Vol. 38 (2007) *ACTA PHYSICA POLONICA B* No 12

CLUSTERING SCALE OF DARK MATTER*

Joanna Jałocha, Łukasz Bratek, Marek Kutschera
Marcin Kolonko

The Henryk Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland

(Received October 24, 2007)

We obtain mass distribution for flattened galaxies M94 and M101 whose rotation curves fail sphericity test in thin disk approximation by applying Iterative Spectral Method. The results obtained suggest smaller amount of dark matter (if any) than predicted by classical approach to modelling of rotation curves customarily assuming massive spherical halo.

PACS numbers: 98.80.–k, 95.35.+d, 98.35.Ce

1. Introduction

According to the Λ-CDM cosmological model, 22% of total energy content in the Universe is in the form of non-baryonic dark matter. If this is so, then the question arises how is the alleged exotic matter distributed and what is its clustering scale? Is this scale comparable with the characteristic size of galaxies, of a cluster of galaxies or maybe even greater?

Various clustering scales translate to different physical parameters of dark matter, such as mass density or temperature. At the galactic scale the expected density should be many orders of magnitudes higher than at the scale of clusters of galaxies. Knowing the scale is indispensable to estimate properly both our chances to discover dark matter in labs and the risk of wasting money for such experiments.

* Presented at the XLVII Cracow School of Theoretical Physics, Zakopane, Poland, June 14–22, 2007.

2. Standard approach to modelling of rotation curves of spiral galaxies

In the most widely used approach to modelling of mass distribution in spiral galaxies, the starting point is the splitting of luminosity profile of a galaxy into two subcomponents — a central bulge and a galactic disk with their corresponding constant mass-to-luminosity ratios that are determined by finding the best fit to observations. This allows for obtaining mass distribution of luminous matter, and hence, the corresponding rotational velocity one would expect for such mass distribution. Such obtained rotational velocity falls off with radial distance much faster than the observed rotation curve which is usually flat for large radii, therefore the third component — a massive spherical halo composed of non-baryonic dark matter — is introduced to account for the observed rotation curve. The effect of such procedure applied to a spiral galaxy M94 is shown in figure 1 prepared according to Kent's results [1]. This fit is still unsatisfactory at larger radii even though

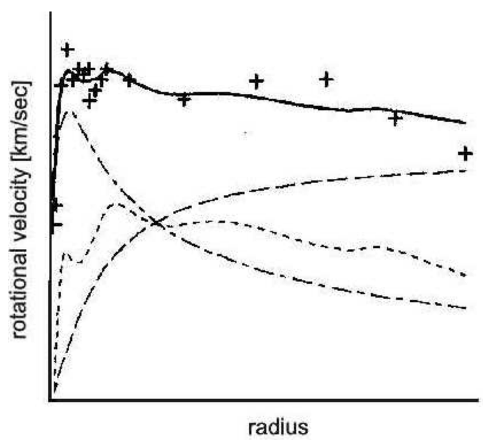


Fig. 1. The result of reconstruction of rotation curve of galaxy M94 in the framework of standard rotation curve modelling that assumes a massive spherical halo of dark matter. The dash-dotted line represents the contribution to galaxy rotation from the central bulge, the dotted line — from the disk component and the dashed line — from the massive dark halo component. The figure was prepared according to Kent's original results published in [1].

dark halo was introduced. But this should not surprise as rotation curve of the galaxy cannot be explained under spherical symmetry with massive spherical halo as this would require a function $r\,v^2(r)$ to be non-decreasing for larger radii (we shall come to this issue later on).

The weakest point of this generally accepted method of finding matter distribution in spiral galaxies is the arbitrary assumption that mass density in the central bulge and in the galactic disk must be proportional to luminosities of these components and that the corresponding mass to luminosity ratios which are proportionality factors between luminosity and mass density, are in average constant throughout a galaxy, even though the opposite is observed: different galaxy regions are dominated by different star populations. Moreover, the obtained mass distributions are strongly model-dependent, showing that galaxy dynamics is still not well understood. The best illustration of this is the fact that a galaxy that is dark-matter dominated in one model can be described without dark matter by another model. In this paper we report on two such examples.

