

NEUTRINO PHYSICS AND CP VIOLATION*

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We review some aspects of neutrino physics and CP violation both in the quark and lepton sectors.

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1. Introduction

In these lectures, we cover topics related to neutrino physics and CP violation. The treatment of these topics is not extensive. For a thorough treatment of the above topics, the reader is advised to consult the excellent books and review articles which exist in the literature covering neutrino physics and CP violation. Some books are cited in what follows. A list of a few review articles can be found in Ref. [1]. These lectures are organised as follows. In the next section we describe some of the minimal extensions of the SM which can incorporate non-vanishing neutrino masses, with special emphasis on the seesaw mechanism [2]. In Section 3 we cover CP violation both in the quark and lepton sector. In the lepton sector, we discuss CP violation both at low and high energies in the minimal seesaw mechanism. In the last subsection we briefly describe the generation of the baryon asymmetry of the Universe through leptogenesis.

2. Minimal extensions of the SM incorporating neutrino masses

In the leptonic sector of the SM the fermionic field content is:

$$L_{Li} = \begin{pmatrix} \nu^0 \\ l^0 \end{pmatrix}_{Li}, \quad l_{Ri}^0, \quad (i = 1, 2, 3), \quad (1)$$

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where L_{Li} denote the lefthanded leptonic doublets, containing neutrinos and charged leptons. The righthanded components of the charged leptons, l_{Ri}^0 , are SU(2) singlets. No righthanded components for the neutrino fields are introduced in the SM.

The charged leptons acquire mass through Yukawa terms of the form:

$$\mathcal{L}_Y = f_{ij}^e \bar{L}_{Li} l_{Rj}^0 \phi + \text{h.c.}, \quad (2)$$

with ϕ a scalar Higgs doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (3)$$

Due to the absence of righthanded singlet fields ν_{Ri}^0 , it is not possible to have a Dirac mass term for the neutrinos.

In general, Dirac mass terms are of the form

$$-\mathcal{L}_D^m = m_D \bar{\psi} \psi = m_D (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) \quad (4)$$

and are invariant under U(1) transformations, *i.e.*, they conserve any charge carried by ψ associated to a U(1) symmetry (*e.g.*, electrical charge, lepton number, *etc.*). Upon spontaneous gauge symmetry breaking (SSB) the neutral component of ϕ acquires a vacuum expectation value

$$\langle \phi^0 \rangle = \frac{v}{\sqrt{2}}. \quad (5)$$

As a result, Dirac mass terms for the charged leptons are generated from the Yukawa couplings, given by:

$$\mathcal{L}_{m_l} = \frac{v}{\sqrt{2}} f_{ij}^e \bar{l}_{Li}^0 l_{Rj}^0 + \text{h.c.} \equiv m_{lij} \bar{l}_{Li}^0 l_{Rj}^0 + \text{h.c.} . \quad (6)$$

Neutrinos have the very special feature of being the only known fermions which have zero electrical charge. As a result, neutrinos can have Majorana mass terms, which are of the form

$$-\mathcal{L}_M^m = \frac{1}{2} [\psi_L^T C m_L \psi_L + \text{h.c.}] , \quad (7)$$

where C is the charge-conjugation matrix defined by:

$$C^{-1} \gamma_\mu C = -\gamma_\mu^T \quad (8)$$

with γ_μ denoting the Dirac matrices.

With the fermionic content of the Standard Model this would correspond to terms of the form $\nu_{Li}^{0T} C \nu_{Lj}^0$. However, in the SM these terms cannot be introduced at the Lagrangian level, because they are not gauge invariant. Also, due to exact $B - L$ conservation, they cannot be generated neither radiatively in higher orders, nor nonperturbatively. As a result, neutrinos are *strictly massless* in the SM.

The charged lepton mass matrix given in Eq. (6) can be diagonalised through the biunitary transformation:

$$U_{lL}^\dagger m_l U_{lR} = \text{diag} (m_e, m_\mu, m_\tau). \quad (9)$$

After this transformation, the matrix U_{lL} would appear in the charged leptonic interactions:

$$\mathcal{L}_W = \frac{v}{\sqrt{2}} \bar{l}_L \gamma_\mu U_{lL}^\dagger \nu_L W^{+\mu} + \text{h.c.} . \quad (10)$$

However, since in the SM neutrinos are strictly massless, the matrix U_{lL}^\dagger can always be eliminated through a redefinition of the neutrino fields. Therefore, in the SM there is *no leptonic mixing*, which leads to separate conservation of the flavour lepton numbers L_e, L_μ, L_τ . Consequently the recent observation of neutrino oscillations provides clear evidence for *physics beyond the SM*.

2.1. Generating neutrino masses through extensions of the scalar sector

There are various ways of generating neutrino masses through extensions of the SM involving the scalar sector [3]. Since ν_L are part of a doublet, one of the simplest ways of generating such mass terms is by extending the SM through the introduction of a scalar Higgs triplet [4] \vec{H} which would allow for the following renormalizable Yukawa term

$$-\mathcal{L}_Y^H = f_{ij} L_{Li}^T C(i\tau_2)(\vec{\tau} \cdot \vec{H}) L_{Lj} + \text{h.c.}, \quad (11)$$

still conserving lepton number, where

$$\vec{\tau} \cdot \vec{H} = \begin{pmatrix} H^+ & \sqrt{2}H^{++} \\ \sqrt{2}H^0 & -H^+ \end{pmatrix}. \quad (12)$$

When \vec{H} develops a vacuum expectation value (VEV) lepton number is violated and Majorana mass terms for the neutrinos are generated. Notice that $[f_{ij}]$ is a symmetric matrix due to anticommutation of the fermion fields, the antisymmetric property of the charge conjugation matrix and the symmetric character of $(i\tau_2)\vec{\tau}$. Note that even in the context of the SM, it would be possible to construct a composite triplet Higgs operator out

of two Higgs doublets. Of course, such a term $(L_L^T C i\tau_2 \bar{\tau} L_L)(\phi^T i\tau_2 \bar{\tau} \phi)$ has dimension five and would be non renormalizable. However, it cannot be effectively generated in the Standard Model since it violates $B - L$ which is an accidental exact symmetry of the Standard Model.

An alternative simple way of generating Majorana mass terms for left-handed neutrinos is, for instance, the introduction of a singly charged scalar singlet h^+ as proposed by Zee [5] allowing for a Yukawa coupling of the form:

$$-\mathcal{L}_Y^H = f_{ij} L_{Li}^T C(i\tau_2) L_{Lj} h^+ + \text{h.c.} \quad (13)$$

in this case $[f_{ij}]$ must be an antisymmetric matrix, since $i\tau_2 \equiv \varepsilon$ is antisymmetric. This coupling by itself does not violate $B - L$, since one has the freedom to assign $B - L$ quantum number (-2) to the field h^+ . In order to generate neutrino masses, one needs at least two Higgs doublets and a cubic coupling of the form

$$M_{\alpha\beta} \varepsilon_{ij} \phi_\alpha^i \phi_\beta^j h^-, \quad (14)$$

where the indices α, β distinguish between the Higgs doublets and i, j are SU(2) indices. The coupling $M_{\alpha\beta}$ has dimension of mass and is antisymmetric. The simultaneous presence of the two couplings (13) and (14) in the theory violates explicitly $B - L$ by two units and leads to finite and calculable one loop contributions to neutrino masses.

These are just two of the simplest examples considered in the literature, where neutrino masses are generated via extensions of the scalar sector.

2.2. Generating neutrino masses through the introduction of righthanded neutrinos

In these lectures we are mainly concerned with extensions of the Standard Model where only $SU(2) \times U(1)$ singlet righthanded neutrinos are added to its spectrum. Indeed, one may view the simple addition of righthanded neutrino components to the SM as the most straightforward way of incorporating neutrino masses. In this case the number of fermionic degrees of freedom for neutrinos equals those of all other fermions in the theory provided that three righthanded neutrinos are introduced. It is well known that such an extension of the SM allows for the seesaw mechanism [2] to operate, giving rise to three light and three heavy neutrinos of Majorana character, as well as leptonic mixing and the possibility of CP violation in the couplings of the neutrinos to the charged leptons. Low energy physics (the decoupling limit) in this framework, is described by an effective lefthanded Majorana mass matrix as is the case in models where only the scalar sector of the SM is enlarged. Yet, the seesaw mechanism plays an important rôle in explaining in a natural way the “extreme” smallness of neutrino masses

when compared to the masses of the other fermions. Furthermore, CP violation in the decay of the heavy neutrinos may lead to a lepton asymmetry which is subsequently transformed into a baryon asymmetry, thus providing an explanation for the observed baryon asymmetry of the Universe through leptogenesis. The lepton number asymmetry thus produced can be fully parametrised in terms of neutrino mass matrices. In flavour models where the number of free parameters is reduced through the introduction of family symmetries or the imposition of special ansätze, it is often possible to establish a direct connection between low energy and high energy physics in the leptonic sector.

