma_t = 1.516\ \mathrm{GeV}\,, \qquad \Gamma_W = 2.05\ \mathrm{GeV}\,, \qquad \Gamma_H = 7.97\ \mathrm{MeV}\,, \tag{7}$$

for the top quark, W-boson and Higgs boson widths, respectively. The Z-boson width, whose actual value plays practically no role in the calculation is fixed at $\Gamma_Z = 2.4998\,\mathrm{GeV}$.

In order to estimate the pure off shell effect we calculate the cross section of reaction (3) with the "signal" Feynman diagrams of Fig. 1 and compare it with the cross in the narrow width approximation (NWA) for the top and antitop. The total cross section of reaction (3) in the NWA is defined in the following way

$$\sigma_{\mathrm{NWA}} = \sigma(e^+e^- \to t\bar{t}H) \times \frac{\Gamma_{W^+\to u\bar{d}}}{\Gamma_W} \times \frac{\Gamma_{W^-\to \mu^-\bar{\nu}_\mu}}{\Gamma_W}\,, \tag{8}$$

where $\sigma(e^+e^- \to t\bar{t}H)$ denotes the total cross section of (1) and we have assumed that

$$\frac{\Gamma_{t\to bW^+}}{\Gamma_t} = \frac{\Gamma_{\bar{t}\to \bar{b}W^-}}{\Gamma_t} = 1\,.$$

The corresponding amplitudes are calculated with the helicity amplitude method of [8] and computed with the use of program libraries described in [9]. Our results illustrating the pure off shell effect are shown in Table I and in Fig. 2. We see that the effect is at the level of a few percent. They are positive for energies just above the threshold and become negative for higher energies.

TABLE I

Lowest order cross sections of (3): the "signal" cross section σ_{signal}, the cross section in NWA σ_{NWA} and the corresponding relative correction $\delta = (\sigma_{\mathrm{NWA}} - \sigma_{\mathrm{signal}})/\sigma_{\mathrm{signal}}$.

$\sqrt{s}$ [GeV]	σ_{signal} [ab]	σ_{NWA} [ab]	δ [%]
500	5.62(1)	5.82(1)	3.6
800	79.9(1)	82.2(1)	2.9
1200	58.8(1)	58.7(1)	−0.1
2000	30.1(2)	28.4(1)	−5.7

To see the off resonance background effect we compare the cross sections of reaction (3) calculated with the full set of the lowest order Feynman diagrams with the "signal" cross section. The corresponding results are shown in Table II. We see that the background effect is positive, of the order of a few per cent.

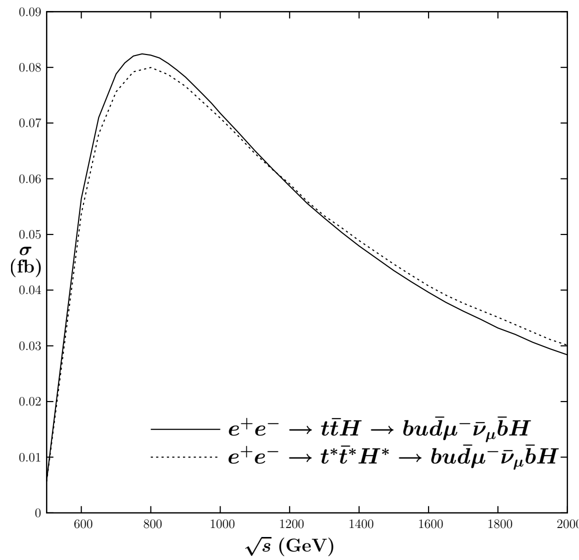

Fig. 2. Total cross sections of reaction (3) as functions of the CMS energy: the "signal" cross section $\sigma_{\rm signal}$ (short-dashed line) and the cross section in NWA $\sigma_{\rm NWA}$ (solid line).

TABLE II

Lowest order cross sections of reaction (3): the "signal" cross section $\sigma_{\rm signal}$, the cross section calculated with the complete set of the Feynman diagrams σ and the corresponding relative correction $\delta = (\sigma - \sigma_{\rm signal})/\sigma_{\rm signal}$.

$\sqrt{s}$ [GeV]	$\sigma_{\rm signal}$ [ab]	σ [ab]	δ [%]
500	5.62(1)	5.92(4)	5.3
800	79.9(1)	81.7(1)	2.4
1200	58.8(1)	61.8(1)	5.1
2000	30.1(2)	32.6(1)	8.3

4. Conclusions

We have discussed the off mass shell and off resonance background effects in the reaction of associated production of the top quark pair and the light Higgs boson at the ILC. Both effects are at the level of a few per cent and therefore might be relevant for the analysis of the future data.

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