Every galaxy model provides some correspondence between mass distribution and rotation curve and serves only as a tool for obtaining an approximation of galactic gravitational potential. We see no direct connection of the problem of stability of the assumed matter configuration in a particular model, with efficiency of the model in approximating real gravitational potential. The vast majority of people engaged with rotation curve modelling believe that spiral galaxies more resemble balls, the other group prefer thinking about spiral galaxies as they are observed — flattened disk-like structures. On the one hand, the first group is happier as spherical symmetry stabilises systems and puts aside many awkward problems, but the group is also constantly starved for more and more new exotic physics, on the other. The other, tiny group, however, seems to perform better in relating mass distributions to rotation curves without new physics. But which limit, spherical or flat, better approximates the correspondence between mass distribution and rotation of a real spiral galaxy?

3. The disk model

We apply the infinitesimally thin disk model known from standard textbooks on galaxy dynamics [2] for approximate description of gravitational field of matter in spiral galaxies. We assume that the matter is grouped mainly in the neighbourhood of galactic plane $z = 0$. Although the disk model has been known for decades, the method of determining mass distribution we have developed [3] in the framework of this model is quite novel and overcomes difficulties arising in handling observational data in this model.

In the disk model approximation, gravitational potential outside the galactic plane is determined by solving the Laplace equation $\Delta\Phi = 0$. It is assumed that solutions must be cylindrically symmetric and reflection symmetric with regard to the galactic plane. Having found a solution that vanish at infinity sufficiently fast, mass distribution in the disk plane can be found from a Gauss law and reads

$$\sigma(\rho) = \frac{1}{2\pi G}\partial_z \Phi\Big|_{z=0} . \tag{3.1}$$

The problem of finding gravitational potential in this case is simple and quite analogous to finding electric potential in the presence of electric charge distributed in the plane $z = 0$. In particular, internal solution of a boundary problem with $\Phi = \text{const.}$ on the side wall of a cylinder of radius R is a superposition $\Phi = -\sum_n C_n J_0(\lambda_n \rho) e^{-|z|\lambda_n}$ for $\rho \le R$ and $z \ne 0$, were λ_n's are consecutive zeros of equation $J_1(\lambda_n R) = 0$. Assume that C_n's are such that

$$v^2(\rho) = \rho \sum_n \lambda_n C_n J_1(\lambda_n \rho) \tag{3.2}$$

which follows from the requirement that the condition of circular orbits $\frac{v^2}{\rho} = \partial_\rho \Phi$ at $z = 0$ be satisfied for $\rho < R$. The coefficients can be found by the inverse transform as described in [3] or even by the least square method as higher frequency terms can be in practice usually neglected. One expects that such determined Φ should well approximate galactic gravitational potential close to $z = 0$ and for sufficiently small ρ, such that the influence of the simplifying boundary condition $\Phi = \text{const.}$ on the side wall $\rho = R$ could be neglected (for our method this boundary condition is irrelevant, however). Then also surface density (3.1) should well approximate column density in galactic disk, at least sufficiently close to $\rho = 0$. Therefore, we assume calculation of (3.1) as our starting point and the first coarse estimation of mass distribution. For larger ρ's and outside R, where rotation curve is not known from observational reasons, the problem of finding rotation curve globally and consistently with mass distribution of hydrogen can be solved by iterations. For several galaxies that were considered dark matter dominated this can be done with the amount of invisible matter that is acceptable as ordinary non-shining matter. By consistent we mean that the integral