With the introduction of righthanded neutrino fields, the most general leptonic mass term after SSB is of the form:

$$\begin{aligned}\mathcal{L}_m &= - \left[\frac{1}{2} \nu_L^{0T} C m_L \nu_L^0 + \overline{\nu}_L^0 m_D \nu_R^0 + \frac{1}{2} \nu_R^{0T} C M_R \nu_R^0 + \overline{l}_L^0 m_l l_R^0 \right] + \text{h.c.} \\ &= - \left[\frac{1}{2} n_L^T C \mathcal{M}^* n_L + \overline{l}_L^0 m_l l_R^0 \right] + \text{h.c.}\end{aligned}\quad (15)$$

with the 6×6 matrix \mathcal{M} given by:

$$\mathcal{M} = \begin{pmatrix} m_L^* & m_D \\ m_D^T & M_R \end{pmatrix}. \quad (16)$$

As already explained the appearance of the term $\nu_L^{0T} C m_L \nu_L^0$ would require further enlargement of the scalar sector of the Lagrangian. In what follows, we discuss the minimal seesaw framework where this term is not present. The terms in m_D are generated through Yukawa couplings and, therefore, cannot be of a scale larger than the electroweak scale. However, the terms in M_R are $SU(2) \times U(1)$ invariant, not protected by any symmetry. Therefore, it is natural to assume that their scale is much larger than the electroweak scale. The origin of the term “seesaw” is based on the implications of choosing the scale of M_R much larger than the scale of m_D , as illustrated below. In fact, the existence of these two very different scales gives rise to two sets of neutrinos of different mass scales, one large, of order of M_R , and another one much suppressed by comparison to the electroweak scale. This provides a natural explanation for the observed smallness of neutrino masses.

After spontaneous symmetry breaking, but before diagonalisation of the fermion mass terms, the leptonic charge gauge interactions are still diagonal and, therefore, can be written as

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \overline{l}_L^0 \gamma^\mu \nu_L^0 + \text{h.c.} \quad (17)$$

this basis is usually called a weak basis (WB). WB transformations are defined as transformations of the fermion fields that leave the gauge currents flavour diagonal. In the present extension of the SM the most general such transformations are of the form:

$$\begin{aligned} l_L^0 &\longrightarrow U' l_L^0, & \nu_L^0 &\longrightarrow U' \nu_L^0, \\ l_R^0 &\longrightarrow V' l_R^0, & \nu_R^0 &\longrightarrow W' \nu_R^0, \end{aligned} \quad (18)$$

where $U', V' W'$ are arbitrary unitary matrices. The lefthanded fields l_L^0, ν_L^0 must transform in the same way in order to leave the charged weak current of Eq. (17), diagonal. Since there are no righthanded gauge currents mediated by W , in this extension of the SM this constraint does not exist for the righthanded fields. Physics does not depend on the choice of WB, in particular all WB lead to the same fermion masses and mixing. Clearly it is always possible to choose without loss of generality a WB where m_l is real diagonal and positive. In this basis the matrix V that diagonalises \mathcal{M} has physical meaning. The diagonalisation of the matrix \mathcal{M} is then performed via the unitary transformation

$$V^T \mathcal{M}^* V = \mathcal{D}, \quad (19)$$

where $\mathcal{D} = \text{diag}(m_1, m_2, m_3, M_1, M_2, M_3)$, with m_i and M_i denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write V and \mathcal{D} in the following block form:

$$V = \begin{pmatrix} K & G \\ S & T \end{pmatrix}, \quad (20)$$

$$\mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix}. \quad (21)$$

The neutrino weak-eigenstates are related to the mass eigenstates by:

$$\nu_{iL}^0 = V_{i\alpha} \nu_{\alpha L} = (K, G) \begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix} \quad \begin{pmatrix} i = 1, 2, 3 \\ \alpha = 1, 2, \dots, 6 \end{pmatrix} \quad (22)$$

and thus the leptonic charged current interactions are given by:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\overline{l_{iL}} \gamma_\mu K_{ij} \nu_{jL} + \overline{l_{iL}} \gamma_\mu G_{ij} N_{jL}) W^\mu + \text{h.c.} \quad (23)$$

with K and G being the charged current couplings of charged leptons to the light neutrinos ν_j and to the heavy neutrinos N_j , respectively. From Eqs. (19), (20), (21) and for

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \quad (24)$$

one obtains:

$$S^\dagger m_D^T K^* + K^\dagger m_D S^* + S^\dagger M_R S^* = d, \quad (25)$$

$$S^\dagger m_D^T G^* + K^\dagger m_D T^* + S^\dagger M_R T^* = 0, \quad (26)$$

$$T^\dagger m_D^T G^* + G^\dagger m_D T^* + T^\dagger M_R T^* = D. \quad (27)$$

In the context of seesaw with M_R of a scale M , much larger than the weak scale, v , the following relations can be derived from these equations, valid to an excellent approximation:

$$S^\dagger = -K^\dagger m_D M_R^{-1}, \quad (28)$$

$$-K^\dagger m_D \frac{1}{M_R} m_D^T K^* = d. \quad (29)$$

It is clear from Eq. (28) that S is of order m_D/M_R and, therefore, is very suppressed. Eq. (29) is the usual seesaw formula with the matrix K frequently treated as being equivalent to U_{PMNS} , the Pontecorvo, Maki, Nakagawa, Sakata (PMNS) matrix [6]. Although the block K in Eq. (20) is not a unitary matrix its deviations from unitarity are of the order m_D^2/M_R^2 . It is from Eq. (29) that the low energy physics of the leptonic sector is derived. The decoupling limit corresponds to an effective theory with only lefthanded neutrinos and a Majorana mass matrix, m_{eff} defined as:

$$m_{\text{eff}} = -m_D \frac{1}{M_R} m_D^T \quad (30)$$

showing that for m_D of the order of the electroweak scale and M_R of the scale of grand unification, the smallness of light neutrino masses is a natural consequence of the seesaw mechanism [2]. From the relation $\mathcal{M}^* V = V^* \mathcal{D}$ and taking into account the zero entry in \mathcal{M} one derives the following exact relation

$$G = m_D T^* D^{-1}. \quad (31)$$

This equation plays an important rôle in the connection between low energy and high energy physics in the leptonic sector, and shows explicitly that the suppression in the matrix G is of the same order of the suppression in S as required by unitarity of the matrix V .

There are in the literature excellent reviews [3, 7] on the seesaw mechanism, showing explicitly that under this mechanism the resulting physical fermions are in general Majorana spinors. The left and the righthanded components of Majorana spinors are not independent. Out of two independent spinor components Ψ_L and Ψ_R one can form a Dirac spinor:

$$\Psi = \Psi_L + \Psi_R, \quad (32)$$

or two Majorana spinors:

$$\chi = \Psi_L + \Psi_L^c, \quad \Psi_L^c \equiv C \bar{\Psi}_L^T, \quad (33)$$

$$\omega = \Psi_R + \Psi_R^c, \quad \Psi_R^c \equiv C \bar{\Psi}_R^T. \quad (34)$$

Majorana spinors have the property of being self-conjugate, that is:

$$\chi^c = \chi, \quad \omega^c = \omega, \quad \left(\Psi^c \equiv C \bar{\Psi}^T = C \gamma_0^T \Psi^* \right). \quad (35)$$

The most general definition of a Majorana spinor allows for a relative phase in the components of χ and of ω which would manifest itself in the self conjugate condition.

3. CP violation in the quark and lepton sectors

3.1. The quark sector

A thorough discussion of CP violation in the SM and in some of its extensions can be found in [8]. Here we only address a selected number of topics.

