Vol. 38 (2007) ACTA PHYSICA POLONICA B No 12

MATTER STABILITY IN THE NEUTRON STAR INTERIOR*

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(Received October 29, 2007)

The nuclear symmetry energy is directly connected to stability of matter in neutron star interior. At low density symmetry energy determines the crust-core transition. At higher densities, it appears that low values of symmetry energy leads to phase separation. The two phases may coexist and lead to new, unexpected structure of neutron star interior.

PACS numbers: 26.60.+c, 21.30.Fe, 21.65.+f, 97.60.Jd

1. Introduction

In a neutron star, the prevailing part of its interior is filled with matter which is in the state of the beta equilibrium [1]. Few meters below neutron star surface, at densities of $\sim 10^7$ g/cm^3 the degenerated electrons become relativistic and they easily penetrate the nuclei. When their chemical potential μ_e (or Fermi energy) is greater than neutron–proton mass difference $\Delta m = 1.3$ MeV, electrons are able to convert protons to neutrons. One may say that nucleons and electrons are in chemical equilibrium with respect to beta decay and capture reactions:

$$n \leftrightarrow p + e \tag{1}$$

(we neglect neutrinos here as they leave the system freely). This state of matter is a beta equilibrium and may be expressed by the constraint on chemical potentials of particles taking part in the cycle:

$$\mu_n - \mu_p = \mu_e \,. \tag{2}$$

At these conditions the most stable nuclei become more neutron rich than the most stable terrestrial nucleus ^{56}Fe. For both nucleons their chemical potentials are smaller than their masses[1], $\mu_n < m_n$ and $\mu_p < m_p$, what

* Presented at the XLVII Cracow School of Theoretical Physics, Zakopane, Poland, June 14–22, 2007.

[1] We mean here the *total* chemical potential *i.e.* including the rest mass of a particle.

means they are confined to the nucleus volume. With increasing density, the neutron chemical potential increases, whereas for protons it falls down. At the density $\rho_{\rm drip} \sim 10^{11}$ g/cm^3 the neutron energy is sufficiently high, $\mu_n > m_n$, and neutrons are allowed to drip out of nuclei [2]. Above the neutron drip density one may treat the neutron star matter as a two-phase system in phase equilibrium although not all Gibbs conditions are fulfilled, because protons are still confined to nuclei:

$$P^{\rm d} = P^{\rm nuc}\,, \tag{3}$$

$$\mu_e^{\rm d} = \mu_e^{\rm nuc}\,, \tag{4}$$

$$\mu_n^{\rm d} = \mu_n^{\rm nuc}\,, \tag{5}$$

but $\mu_p^{\rm d} \neq \mu_p^{\rm nuc}$, where superscripts correspond respectively to the dripped and nuclear phase. In the dripped phase neutrons are stable due to degenerated electrons presence what is expressed by the relation for chemical potentials: $\mu_n^{\rm d} < \mu_p^{\rm d} + \mu_e^{\rm d}$. At higher density, close to the saturation density, the proton drip occurs as well. The energy of protons, inside nuclei and out of them is equal

$$\mu_p^{\rm d} = \mu_p^{\rm nuc} \tag{6}$$

and above this density one gets a two-phase system with all Gibbs conditions Eqs. (3)–(6) satisfied. The two phases have different properties like baryon density ρ and proton fraction x or charge. The density range where the two-phase matter appears is rather small, soon after the proton drip the differences in ρ and x vanish and the star matter represents *npl* matter — a homogeneous liquid of nucleons and leptons (also muons start to be produced at these densities). In the two-phase system, the nuclei form a lattice leading to a matter with solid state properties, unlike the fluid homogeneous system. In this way a neutron star has a dense, liquid core covered by a solid crust with smaller density. After the inclusion of finite-size effects, the transition between solid crust and liquid core appears not to be sharp. The competition between Coulomb and surface energy leads to deformation of nuclei into non-spherical structures like rods an plates. It is likely that in the transitional region such "funny phases" make smooth passage between solid crust and liquid core. In this article we limit ourselves to the bulk approximation and show that the phase separation is directly connected to the nuclear symmetry energy. I this way the placement of crust-core transition is sensitive to symmetry energy behavior. The same analysis applied to higher density region reveals the possibility of solidification in the central part of liquid core and formation of new interesting structure in the central part of neutron star.

2. The symmetry energy

The symmetry energy is the quantity describing the strength of nuclear interaction in isovector channel. The energy per baryon for infinite matter consisting only of nucleons may be expressed by the following expansion

$$E/B = u^N(n,x) = V(n) + E_{\mathrm{s}}(n)(1-2x)^2 \; + \mathcal{O}(1-2x)^4\,, \qquad (7)$$

where n and x is the baryon number density and x is the proton fraction $x = n_p/n$. The isospin symmetry allows for the expansion in even powers of $(1-2x)$ but terms higher than quadratic can be neglected.