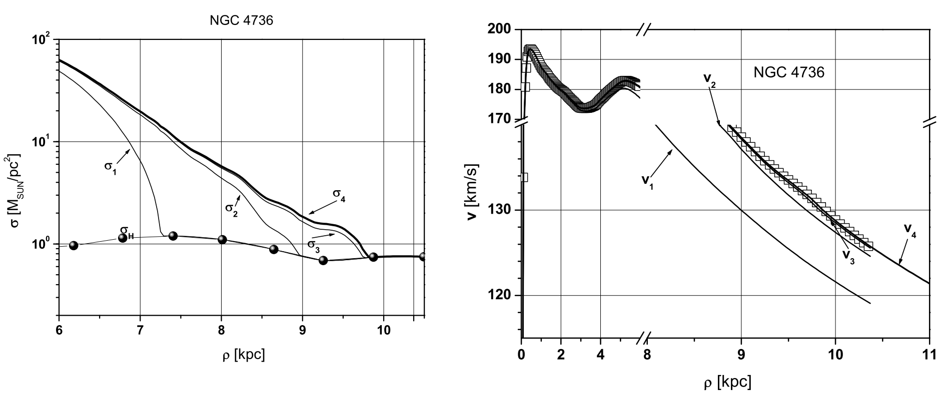
$$\frac{v^2(\rho)}{4G\rho} = \mathcal{P}\left(\int_0^\rho \sigma(\chi)\frac{\chi E\left(\frac{\chi}{\rho}\right)}{\rho^2 - \chi^2} d\chi - \int_\rho^\infty \sigma(\chi)\left[\frac{\chi^2 E\left(\frac{\rho}{\chi}\right)}{\rho(\chi^2 - \rho^2)} - \frac{K\left(\frac{\rho}{\chi}\right)}{\rho}\right] d\chi\right) \tag{3.3}$$

(understood in the principal value sense, hence the '$\mathcal{P}$' symbol) that relates rotation velocity to mass distribution in the $z = 0$ plane in disk model, should be globally invertible and, in addition, that $\sigma(\rho)$ should agree with the observed amount of hydrogen for $\rho > R$ and $v(\rho)$ should agree with the observed part of rotation curve. In the above integral E and K are elliptic functions.

3.1. The iterative spectral method

To apply the method we have to know the distribution of visible matter beyond the last point $\rho = R$ of the observed rotation curve (R is called here 'the cutoff radius'). The matter provides the lower bound for surface mass density for $\rho > R$. In practice we make use of the measurements of neutral hydrogen (H_I) which often stretch beyond cutoff radius and if required we may multiply its amount by the factor 4/3 to include helium. The application of our method is demonstrated for the example galaxy M94. Rotation curve of this galaxy was taken from Y. Sofue's web page [4].

The initial step in our method is to find the approximate density distribution as described in the previous paragraph. The result is shown in figure 2(a) as the curve marked with σ_0. Neutral hydrogen distribution that was taken from [5] was marked in that picture with 'σ_H'. In the first step we assume that global surface density σ_1 equals to σ_0 everywhere up to the first point where σ_0 becomes less than H_I's density. Beyond that point we assume that σ_1 equals to the observed column density of H_I, we assume also $\sigma_1 = 0$ for greater radii where nothing can be detected. In this way we obtain the first approximation of column mass distribution in the whole space. Then by substituting σ_1 into the exact formula (3.3) we obtain the corresponding rotation curve and denote it by v_1. As is seen in figure 2(b) this curve does not overlap ideally with the observed rotation curve yet, thus more matter has to be added. Surface density of missing matter is proportional to the



(a) (b)

Fig. 2. (a) Consecutive iteration steps for column mass density in galaxy M94, only a fragment is shown; (b) Rotation curve reconstruction — consecutive iteration steps. The v_4 line is the global rotation curve obtained from the global mass distribution found in the 4-th iteration step and agrees perfectly with the observed part of rotation curve of galaxy M94.