Gauge invariance does not constrain the flavour structure of Yukawa interactions. As a result, in the SM quark masses and mixing are arbitrary. It has been shown that gauge theories with fermions, but without scalar fields, do not break CP symmetry [9]. A Higgs doublet is used in the SM to break both the gauge symmetry and generate fermion masses through Yukawa interactions. Yukawa couplings have the special feature of being the only couplings of the SM which can be complex. All other couplings are constrained to be real, by hermiticity. This is the essential reason why, in the context of the SM, Yukawa couplings play a crucial rôle in generating CP violation. Indeed CP violation in the SM can only arise from the simultaneous presence of Yukawa and gauge interactions. For three or more fermion generations CP violation can be broken at the Lagrangian level. In the SM where a single Higgs doublet is introduced, it is not possible to have spontaneous CP violation since any phase in the vacuum expectation value of the neutral Higgs can be eliminated by rephasing the Higgs field. Furthermore, in the SM it is also not possible to violate CP explicitly in the Higgs sector since gauge invariance together with renormalizability restrict the potential:

$$V = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi \right)^2 + \text{h.c.} \quad (36)$$

to have only quadratic and quartic couplings and hermiticity constrains both of these terms to be real.

The Yukawa interactions for the quark sector can be written as:

$$\mathcal{L}_Y(\text{quarks}) = g_{ij} \overline{q_{L_i}^0} \tilde{\phi} u_{R_j}^0 + f_{ij} \overline{q_{L_i}^0} \phi d_{R_j}^0 + \text{h.c.} \quad (37)$$

with q_L^0 the lefthanded quark doublets and $\tilde{\phi} = i\tau_2 \phi^*$. After SSB the following quark mass terms are generated

$$\mathcal{L}_m(\text{quarks}) = -\overline{u_L^0} m_u u_R^0 - \overline{d_L^0} m_d d_R^0 + \text{h.c.} \quad (38)$$

We are still in a WB, so the charged current is diagonal, of the form:

$$\mathcal{L}_W(\text{quarks}) = -\frac{g}{\sqrt{2}} W_\mu^+ \overline{u_L^0} \gamma^\mu d_L^0 + \text{h.c.} \quad (39)$$

The mass matrices are general complex matrices and may be diagonalized through a bi-unitary transformation:

$$\begin{aligned} u_L &= U_L^u u_L^0, & u_R &= U_R^u u_R^0, \\ d_L &= U_L^d u_L^0, & d_R &= U_R^d d_R^0, \end{aligned} \quad (40)$$

such that:

$$U_L^{u\dagger} m_u U_R^u = \text{diag} (m_u, m_c, m_t), \quad (41)$$

$$U_L^{d\dagger} m_d U_R^d = \text{diag} (m_d, m_s, m_b). \quad (42)$$

After this transformation the charged currents are no longer diagonal. In terms of quark mass eigenstates the charged currents are now given by:

$$\mathcal{L}_W(\text{quarks}) = -\frac{g}{\sqrt{2}} W_\mu^+ \overline{u_L} \gamma^\mu V_{\text{CKM}} d_L + \text{h.c.}, \quad (43)$$

where $V_{\text{CKM}} = U_L^{u\dagger} U_L^d$, denotes the Cabibbo–Kobayashi–Maskawa (CKM) matrix. The appearance of a nontrivial CKM matrix in the charged currents reflects the fact that the Hermitian matrices H_u and H_d defined as:

$$H_u = m_u m_u^\dagger, \quad H_d = m_d m_d^\dagger, \quad (44)$$

are in general diagonalized by different unitary matrices:

$$U_L^{u\dagger} H_u U_L^u = \text{diag} (m_u^2, m_c^2, m_t^2), \quad (45)$$

$$U_L^{d\dagger} H_d U_L^d = \text{diag} (m_d^2, m_s^2, m_b^2). \quad (46)$$

In fact, of the four unitary matrices appearing in Eqs. (41) and (42) only the matrices U_L^u and U_L^d play a rôle in generating V_{CKM} which encodes the

physical quark mixing and CP violation. In the SM, one can use the freedom to make WB transformations to choose, without loss of generality, a basis where m_u , m_d are hermitian. Furthermore, one may also choose without loss of generality a basis where m_u (or m_d) is diagonal and m_d (or m_u) are hermitian.

Given a Lagrangian, obtained for instance from model building, one may ask whether or not it violates CP. In the context of the SM one may always investigate the CP properties by going to the physical basis and analysing the CKM matrix. However, it may be useful to try to answer the same question still in a WB without requiring cumbersome changes of basis. In this case the relevant information is contained in the matrices m_u and m_d . The general method allows for construction of weak basis invariants which have to vanish in order for CP symmetry to hold and was first proposed in [10] for the Standard Model. Weak basis invariant conditions relevant for CP violation in the leptonic sector were later developed and are discussed in some detail in the next subsection. This approach has been widely applied in the literature to study CP violation in many other scenarios [11]. The strategy is to apply the most general CP transformation for fermion fields in a WB, *i.e.*, leaving the gauge interaction invariant:

$$\begin{aligned} \text{CP}u_L^0(\text{CP})^\dagger &= U'\gamma^0 C \overline{u_L^0}^T, & \text{CP}u_R^0(\text{CP})^\dagger &= V'\gamma^0 C \overline{u_R^0}^T, \\ \text{CP}d_L^0(\text{CP})^\dagger &= U'\gamma^0 C \overline{d_L^0}^T, & \text{CP}d_R^0(\text{CP})^\dagger &= W'\gamma^0 C \overline{d_R^0}^T, \\ \text{CP}W_\mu^+(\text{CP})^\dagger &= -(-1)^{\delta_{0\mu}} W_\mu^-, \end{aligned} \quad (47)$$

where U' , V' , W' are unitary matrices acting in flavour space not related to those introduced in the previous section. This transformation can be viewed as a combination of the CP transformation of a single fermion field with a WB transformation [12]. Invariance of the mass terms under the above CP transformation, requires that the following relations have to be satisfied [10]:

$$U'^\dagger m_u V' = m_u^*, \quad (48)$$

$$U'^\dagger m_d W' = m_d^*. \quad (49)$$

It can be easily seen that if there are unitary matrices U' , V' , W' satisfying Eqs. (48), (49) in one particular WB, then a solution exists for any other WB. It is also clear that for m_u and m_d real these conditions are trivially satisfied for U' , V' , W' equal to the identity matrix. This shows that the existence of CP violation in the SM does require Yukawa couplings to be complex. In this form these conditions are not yet very useful since at this stage one just replaced the requirement of diagonalizing the mass matrices

by that of finding these three unitary matrices. However, combining these equations in such a way as to end up with similarity transformations one may be rid of the unitary matrices and derive necessary and sufficient conditions for CP invariance, expressed in terms of invariants (traces, determinants). In this way one may derive the following condition

$$\text{Tr} [H_u, H_d]^3 = 0, \quad (50)$$

which is a necessary and sufficient condition for CP invariance in the SM with three generations [10]. This invariant condition can be applied in any WB, as was already stressed. It can also be expressed in terms of physical quantities in the form:

$$\begin{aligned} \text{Tr} [H_u, H_d]^3 = & 6i (m_t^2 - m_c^2) (m_t^2 - m_u^2) (m_c^2 - m_u^2) (m_b^2 - m_s^2) \\ & \times (m_b^2 - m_d^2) (m_s^2 - m_d^2) \text{Im} (V_{12} V_{23} V_{13}^* V_{22}^*), \end{aligned} \quad (51)$$

where V_{ij} denote the entries of V_{CKM} . For three generations the condition of Eq. (50) is equivalent to:

$$\det [H_u, H_d] = 0. \quad (52)$$

This expression was first given in [13]. Note that the condition of Eq. (52) only applies to an odd number of generations, while Eq. (50) is a necessary condition for CP invariance, in any number of generations.

In the physical basis, *i.e.* after diagonalisation of the quark mass matrices, the only terms of the SM Lagrangian that may violate CP are the charged current interactions which are parametrised by the V_{CKM} matrix. CP can only be violated if V_{CKM} is a complex matrix. However, not all of its phases have physical meaning since there is freedom to rephase the quark fields:

$$u_i = e^{i\alpha_i} u'_i, \quad d_j = e^{i\beta_j} d'_j, \quad V'_{ij} = e^{i(\beta_j - \alpha_i)} V_{ij}. \quad (53)$$

This allows to eliminate five phases out of the nine that may in principle be present in V_{CKM} . Physically meaningful quantities must be invariant under rephasing of the fields. Rephasing invariants involve products of several elements of V_{CKM} and its complex conjugate. In the absence of zero entries all rephasing invariants may be expressed in terms of the simplest ones which are the moduli of matrix elements $|V_{ij}|$ and the terms called quartets which are of the form:

$$Q_{ijkl} \equiv V_{ij} V_{kl} V_{il}^* V_{kj}^* \quad (54)$$

for $i \neq k$ and $j \neq l$.