For neutron star matter which includes nucleons and leptons subject to beta equilibrium, Eq. (2), the symmetry energy is the key quantity fixing the proportion between particles present in the system. The actual density dependence of symmetry energy is not exactly known. We know only its value at saturation point, $n_0 = 0.16\ \mathrm{fm}^{-3}$, where it is about 30 MeV. Recent analysis of the isospin diffusion in heavy-ion collisions constrained significantly the slope of $E'_{\mathrm{s}}(n_0)$ and the stiffness $E''_{\mathrm{s}}(n_0)$ at saturation point [5], however, these results do not determine the high density behavior definitely. There are no experimental data on values of E_{s} at very high density which is available in the central parts of a neutron star. In such extrapolations we must rely on the model calculations. For all of them the symmetry energy at saturation point has positive slope but at higher densities they lead to different conclusions. For most cases the E_{s} is monotonically increasing function of n but some models lead to the E_{s} which saturates at higher densities or even bends down at some point and goes to zero [6, 7].

The shape of E_{s} at high density is strictly connected to the thermal story of neutron star. Low E_{s} makes low x and blocks fast cooling by so-called direct URCA cycle [8]. So, the presence of at least one very cold isolated neutron star should rule out this type of symmetry energy. However, this conclusion is simplified because of other possibilities of fast cooling in the presence of kaon or pion condensates (see [9] for review). In this work we predict effects of low E_{s} which concern not only thermal properties of neutron star. In this way we acquire another tool in constraining the shape of symmetry energy.

The behavior of E_{s} at low density is also a subject of our interest. As was mentioned above, recent experimental results constrained the values of E_{s} to some extend. In this work we show that a relation between the thickness of neutron star crust and the shape of $E_{\mathrm{s}}(n)$ below saturation point can be found. The observations of pulsar glitching allow to estimate the size of the crust and may constrain, in this way, the symmetry energy at low densities.

3. Stability conditions

Having neglected the temperature, important only for a young hot star, the total energy U becomes a function of volume and conserved numbers: the total charge and the baryon number B. In order to consider stability of single phase one need to introduce intense (local) quantity $u = U/B$. In the case of *npl* matter the total energy per baryon is a sum of nucleon and lepton contributions $u = u^N + u^L$ and may be expressed as a function of quantities taken per baryon number $v = V/B$ and $q = Q/B$. The first principle of thermodynamics takes the following form

$$du = -P\, dv - \mu\, dq\,, \tag{8}$$

where P is the pressure and μ chemical potential of electric charge. The minus sign before μ in (8) comes from convention that μ is equal to μ_e chemical potential of electron which carries the negative charge. The stability of any single phase, also called the *intrinsic* stability, is ensured by the convexity of $u(v, q)$ [3]. Thermodynamical identities allow to express this requirement in terms of following inequalities [4]

$$-\left(\frac{\partial P}{\partial v}\right)_q > 0\,, \qquad -\left(\frac{\partial \mu}{\partial q}\right)_P > 0\,, \tag{9}$$

or

$$-\left(\frac{\partial P}{\partial v}\right)_\mu > 0\,, \qquad -\left(\frac{\partial \mu}{\partial q}\right)_v > 0\,. \tag{10}$$

Usually, only the positive compressibility is examined, in particular, it is required for locally neutral matter *i.e.* $-(\partial P/\partial v)_{q=0} > 0$. However, the second type of inequalities: $(\partial\mu/\partial q)$ is of the same importance. It concerns the stability of charge fluctuations and it is connected to the positive value of the screening length in matter.

For further discussion we introduce the compressibility and electric capacitance as

$$K_i \;=\; -v^2\left(\frac{\partial P}{\partial v}\right)_i = \left(\frac{\partial P}{\partial n}\right)_i\,, \qquad i = q, \mu\,, \tag{11}$$

$$\chi_j \;=\; -\left(\frac{\partial q}{\partial \mu}\right)_j\,, \qquad j = P, v\,. \tag{12}$$

Then, the second pair of inequalities (10) applied to the expression for the nucleon energy, Eq. (7) may be written as [17]

$$K_\mu \;=\; n^2\left(E_{\rm s}''(1-2x)^2 + V''\right) + 2\,n\left(E_{\rm s}'(1-2x)^2 + V'\right) - \frac{2(1-2x)^2 E_{\rm s}'^2 n^2}{E_{\rm s}} > 0\,, \tag{13}$$

$$\chi_v \;=\; \frac{1}{8E_{\rm s}(n)} + \frac{\mu(k_e + k_\mu)}{n\pi^2} > 0\,. \tag{14}$$

We used the second pair of stability conditions as they finally lead to much simpler formulae than the first one and, moreover, it can be shown that [17]

$$K_\mu < K_q \quad \text{and} \quad \chi_v < \chi_P \,. \tag{15}$$

so K_μ is more proper as it changes its sign before K_q.