difference of squares of v_1 and v_0, where v_0 is the observed rotation curve. This difference is substituted to equation (3.2) in place of $v^2(\rho)$ from which the corresponding spectral coefficients can be found and next the density of missing matter is calculated from equation (3.1). Such obtained correction is then added to σ_1 and the result is assumed as the second approximation of global surface density up to the point where it becomes less then the density of hydrogen, beyond that point we assume it is equal to the column density of hydrogen. In this way we obtain the global surface density in the second iteration step and denote it by σ_2. Then again, we find from equation (3.3) the corresponding rotation curve and similarly as in the previous step we look for the next correction to global density. We repeat the procedure again and again until satisfactory agreement with the observed rotation curve is obtained. For the particular galaxy we obtain such agreement in the 4^{th} iteration step, which can be seen in figure 2(b). In this way, as the result of the application of the Iterative Spectral Method in the framework of disk model, we have found the density distribution which accounts for the measured rotation curve perfectly and also agrees with the observed amount of matter in distant regions of the galaxy.

3.2. Results

For a given luminosity profile, we may check how large is the corresponding mass-to-luminosity ratio and how its dependence on radius looks like. Luminosity profiles of galaxy M94 obtained in three different filters were adopted from [6] and are presented in figure 3(a) (after rescaling to currently accepted distances), and shown together with the distribution of total surface mass density and hydrogen column density. As is seen in figure 3(b), the mass-to-luminosity ratio is low for this galaxy and is the lowest in the I-band. In this band the maximum is attained at 2.5. We thus infer that the region in the neighbourhood of the maximum should be populated mainly by stars with masses of about 0.8 of the solar mass. What is also important, the mass-to-luminosity ratio decreases towards the galactic edge, in contrary to what the currently accepted approach to modelling of rotation curves arbitrarily assumes. The mass-to-luminosity ratio we have obtained, varies also with radius and is always low, as one would expect for a spiral galaxy. It thus follows that the assumption of a massive, spherical halo is not necessary for galaxy M94 — by applying the disk model, we obtained mass distribution which perfectly accounts for its rotation curve without the massive dark halo, thus there is no apparent reason for introducing dark halo of non-baryonic matter to the galaxy.

There is also another very important and simple test that excludes the presence of a massive and spherical halo in galaxy M94 that would dominate