In the physical basis the most general CP transformation for the quarks and for the W boson are of the form [8]:

$$\begin{aligned}\text{CP } u_i \text{ (CP)}^\dagger &= e^{(i\sigma_i)} \gamma^0 C \bar{u}_i^T, \\ \text{CP } d_j \text{ (CP)}^\dagger &= e^{(i\kappa_j)} \gamma^0 C \bar{d}_j^T, \\ \text{CP } W_\mu^+ \text{ (CP)}^\dagger &= -e^{(i\rho_w)} W_\mu^-.\end{aligned}\tag{55}$$

Note that the CP transformation no longer mixes fermion generations since the quark mass terms are already diagonal and there is no mass degeneracy. Invariance of the Lagrangian under this transformation requires

$$V_{ij}^* = e^{i(\rho_w + \kappa_j - \sigma_i)} V_{ij}.\tag{56}$$

This equation can always be made to hold if one considers a single matrix element of V_{CKM} , because the CP transformation phases σ_i , κ_j and ρ_w , are arbitrary. Obviously for a real V_{CKM} this condition is trivially verified. However, imposing this condition on each element of V_{CKM} simultaneously forces the quartets and all other rephasing-invariant functions of V_{CKM} , to be real. In general, [8] *there is CP violation in the SM if and only if any of the rephasing-invariant functions of the CKM matrix is not real.*

It can be easily shown that as a consequence of the orthogonality of any pair of different rows or columns of the CKM matrix the imaginary parts of all quartets are equal up to their sign. Let us count the number of independent parameters in V_{CKM} . An $n \times n$ unitary matrix has n^2 independent parameters. Taking into account that $(2n - 1)$ phases can be removed from V_{CKM} , through rephasing of the $2n$ quark fields (note that an overall rephasing of quark fields does not affect V_{CKM}) the number of physical parameters in V_{CKM} is

$$N_{\text{param}} = n^2 - (2n - 1) = (n - 1)^2.\tag{57}$$

An orthogonal $n \times n$ matrix $O(n)$ is parametrised by $n(n - 1)/2$ rotation angles which are sometimes called Euler angles. A unitary matrix is a complex extension of an orthogonal matrix. Therefore, out of the N_{param} parameters of V_{CKM} ,

$$N_{\text{angle}} = \frac{1}{2}n(n - 1)\tag{58}$$

should be identified with rotation angles. The remaining

$$N_{\text{phase}} = N_{\text{param}} - N_{\text{angle}} = \frac{1}{2}(n - 1)(n - 2)\tag{59}$$

parameters of V_{CKM} are physical phases. For $n = 3$ there is one phase and three mixing angles, and V_{CKM} can be written as:

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (60)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ and δ is the only phase. δ is called a Dirac type phase because it is the Dirac character of the quarks that allows to rephase away all other phases leaving only δ . This is the so-called standard parametrisation of V_{CKM} [14]. The mechanism just described for CP violation is the Kobayashi–Maskawa (KM) mechanism. In the SM this is the only source of CP violation. Notice that δ is not a rephasing invariant quantity, it is only meaningful within a given parametrisation.

A particularly useful phase convention for V_{CKM} only in terms of rephasing invariant quantities is [8]:

$$V_{\text{CKM}} = \begin{pmatrix} |V_{ud}| & |V_{us}| e^{i\chi'} & |V_{ub}| e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| e^{-i\beta} & -|V_{ts}| e^{i\chi} & |V_{tb}| \end{pmatrix}, \quad (61)$$

where the CP-violating phases introduced in Eq. (61) are defined by:

$$\begin{aligned} \beta &= \arg(-V_{cd}V_{cb}^*V_{td}^*V_{tb}) , & \gamma &= \arg(-V_{ud}V_{ub}^*V_{cd}^*V_{cb}) , \\ \chi &= \arg(-V_{ts}V_{tb}^*V_{cs}^*V_{cb}) , & \chi' &= \arg(-V_{cd}V_{cs}^*V_{ud}^*V_{us}) . \end{aligned} \quad (62)$$

Without imposing the constraints of unitarity, the four rephasing invariant phases, together with the nine moduli are all the independent physical quantities contained in V_{CKM} . In the SM, where unitarity holds, these quantities are related by a series of exact relations which provide a stringent test of the SM [15].

Unitarity of V_{CKM} implies orthogonality of rows and columns. Let us consider the orthogonality between the first and third column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 . \quad (63)$$

This equation may be interpreted as representing a triangle in the complex plane. One may in principle build in this way three different triangles from orthogonality of columns and three other triangles from orthogonality of rows. Out of the six unitarity triangles, only two have sides of comparable size, the one corresponding to Eq. (63) and the one corresponding to orthogonality of the first and the third rows. A remarkable feature of the unitarity triangles is the fact that all of them have the same area. Phenomenologically the most interesting triangle is the one depicted in Fig. 1, corresponding to

Eq. (63) which is often referred to in the literature as *the* unitary triangle. The angle α , represented in the figure, is defined as $\alpha \equiv \arg(-V_{td}V_{tb}^*V_{ud}V_{ub})$ and obeys by definition the relation $\alpha = \pi - \beta - \gamma$. Rephasing of the CKM matrix, as in Eq. (53), rotates the triangle as a whole, since under rephasing

$$V'_{ij}V'^*_{ik} = e^{i(\beta_j - \beta_k)} V_{ij}V^*_{ik}, \quad (64)$$

therefore, the orientation of the triangle is physically meaningless. However, the shape of the triangle remains unchanged because both its inner angles and the lengths of its sides are rephasing invariant.

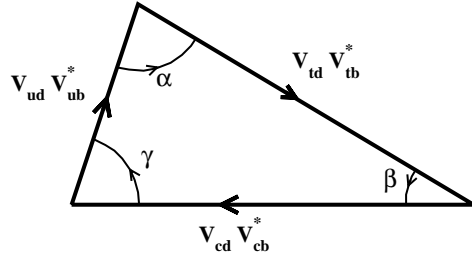


Fig. 1. The Standard Model unitary triangle.

In the SM neutrinos are massless and there is no leptonic mixing. Furthermore, in the SM there are no flavour changing neutral currents (FCNC) at tree level neither mediated by the Z nor by neutral scalar fields. Therefore, the only source of CP violation in the SM is the KM mechanism just described.

3.2. The lepton sector

In the previous section the seesaw mechanism was explained, working on the WB where the mass matrix m_l was chosen to be real and diagonal. In order to discuss CP violation in this framework it is useful to adopt as a starting point a WB where, in addition to m_l , the matrix M_R is also chosen to be real and diagonal. It should be clear from Eqs. (15) and (18) that this is indeed a possible choice of WB. In this case, since we are working in a framework where m_L is not introduced, all phases appear in the matrix m_D and the leptonic mass matrix becomes:

$$\mathcal{L}_m = -\overline{\nu}_L^0 m_D \nu_R^0 - \frac{1}{2} \nu_R^{0T} C D_R \nu_R^0 - \overline{l}_L^0 d_l l_R^0 + \text{h.c.} . \quad (65)$$

The matrix D_R coincides with the matrix D of Eq. (21) up to negligible corrections. This can be seen from Eq. (27) since the matrix G is very

suppressed and m_D is of order much smaller than M_R . In this WB, the exact relation given by Eq. (31) is very well approximated by

$$G = m_D D^{-1}. \quad (66)$$

The matrix d_l is also diagonal and contains the masses of the charged leptons, therefore, we could have considered dropping the 0 upper index for these fields since these are already physical fields up to phase redefinitions. The matrix m_D is perfectly general, it contains nine real parameters — the moduli of each entry — and nine phases. However, there is still freedom to rotate away three of these phases through the rephasing of the ν_L^0 fields. These phases would appear in \mathcal{L}_W of Eq. (17), however, they can be eliminated by rotating l_L^0 . Finally a rotation of the fields l_R^0 would also eliminate these phases from d_l . We are thus left with six real parameters in D_R and d_l plus nine real parameters in m_D and six phases. The following special possible parametrisations for m_D :

$$m_D = UY_\Delta \quad \text{or} \quad m_D = UH, \quad (67)$$

where U is an unitary matrix, H is an Hermitian matrix and Y_Δ is a lower triangular matrix, have revealed themselves particularly useful in model building. The number of parameters in this WB equals the number of physical parameters — in the form of masses and mixing — obtained after diagonalisation of the mass matrices. In this case there are the nine masses of the three charged leptons, the three light neutrinos and the three heavy neutrinos, together with six mixing angles required to parametrise the 3×6 physical block (K, G) of the 6×6 unitary matrix V [16] as well as six phases [17]. In general, with m_L different from zero one would have twelve independent phases [16] in the mixing matrix. It is easy to understand why having m_L equal to zero reduces the number of CP-violating phases. Notice that m_L is in general a complex symmetric matrix and, therefore, would have six phases in the case of three generations. Once m_L is equal to zero, from $\mathcal{M}^* = V^* \mathcal{D} V^\dagger$, the zero entry in the upper left block of \mathcal{M} implies:

$$K^* d K^\dagger + G^* D G^\dagger = 0. \quad (68)$$

providing additional constraints for the matrices K and G beyond those derived from unitarity of the matrix V .