The leptonic contribution u^L to the total energy does not appear in the Eqs. (13) and (14) explicitly, however, lepton manifests itself by the presence of the last term in (13). First two terms in (13) stand for the "usual" compressibility of pure nucleonic matter and are always positive, whereas the last term contributes negatively and may break the positivity condition. Hence, one may conclude that it is the lepton presence which may get matter unstable. Above equations show explicitly the importance of symmetry energy in the stability considerations.

4. Nuclear models

In order to present the role played by the symmetry energy we apply a set of nuclear models. At low densities the isoscalar part is kept the same, whereas the symmetry energy takes different forms. The isoscalar potential $V(n)$, was taken from [11] which leads to the compressibility of symmetric matter equal to 240 MeV at saturation point. For $E_{\rm s}$ we used shapes applied by Chen *et al.* in [5]. Here we named them by (a), (b), (c), (d). The model (e), not belonging to the above family, imitates an interesting result [10] which shows that the symmetry energy does not vanish at zero density but rather saturates at about 10 MeV. All the shapes of $E_{\rm s}$ at lower densities are presented in Fig. 1.

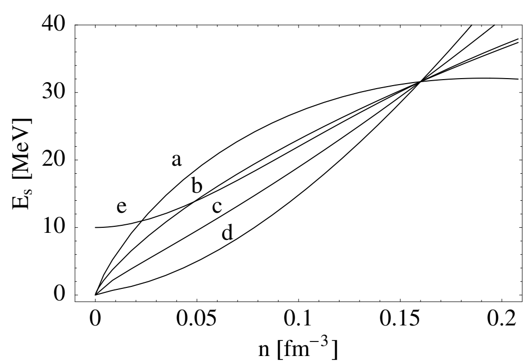


Fig. 1. The different shapes of the symmetry energy at densities below saturation point.

At higher densities, much above n_0, we introduce two kinds of isoscalar potential $V(n)$, one from [11] (the same as in low density regime) and the other from [12]. The isoscalar potential mainly influences the stiffness of equation of state. In this way one may test how the instability point is affected by the stiffness of EOS. The latter potential is stiffer and leads to stars with higher maximal mass and is in better agreement with recent observations of pulsar with mass $2.1 \pm 0.2 M_{\odot}$ [13]. For $E_{\rm s}$ we applied a "bent down" function. This type of symmetry energy with low values at high density was typical in the past variational calculations based on realistic potentials [6]. This kind of behavior is not obtained in more recent realistic potential calculations like in [12]. Nevertheless, there are also other modern approaches based on chiral dynamics [7] and Skyrme effective forces [14] or relativistic mean field [15], where very low values of $E_{\rm s}$ were obtained. Here, for numerical simplicity, we introduced the simple polynomial (for details see [17]) which imitate results of works mentioned above. The shapes of these functions, named A, B, C are shown in the Fig. 2.

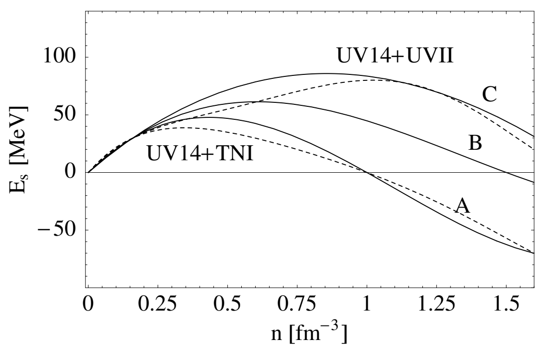


Fig. 2. Three different shapes of the symmetry energy at densities above saturation point (solid lines). For comparison the results of realistic potentials (dotted lines).

5. Results

The transition between liquid core and solid crust corresponds to the breaking one of the conditions (13) and (14). Fig. 3 shows the compressibility under constant μ and its two contribution: "nuclear" $K_\mu^{\rm nuc}$ — the two first terms in (13), and "beta" K_μ^β — the last term in (13), which comes from the leptons presence. The "beta" contribution is always negative, hence there is always a competition between the positive "nuclear" compressibility and the beta reactions which tends to destabilize the matter. At some critical point, $n_{\rm c}$, the compressibility K_μ vanishes and below $n_{\rm c}$ the matter splits into two phases. The point of actual splitting does not occur exactly at

n_c, but slightly above n_c because the system must find a state where the two charged phases may coexist to ensure global neutrality. However, the correction is very tiny so n_c may be treated as a good estimation for the boundary of the liquid core in neutron star.