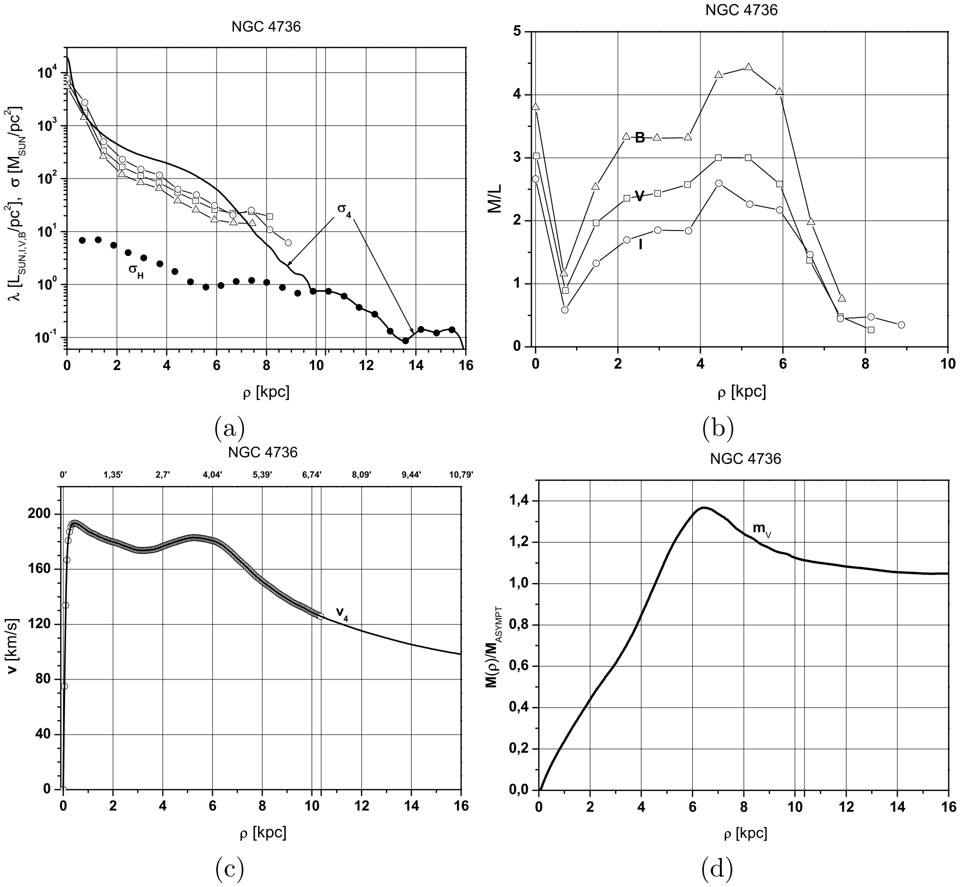


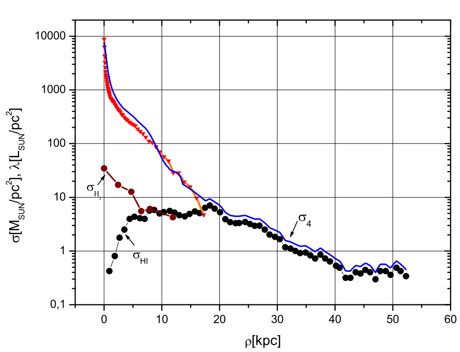
Fig. 3. (a) Global surface mass density (σ_4), neutral hydrogen surface density (σ_{H}), and luminosity profiles in three bands B, V and I (B — triangles, V — squares, I — circles) for galaxy M94. (b) Mass-to-light ratio as a function of galactic radius for three different filters. (c) Rotation curve of the M94. Open circles — observational data points (from [4]). Solid line — rotation curve reconstructed based on global surface density obtained using iteration method. (d) Negative sphericity test for galaxy M94.

gravitationally at large radii. This argument is based on the analysis of rotation curve of the galaxy. If we assume for contradiction that the gravitating mass in the galaxy has spherically symmetric distribution, then its rotation curve (under the assumption of circular orbits) must satisfy the condition that Keplerian mass function $\rho\, v^2(\rho)/G$ is non-decreasing

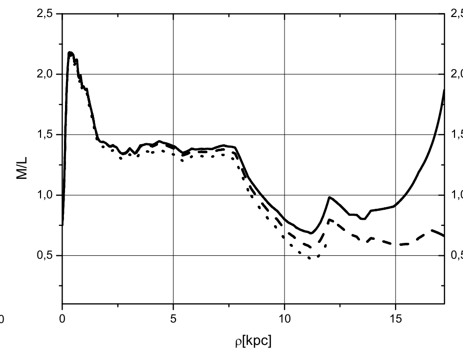
$$\frac{d}{d\rho}\left(\rho v^2(\rho)\right) \geq 0\,. \tag{3.4}$$

This condition is apparently broken at larger distances from the centre in galaxy M94 as is seen in figure 3(d).

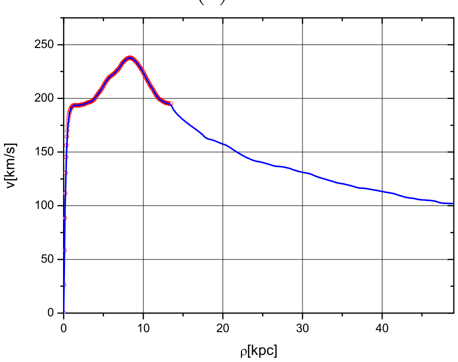
The next example is the M101 galaxy. Its rotation curve was taken from Sofue web page [4]. In order to find mass distribution in the galaxy we proceed analogously to the way presented in the previous example. The distribution of neutral hydrogen in the galaxy [7] was multiplied by 4/3 so as to take helium into account. Luminosity profile of the galaxy was measured in a K-filter [8]. The sphericity test in figure 4(d) shows that rotation curve excludes the massive spherical dark halo also for this galaxy. Therefore the