It is quite straightforward to determine the number of independent CP restrictions, by making use of the WB basis chosen above, for the general case of n generations [18]. Invariance of the mass terms under the most general CP transformation which leaves the gauge interaction invariant:

$$\begin{aligned} \text{CP} l_L^0 (\text{CP})^\dagger &= U' \gamma^0 C \bar{l}_L^{0T}, & \text{CP} l_R^0 (\text{CP})^\dagger &= V' \gamma^0 C \bar{l}_R^{0T}, \\ \text{CP} \nu_L^0 (\text{CP})^\dagger &= U' \gamma^0 C \bar{\nu}_L^{0T}, & \text{CP} \nu_R^0 (\text{CP})^\dagger &= W' \gamma^0 C \bar{\nu}_R^{0T}, \end{aligned} \quad (69)$$

requires that the following relations have to be satisfied:

$$W'^T D_R W' = -D_R^*, \quad (70)$$

$$U'^\dagger m_D W' = m_D^*, \quad (71)$$

$$U'^\dagger d_l V' = d_l^*. \quad (72)$$

From Eq. (70) W' is constrained to be of the form

$$W' = \text{diag}(\exp(i\alpha_1), \exp(i\alpha_2), \dots, \exp(i\alpha_n)) \quad (73)$$

and the α_i have to satisfy:

$$\alpha_i = (2p_i + 1)\frac{\pi}{2} \quad (74)$$

with p_i integer numbers. Multiplying Eq. (72) by its Hermitian conjugate, one concludes that U' has to be of the form:

$$U' = \text{diag}(\exp(i\beta_1), \exp(i\beta_2), \dots, \exp(i\beta_n)) , \quad (75)$$

where β_i are arbitrary phases. From Eqs. (71), (73), (75) it follows then that CP invariance constrains the matrix m_D to satisfy:

$$\arg(m_D)_{ij} = \frac{1}{2}(\beta_i - \alpha_j) . \quad (76)$$

Note that the α_i are fixed by Eq. (74), up to discrete ambiguities. Therefore, CP invariance constrains the matrix m_D to have only n free phases β_i . Since m_D is in an arbitrary matrix, with n^2 independent phases, it is clear that there are $n^2 - n$ independent CP restrictions. This number equals, of course, the number of independent CP-violating phases which appear in general in this model. In the WB which we are considering, these phases appear as $n(n - 1)$ phases which cannot be removed from m_D . It should be pointed out that it is also possible to generate neutrino masses in such a framework without requiring the number of righthanded and lefthanded neutrinos to be equal. When the number of righthanded neutrinos is n' different from n , the matrix m_D has dimension nn' , in this case the number of CP-violating phases is equal to $nn' - n$.

In the context of seesaw, CP violation occurs both at low and high energies. It is clear from Eq. (23) that CP violation at high energies will manifest itself in the decays of heavy neutrinos. These decays provide a possible source for the generation in the early Universe of the baryon asymmetry of the Universe (BAU) through leptogenesis [19]. A detailed analysis on the present theoretical and experimental situation in neutrino physics and on where it is going in the future is done in Ref. [20].

3.2.1. CP violation at low energies

We start by summarising what is presently known about neutrino masses and leptonic mixing. For a detailed account of the present experimental status of neutrino physics see the contribution of D.L. Wark, “Experimental neutrino physics” in this volume. It is by now experimentally established that neutrinos have masses and that there is mixing in the leptonic sector. At low energies, only the first term of Eq. (23) involving charged leptons and light neutrino couplings to the W boson is relevant, since heavy neutrinos in the seesaw framework are expected to have masses that may be of order 10^{13} GeV or even larger. Such heavy neutrinos cannot be produced at present colliders and would have decayed in the early Universe.

In the seesaw framework, described before, m_{eff} given by Eq. (30) is an effective Majorana mass matrix and the mixing matrix K can be treated as the unitary matrix that diagonalises m_{eff} in Eq. (29). Deviations from unitarity cannot be experimentally observed at the level predicted in this framework. With the usual conventions, where the Majorana mass term is given by $\nu_L^{0T} C m_\nu \nu_L^0$, and the PMNS matrix defined by $U_{\text{PMNS}}^T m_\nu U_{\text{PMNS}} = \text{diag}(m_1, m_2, m_3)$, we have the following correspondence:

$$m_{\text{eff}} = m_\nu^* \quad \text{and} \quad U_{\text{PMNS}} = K^* . \quad (77)$$

This effective low energy physics corresponds to integrating out the heavy neutrinos. Majorana mass terms are symmetric by construction. In fact, anticommutation of fermion fields together with the property that C is an antisymmetric matrix, $C^T = -C$, which follows from its definition, leads to $\nu_{Li}^{0T} C \nu_{Lj}^0 = \nu_{Lj}^{0T} C \nu_{Li}^0$.

In the physical basis the mass terms and the leptonic charged currents in the low energy effective theory are of the form:

$$\mathcal{L}_{\text{eff}}^{\text{phys}} = \nu_L^T C d \nu_L + \overline{l_L} d_l l_R + \frac{g}{\sqrt{2}} \overline{l_{iL}} \gamma_\mu U_{ij} \nu_{jL} W^{+\mu} + \text{h.c.} \quad (78)$$

with d and d_l diagonal real and positive matrices. For simplicity, we have dropped the index PMNS in the mixing matrix U . The 3×3 unitary matrix U is in general parametrised by six phases and three mixing angles. Three of these phases can be factored out to the left and rotated away through the redefinition of the charged leptons l_L . The phases thus appearing in d_L can be eliminated by the simultaneous redefinition of the fields l_R . Another two of the six phases of U can be factored out to the right. However, in this case, these two phases are physical, since rotating them away from U simply corresponds to transferring them to the mass term of the light neutrinos. This is an important difference from the quark sector resulting from the fact that in the seesaw framework neutrinos have Majorana masses and are Majorana

particles, unlike quarks. These factorizable phases that cannot be removed from the theory are called Majorana phases. As a result, in the seesaw framework, in the effective low energy theory, there are additional sources for CP violation beyond the Kobayashi–Maskawa mechanism of the hadronic sector, one is a Dirac type CP-violating phase appearing in the leptonic sector, analogous to the one of the quark sector, together with two additional Majorana type phases. The PMNS matrix may be parametrised by a matrix of the same form as the one given in Eq. (60) for V_{CKM} , multiplied by a diagonal matrix P with two phases:

$$P = \text{diag} (1, e^{i\alpha}, e^{i\beta}), \quad (79)$$

with α and β denoting phases associated to the Majorana character of neutrinos.

It is important to notice that including a phase γ in the mass term is equivalent to work with a Majorana spinor field χ_ν defined by:

$$\chi_\nu = \nu_L + e^{i\gamma} \nu_L^c \quad (80)$$

since in this case the Majorana mass term is given by:

$$\begin{aligned} \mathcal{L}_M(\chi_\nu) \equiv m_M \overline{\chi_\nu} \chi_\nu &= m_M (\overline{\nu_L} + e^{-i\gamma} \overline{\nu_L^c}) (\nu_L + e^{i\gamma} \nu_L^c) \\ &= m_M e^{-i\gamma} \nu_L^T C \nu_L + \text{h.c.} . \end{aligned} \quad (81)$$

A Majorana spinor defined as χ_ν obeys the following self-conjugate relation:

$$\chi_\nu^c = \nu_L^c + e^{-i\gamma} \nu_L = e^{-i\gamma} \chi_\nu . \quad (82)$$

Experimentally it is not yet known whether any of the three CP-violating phases of the leptonic sector is different from zero. The current experimental bounds on neutrino masses and leptonic mixing are [14]:

$$\Delta m_{21}^2 = 8.0_{-0.3}^{+0.4} \times 10^{-5} \text{ eV}^2, \quad (83)$$

$$\sin^2(2\theta_{12}) = 0.86_{-0.04}^{+0.03}, \quad (84)$$

$$|\Delta m_{32}^2| = (1.9 \text{ to } 3.0) \times 10^{-3} \text{ eV}^2, \quad (85)$$

$$\sin^2(2\theta_{23}) > 0.92, \quad (86)$$

$$\sin^2 \theta_{13} < 0.05, \quad (87)$$

with $\Delta m_{ij}^2 \equiv m_j^2 - m_i^2$. The angle θ_{23} may be maximal, meaning 45° , whilst θ_{12} is already known to deviate from this value. At the moment, there is

only an experimental upper bound on the angle θ_{13} . All present data is consistent with the Harrison, Perkins and Scott (HPS) mixing matrix [21]:

$$\begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \quad (88)$$

which exhibits a so-called tri-bimaximal mixing. There have been various attempts at introducing family symmetries leading to this structure. Some examples can be found in Ref. [22].

It is also not yet known whether the ordering of the light neutrino masses is normal, *i.e.*, $m_1 < m_2 < m_3$ or inverted $m_3 < m_1 < m_2$. The scale of the neutrino masses is also not yet established. Direct kinematical limits from Mainz [23] and Troitsk [24] place an upper bound on m_β defined as:

$$m_\beta \equiv \sqrt{\sum_i |U_{ei}|^2 m_i^2} \quad (89)$$

given by $m_\beta \leq 2.3 \text{ eV}$ (Mainz), $m_\beta \leq 2.2 \text{ eV}$ (Troitsk). The proposed KATRIN experiment [25] is expected to be sensitive to $m_\beta > 0.2 \text{ eV}$. The spectrum may vary from extreme hierarchy, between the two lightest neutrino masses, to three quasidegenerate masses. Examples of the possible extreme cases are:

$$m_1 \sim 0 \text{ (or } e.g. \sim 10^{-6} \text{ eV)}, \quad m_2 \simeq 9 \times 10^{-3} \text{ eV}, \quad m_3 \simeq 5 \times 10^{-2} \text{ eV}, \quad (90)$$

corresponding to normal spectrum, hierarchical, or else:

$$m_3 \sim 0 \text{ (or } e.g. \sim 10^{-6} \text{ eV)}, \quad m_1 \simeq m_2 \simeq 0.05 \text{ eV}, \quad (91)$$

corresponding to inverted spectrum, hierarchical, or else:

$$m_1 \simeq 1 \text{ eV}, \quad m_2 \simeq 1 \text{ eV}, \quad m_3 \simeq 1 \text{ eV}, \quad (92)$$

corresponding to almost degeneracy.

The limit of exact mass degeneracy of Majorana neutrinos was studied in [26] where it was shown that it has the remarkable feature of allowing for the existence of mixing and CP violation. In the exact degeneracy limit the leptonic mixing matrix is parametrised by only two angles and one phase and there is no Dirac type CP violation in the leptonic sector. However, there may be Majorana type CP violation.

It is possible to obtain information on the absolute scale of neutrino masses from the study of the cosmic microwave radiation spectrum together with the study of the large scale structure of the universe. For a flat universe, WMAP combined with other astronomical data leads to [27] $\sum_i m_i \leq 0.66 \text{ eV}$ (95% CL).

Neutrino-less double beta decay can also provide information on the absolute scale of the neutrino masses. In the present framework, in the absence of additional lepton number violating interactions, it provides a measurement of the effective Majorana mass given by:

$$m_{ee} = |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2| . \quad (93)$$

The present upper limit is $m_{ee} \leq 0.9 \text{ eV}$ [28] from the Heidelberg–Moscow [29] and the IGEX [30] experiments. There is a claim of discovery of neutrino-less double beta decay by the Heidelberg–Moscow collaboration [31]. Interpreted in terms of a Majorana mass of the neutrino, this implies m_{ee} between 0.12 eV to 0.90 eV. This result awaits confirmation from other experiments and would constitute a major discovery. It would set the scale of the neutrino masses and answer the still open question of whether or not neutrinos are Majorana particles.

Dirac type CP violation occurs whenever the imaginary parts of the quartets similar to those defined by Eq. (54) for the quark sector:

$$I = \text{Im } U_{ij} U_{kl} U_{il}^* U_{kj}^* , \quad (i \neq k, j \neq l) , \quad (94)$$

differ from zero. As previously emphasised, unitarity of U insures that all imaginary parts are equal up to their sign. Therefore, if any entry of the leptonic mixing matrix is zero, there is no Dirac type CP violation in the leptonic sector. On the other hand, it can be easily verified from the structure of the indices of the quartets that Majorana phases always cancel out in the quartets. The simplest rephasing invariants in the leptonic sector include moduli of the matrix U and quartets, as in the quark sector, and, in addition, products of the form $U_{ij} U_{ik}^*$, with no sum implied. Rephasing invariance of these products results from the fact that the only rephasing transformations allowed in this sector are $l_{L_i, R_i} \rightarrow e^{i\lambda_i} l_{L_i, R_i}$. The minimal CP-violating quantities are:

$$S_i \equiv \text{Im } U_{ij} U_{ik}^* , \quad \text{no sum in } i , \quad (95)$$

provided the real part of $U_{ij} U_{ik}^*$ is different from zero [32]. Notice that the S_i are sensitive to the presence of Majorana phases. In the leptonic sector one can construct two types of unitarity triangles [32]. The so-called Dirac triangles, obtained through multiplication of rows of U , are similar to those

in the quark sector. Majorana phases cancel in the product of each term and under rephasing these triangles rotate in the complex plane as:

$$\sum_i U_{ij} U_{kj}^* \rightarrow e^{i(\lambda_i - \lambda_k)} \sum_i U_{ij} U_{kj}^*. \quad (96)$$

Therefore, their orientation has no physical meaning. They share a common area proportional to $|I|$. The vanishing of this area does not imply that the minimal CP-violating quantities S_i are zero and CP can still be violated. The second type of unitarity triangles are constructed through multiplication of columns of U . These are the so-called Majorana triangles given by

$$T_{jk} = U_{ej} U_{ek}^* + U_{\mu j} U_{\mu k}^* + U_{\tau j} U_{\tau k}^*. \quad (97)$$

In this case all terms in the sum are rephasing invariant. These triangles do not rotate under rephasing, and they are sensitive to the presence of Majorana type phases. Fig. 2 depicts an example of a hypothetical Majorana triangle, obviously not based on current experimental observations.

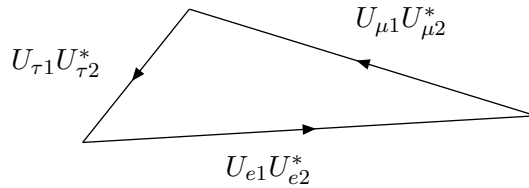


Fig. 2. Majorana unitarity triangle T_{12} . Its orientation is fixed by the Majorana phases and it cannot be rotated in the complex plane.

The Majorana triangles provide the necessary and sufficient conditions for CP conservation [32]:

- (i) Vanishing of their common area;
- (ii) Orientation of all Majorana triangles along the direction of the real or of the imaginary axis.

The first condition implies that the Dirac phase vanishes. The second condition implies that the Majorana phases do not violate CP, provided we are working with a real diagonal d matrix, *i.e.*, provided that the fields of the massive Majorana neutrinos satisfy self-conjugate relations which do not contain phase factors.

CP conservation in the leptonic sector does not require that U be a real matrix. In fact when a Majorana triangle is oriented along the imaginary axis, Majorana phases are present but do not violate CP. This is due to the existence of massive Majorana neutrinos with opposite CP eigenvalues, also

called CP parities [33]. In order to illustrate this point let us consider the following WB in which the charged leptons have already been diagonalised:

$$\mathcal{L}_{\text{mass}} = -\nu_L^{0\text{T}} C m_\nu \nu_L^0 - \bar{l}_L d_l l_R + \text{h.c.} . \quad (98)$$

It is obvious from previous analyses that CP is conserved provided m_ν is real. Since Majorana mass terms are symmetric by construction, the real matrix m_ν can be diagonalised by an orthogonal real transformation O of the form:

$$O^T m_\nu O = \text{diag} (m_1, m_2, m_3) \quad (99)$$

at this stage the m_i are real but may be positive or negative. Two cases are possible:

- (i) all m_i have equal sign;
- (ii) one m_i has a sign different from the other two.