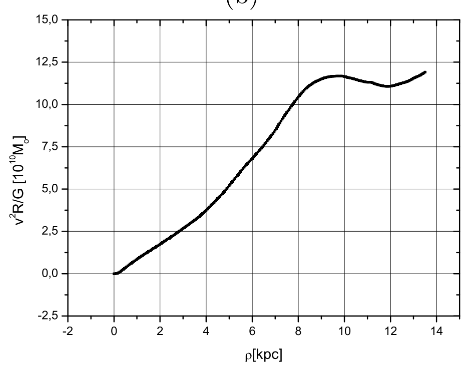
(a)



(b)



(c)



(d)

Fig. 4. (a) Surface mass density including neutral and molecular hydrogen in the M101 galaxy, shown together with K-band surface luminosity profile (triangles). (b) Mass-to-light ratios for the M101 galaxy: solid line for total mass, dashed line for the neutral hydrogen and helium subtracted from total mass, dotted line for the neutral and molecular hydrogen and helium subtracted. (c) Rotation curve of the M101. Open circles — observational data points (from [4]), solid line – curve reconstructed based on surface density obtained using iteration method. (d) Negative sphericity test for galaxy M101 .

disk model can be applied. It is interesting that the mass-to-luminosity ratio profile increases with radius close to the galactic edge, figure 4(b), which might suggest the existence of a flattened dark halo. However, after subtraction of hydrogen and helium from the total mass profile the effect disappears simply because hydrogen and helium are invisible in the K-band. For this galaxy we took into account also molecular hydrogen [9] of which presence reduces the mass-to-light ratio even more, figure 4. To sum up, in galaxy M101 dark matter is not only unnecessary but its introduction would even cause problems.

From the above examples we see that global disk model performs much better than models with spherical halo of dark matter. But most importantly, it follows that the way of thinking about spiral galaxies should be revised as different models lead to quite different predictions for the same galaxies.

3.3. Summary

As was demonstrated in the previous section, the Global Disc Model applied together with Iterative Spectral Method gives reasonable density distributions accounting perfectly for observations. For the two example galaxies, M94 and M101, the mass-to-luminosity ratios are low. Moreover, rotation curves of these galaxies exclude the possibility of existence of dark and massive spherically symmetric components predicted by the standard approach to modelling of rotation curves. Are these galaxies exceptional? This cannot be answered unless we better understand dynamics of spiral galaxies and revise the currently accepted approach to modelling of mass distribution in spiral galaxies. If these galaxies are not exceptions then they would provide a tool for estimation of the lower bound for clustering scale of dark matter. Then the scale may turn out much greater than it is currently thought, maybe as large as the size of clusters of galaxies.

REFERENCES

[1] S.M. Kent, *Astronom. J.* **93**, 816 (1987).
[2] J. Binney, S. Tremain, *Galactic Dynamics*, Princeton Univ. Press, 1987.
[3] Ł. Bratek, J. Jałocha, M. Kutschera, `astro-ph/0611113`.
[4] `http://www.ioa.s.u-tokyo.ac.jp/sofue`
[5] P.S. Mulder, W. van Driel, *Astronom. Astrophys.* **272**, 63 (1993).
[6] C. Munoz-Tunon, M. Prieto, J. Beckman, J. Cepa, *Ap & SS* **156**, 3001 (1989).
[7] R. Braun, *Astrophys. J.* **484**, 637 (1997).
[8] T. Jarrett, 2MASS, NASA/IPAC Informed Science Archive, `http://irsa.ipac.caltech.edu/date/LGA`.
[9] J.S. Young, *Annu. Rev. Astron. Astroph.* **29**, 581 (1991).