Let us consider an example of case (ii), for instance m_2 negative, m_1 and m_3 positive:

$$O^T m_\nu O = \text{diag} (|m_1|, -|m_2|, |m_3|) . \quad (100)$$

Positive masses are obtained by making the transformation:

$$K \text{diag} (|m_1|, -|m_2|, |m_3|) K = \text{diag} (|m_1|, |m_2|, |m_3|) , \quad (101)$$

with $K = \text{diag} (1, i, 1)$. All mass terms are now real positive and diagonal and the mixing matrix U is given by $U = OK$, so that the charged current interaction can be explicitly written as:

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} (\bar{e}, \bar{\mu}, \bar{\tau})_L \gamma^\mu \begin{pmatrix} O_{11} & iO_{12} & O_{13} \\ O_{21} & iO_{22} & O_{23} \\ O_{31} & iO_{32} & O_{33} \end{pmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}_L W_\mu^+ + \text{h.c.} . \quad (102)$$

A CP transformation of the mass eigenstates is of the form:

$$\text{CP} \nu_{iL} (\text{CP})^\dagger = \eta_{i\text{CP}} C \nu_{iL}^* , \quad \text{CP} l_{jL} (\text{CP})^\dagger = \eta_{j\text{CP}}^l C l_{jL}^* . \quad (103)$$

Since d is a Majorana mass matrix, under CP $d \rightarrow -d$, which requires that the $\eta_{\text{CP}} = \pm i$. The relative sign of $\eta_{i\text{CP}}$ is called the relative CP parity of the neutrinos. The structure of the charged weak interactions fixes the relative CP parities. Suppose that all O_{ij} are nonvanishing and take as initial choice, *e.g.*:

$$\text{CP} \nu_{1L} (\text{CP})^\dagger = +i C \nu_{1L}^* , \quad \text{i.e.} \quad \eta_{1\text{CP}} = +i , \quad (104)$$

then O_{11}, O_{21}, O_{31} force $\eta_{\text{CP}}^e = \eta_{\text{CP}}^\mu = \eta_{\text{CP}}^\tau = +i$, the couplings iO_{j2} constrain ν_2 to have $\eta_{2\text{CP}} = -i$, the couplings O_{j3} constrain ν_3 to have $\eta_{3\text{CP}} = +i$.

Therefore, we are forced to assign a CP parity to ν_2 which is different from that of ν_1 and ν_3 . It is clear from this discussion that CP parities can only be defined in the CP conserving case. Furthermore, only relative CP parities have meaning.

It was shown in [26] that, even in the limit of exact degeneracy and in the CP conserving case, leptonic mixing cannot be rotated away provided neutrinos have different CP parities.

From the point of view of model building it is useful to derive WB invariant conditions for CP conservation in the leptonic sector analogous to those derived for the quark sector. The procedure is analogous to the one outlined before and was first applied to the leptonic sector in [34].

Leptonic CP violation at low energies can be detected through neutrino oscillations which are sensitive to the Dirac type phase, but insensitive to the Majorana type phases in the PMNS matrix. The strength of Dirac type CP violation can be obtained from the following low energy WB invariant:

$$\text{Tr} [h_{\text{eff}}, h_l]^3 = -6i \Delta_{21} \Delta_{32} \Delta_{31} \text{Im}\{(h_{\text{eff}})_{12}(h_{\text{eff}})_{23}(h_{\text{eff}})_{31}\}, \quad (105)$$

where $h_{\text{eff}} = m_{\text{eff}} m_{\text{eff}}^\dagger$, $h_l = m_l m_l^\dagger$, and $\Delta_{21} = (m_\mu^2 - m_e^2)$ with analogous expressions for Δ_{31} , Δ_{32} . The right-hand side of this equation is the computation of this invariant in the special WB where the charged masses are real and diagonal. This invariant is analogous to the one presented in Eq. (50), for the quark sector. It can also be fully expressed in terms of physical observables since

$$\text{Im}\{(h_{\text{eff}})_{12}(h_{\text{eff}})_{23}(h_{\text{eff}})_{31}\} = -\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2 I, \quad (106)$$

where I is the imaginary part of an invariant quartet of the leptonic mixing matrix U and is given by:

$$I \equiv \text{Im}[U_{11} U_{22} U_{12}^* U_{21}^*] = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{13}) \sin(2\theta_{23}) \cos(\theta_{13}) \sin \delta. \quad (107)$$

A value for θ_{13} close to the present experimental bound would be good news for the prospects of detection of low energy leptonic CP violation, mediated through a Dirac type phase and would correspond to I of order 10^{-2} for $\sin \delta$ of order one. Note that in the quark sector the corresponding I is of the order 10^{-5} . Many other relations which are necessary conditions for CP invariance can be derived. The Majorana character of the neutrinos provides additional sources for CP violation. Selecting from the necessary conditions a subset of restrictions which are also sufficient for CP invariance is in general not trivial. For three generations it was shown that the following four conditions are sufficient [34] to guarantee CP invariance:

$$\text{Im} \text{Tr} [h_l (m_{\text{eff}} m_{\text{eff}}^*) (m_{\text{eff}} h_l^* m_{\text{eff}}^*)] = 0, \quad (108)$$

$$\text{Im} \text{Tr} [h_l (m_{\text{eff}} m_{\text{eff}}^*)^2 (m_{\text{eff}} h_l^* m_{\text{eff}}^*)] = 0, \quad (109)$$

$$\text{Im Tr} [h_l (m_{\text{eff}} m_{\text{eff}}^*)^2 (m_{\text{eff}} h_l^* m_{\text{eff}}^*) (m_{\text{eff}} m_{\text{eff}}^*)] = 0, \quad (110)$$

$$\text{Im det} [(m_{\text{eff}}^* h_l m_{\text{eff}}) + (h_l^* m_{\text{eff}}^* m_{\text{eff}})] = 0 \quad (111)$$

provided that neutrino masses are nonzero and nondegenerate [35]. It can be easily seen that these conditions are trivially satisfied in the case of complete degeneracy ($m_1 = m_2 = m_3$). Yet there may still be CP violation in this case, as stated before. In this limit a necessary and sufficient condition [26] for CP invariance is:

$$G \equiv \text{Tr} [m_{\text{eff}}^* h_l m_{\text{eff}}, h_l^*]^3 = 0. \quad (112)$$

It is well known that the minimal structure that can lead to CP violation in the leptonic sector is two generations of lefthanded Majorana neutrinos provided that their masses be non degenerate and that none of them vanishes. In this case, it was proved [34] that the condition

$$\text{Im Tr } Q = 0, \quad (113)$$

with $Q = h_l m_{\text{eff}} m_{\text{eff}}^* m_{\text{eff}} h_l^* m_{\text{eff}}^*$ is a necessary and sufficient condition for CP invariance.

A more detailed discussion on WB invariant CP odd conditions relevant for the leptonic sector and neutrino mass models can be found in [36].

3.2.2. Leptogenesis

The observed baryon asymmetry of the Universe (BAU) is given by [37]:

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.1_{-0.2}^{+0.3}) \times 10^{-10}. \quad (114)$$

There have been many attempts at explaining the origin of this asymmetry. Some reviews can be found in Ref. [38] One might wonder whether it could simply result from an initial condition with no need for further explanation. However, presently it seems very likely that our Universe went through a period of inflation and inflation would have erased such a primordial asymmetry. Therefore, this asymmetry must have been generated after inflation. Sakharov [39] conditions require that there be baryon number violation, C and CP violation and out of equilibrium dynamics. It is by now established that the SM of electroweak interactions, where the Kobayashi–Maskawa mechanism is the only source of CP violation, cannot produce a large enough asymmetry [40–45]. Furthermore, a Higgs scalar mass above 80 GeV gives rise to a smooth phase transition and therefore there is no out-of-equilibrium dynamics. The observed BAU requires the existence of

physics beyond the SM. One of the most plausible explanations is Leptogenesis, since it relies on the only aspect of physics beyond the SM that has already been observed, to wit neutrino masses.

In this framework, the initial conditions are $B = 0$ and $L = 0$. A CP asymmetry is generated through out-of-equilibrium L -violating decays of heavy Majorana neutrinos [19] leading to a lepton asymmetry $L \neq 0$ while $B = 0$ is still maintained. Sphaleron processes [46], which are $(B + L)$ -violating and $B - L$ -conserving, partially transform the lepton asymmetry into a baryon asymmetry.

Fig. 3 shows the tree level and one loop diagrams giving rise to a lepton asymmetry, due to CP violation in the decay of the heavy Majorana neutrinos. The lepton number asymmetry ε_{N_j} , thus produced was computed by several authors [47–49]. Summing over all charged leptons one obtains for the asymmetry produced by decay of the heavy Majorana neutrino N_j into the charged leptons l_i^\pm ($i = e, \mu, \tau$):

$$\begin{aligned} \varepsilon_{N_j} &= \frac{g^2}{M_W^2} \sum_{k \neq j} \left[\text{Im} \left((m_D^\dagger m_D)_{jk} (m_D^\dagger m_D)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \\ &\quad \times \frac{1}{(m_D^\dagger m_D)_{jj}} \\ &= \frac{g^2}{M_W^2} \sum_{k \neq j} \left[(M_k)^2 \text{Im} \left((G^\dagger G)_{jk} (G^\dagger G)_{jk} \right) \frac{1}{16\pi} \left(I(x_k) + \frac{\sqrt{x_k}}{1 - x_k} \right) \right] \\ &\quad \times \frac{1}{(G^\dagger G)_{jj}}, \end{aligned} \quad (115)$$

where M_k denote the heavy neutrino masses, the variable x_k is defined as $x_k = M_k^2/M_j^2$ and $I(x_k) = \sqrt{x_k} \{1 + (1 + x_k) \log(x_k/(1 + x_k))\}$. From Eq. (115) it can be seen that, when one sums over all charged leptons, the lepton-number asymmetry is only sensitive to the CP-violating phases appearing in $m_D^\dagger m_D$ in the WB, where M_R is diagonal. Note that this combination is insensitive to rotations of the lefthanded neutrinos. In many

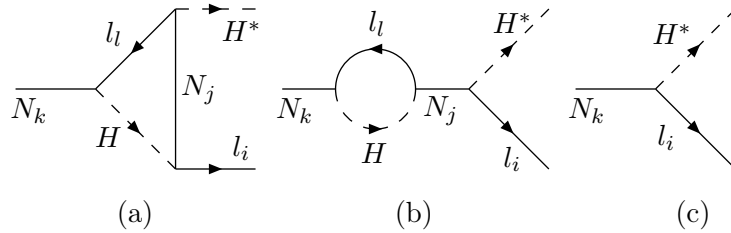


Fig. 3. One-loop and tree diagrams contributing to the asymmetry from the N_k decay.

flavour models the connection between low energy CP violation and leptogenesis is established in this basis, in scenarios where m_D verifies special constraints [50–54]. In the general case it is not possible to establish such a connection [18, 55].

Weak basis invariants relevant for leptogenesis were derived in [18]:

$$I_1 \equiv \text{Im Tr} [h_D H_R M_R^* h_D^* M_R] = 0, \quad (116)$$

$$I_2 \equiv \text{Im Tr} [h_D H_R^2 M_R^* h_D^* M_R] = 0, \quad (117)$$

$$I_3 \equiv \text{Im Tr} [h_D H_R^2 M_R^* h_D^* M_R H_R] = 0, \quad (118)$$

with $h_D = m_D^\dagger m_D$ and $H_R = M_R^\dagger M_R$. These constitute a set of necessary and sufficient conditions in the case of three heavy neutrinos. Different expressions of the same type can be derived following the same procedure.

The simplest leptogenesis scenario corresponds to the case of heavy hierarchical neutrinos where M_1 is much smaller than M_2 and M_3 . In this limit only the asymmetry generated by the lightest heavy neutrino is relevant, due to the existence of washout processes, and ε_{N_1} can be simplified into:

$$\varepsilon_{N_1} \simeq -\frac{3}{16\pi v^2} \left(I_{12} \frac{M_1}{M_2} + I_{13} \frac{M_1}{M_3} \right), \quad (119)$$

where

$$I_{1i} \equiv \frac{\text{Im} [(m_D^\dagger m_D)_{1i}^2]}{(m_D^\dagger m_D)_{11}}. \quad (120)$$

Thermal leptogenesis is a rather involved thermodynamical non-equilibrium process and depends on additional parameters. In the hierarchical case the baryon asymmetry only depends on four parameters [56–59]: the mass M_1 of the lightest heavy neutrino, together with the corresponding CP asymmetry ε_{N_1} in their decays, as well as the effective neutrino mass \widetilde{m}_1 defined as

$$\widetilde{m}_1 = \frac{(m_D^\dagger m_D)_{11}}{M_1} \quad (121)$$

in the weak basis where M_R is diagonal, real and positive. Finally, the baryon asymmetry depends also on the sum of all light neutrino masses squared, $\overline{m}^2 = m_1^2 + m_2^2 + m_3^2$, since it has been shown that this sum controls an important class of washout processes.

Leptogenesis is a non-equilibrium process that takes place at temperatures $T \sim M_1$. This imposes an upper bound on the effective neutrino mass \widetilde{m}_1 given by the “equilibrium neutrino mass” [60–62]:

$$m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} g_*^{1/2} \frac{v^2}{M_{\text{Pl}}} \simeq 10^{-3} \text{ eV}, \quad (122)$$

where M_{Pl} is the Planck mass ($M_{\text{Pl}} = 1.2 \times 10^{19}$ GeV), $v = \langle \phi^0 \rangle / \sqrt{2} \simeq 174$ GeV is the weak scale and g_* is the effective number of relativistic degrees of freedom in the plasma and equals 106.75 in the SM case. Yet, it has been shown [63–66] that successful leptogenesis is possible for $\widetilde{m}_1 < m_*$ as well as $\widetilde{m}_1 > m_*$, in the range from $\sqrt{\Delta m_{12}^2}$ to $\sqrt{\Delta m_{23}^2}$. The square root of the sum of all neutrino masses squared \widetilde{m} is constrained, in the case of normal hierarchy, to be below 0.20 eV [63–65], which corresponds to an upper bound on light neutrino masses very close to 0.10 eV. This result is sensitive to radiative corrections which depend on top and Higgs masses as well as on the treatment of thermal corrections. In [66] a slightly higher value of 0.15 eV is found. From Eq. (119) a lower bound on the lightest heavy neutrino mass M_1 is derived. Depending on the cosmological scenario, the range for minimal M_1 varies from order 10^7 GeV to 10^9 GeV [56, 66].

It was pointed out recently [67–75] that there are cases where flavour matters and the commonly used expressions for the lepton asymmetry, which depend on the total CP asymmetry and one single efficiency factor, may fail to reproduce the correct lepton asymmetry. In these cases, the calculation of the baryon asymmetry produced by thermal leptogenesis with hierarchical righthanded neutrinos must take into consideration flavour dependent washout processes. As a result, in this case, the previous upper limit on the light neutrino masses does not survive and leptogenesis can be made viable with neutrino masses reaching the cosmological bound of $\sum_i m_i \leq 0.66$ eV. The lower bound on M_1 does not move much with the inclusion of flavour effects. The separate lepton i family asymmetry generated from the decay of the k -th heavy Majorana neutrino depends on the combination [69] $\text{Im}((m_{\text{D}}^\dagger m_{\text{D}})_{kk'}(m_{\text{D}}^*)_{ik}(m_{\text{D}})_{ik'})$ as well as on the combination $\text{Im}\left((m_{\text{D}}^\dagger m_{\text{D}})_{k'k}(m_{\text{D}}^*)_{ik}(m_{\text{D}})_{ik'}\right)$ summing over all leptonic flavours i the second term becomes real so that its imaginary part vanishes and the first term gives rise to the combination $\text{Im}\left((m_{\text{D}}^\dagger m_{\text{D}})_{jk}(m_{\text{D}}^\dagger m_{\text{D}})_{jk}\right)$ that appears in Equation (115). Flavour effects bring new sources of CP violation to leptogenesis and the possibility of having a common origin for CP violation at low energies and for leptogenesis [76–79].

We have just referred to the minimal scenario for thermal leptogenesis. For a review including other scenarios see [80, 81]. The case of resonant leptogenesis is a remarkable alternative allowing for much lighter heavy neutrinos, and has raised a lot of interest [70, 82, 83]. One elegant way of obtaining the required smallness of the mass splitting of the heavy neutrinos is through radiative effects induced by renormalisation group running [78, 84–86].

There are many other interesting scenarios for leptogenesis which we do not cover here.

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