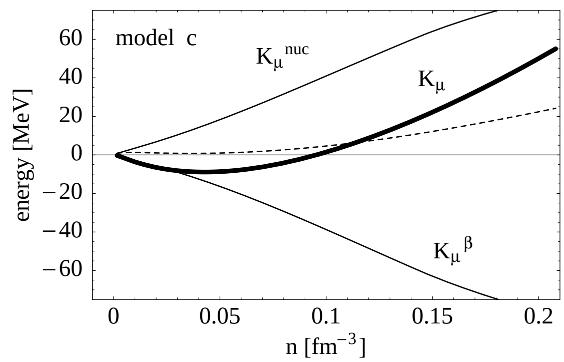


Fig. 3. The compressibility K_μ (thick) and its contributions (thin lines). The dotted line corresponds to the energy per baryon for neutral matter $u(n, 0)$.

Table I shows the critical density. It depends strongly on E_s but does not behave monotonically with the values of E_s because the first and the second derivatives of E_s are essential as well.

TABLE I

The critical density for different models.

Model	a	b	c	d	e
n_c, fm^{-3}	0.119	0.092	0.095	0.160	0.053

Let us pass to high density region. When the symmetry energy is increasing function in the whole range of density the matter is stable indeed. However, the chosen nuclear models, with very low values of E_s, lead again to the same kind of instability as occurs in the crust-core transition region. For all presented models soft and stiff A, B, C there is a critical density n_c where K_μ vanishes. The behavior of the compressibility K_μ for the model B with soft isoscalar potential is shown in Fig. 4.

It is worth to notice that if one looks only on the energy per baryon for homogeneous neutral matter, one may overlook that the matter becomes unstable at some point. The $u(n)$ has always positive curvature ($K_{q=0} > 0$), whereas the K_μ becomes negative at some density and signals the actual phase separation. Table II shows how the value of n_c depends on the symmetry energy model. The lower E_s is at higher density the lower n_c is. The

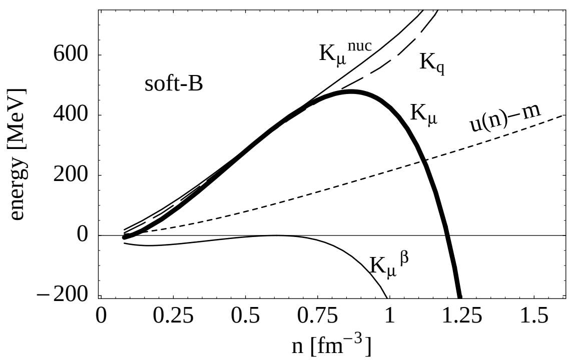


Fig. 4. The compressibility K_μ (thick) and its contributions (thin lines) for soft EOS and B symmetry energy. The dashed line corresponds to K_q and dotted line energy per baryon for neutral matter, $u(n) \equiv u(n,0)$.

stiffness of EOS slightly moves n_c to higher values. The phase separation at higher densities occurs if only n_c is attainable in a neutron star. Table II shows basic neutron star properties: the central density n_cen of a star with maximal mass M_max.

TABLE II

The critical density and neutron star parameters for "soft" and "stiff" equation of state. All densities are in fm^{-3}.

Soft	A	B	C	Stiff	A	B	C
n_c	0.74	1.20	1.43	n_c	0.85	1.40	>1.6
n_cen	1.92	1.32	1.21	n_cen	1.35	1.22	1.17
$M_\mathrm{max}/M_\odot$	1.64	1.73	1.84	$M_\mathrm{max}/M_\odot$	2.02	2.08	2.13

As one may notice, in the case A and B for soft, and in the case A for the stiff EOS the phase instability occurs for the sufficiently massive star. For such star, the central part of its core must contain separated phases. It does indeed. Fig. 5 shows the phase equilibrium for a chosen nuclear model. The line of $K_\mu = 0$ (dashed line) begins at the density $n_\mathrm{c} = 0.85$. Above this line the only stable phase is the phase consisting of neutrons and electrons, for which beta cycle does not work. The second phase, including protons, lays in the stable region where $K_\mu > 0$. The two phases indicated by thick lines fulfill all the Gibbs conditions of phase equilibrium, Eqs. (3)–(6). This is shown by the contours of chemical potentials μ and μ_n (thin lines) which meet at points laying on the phase lines.

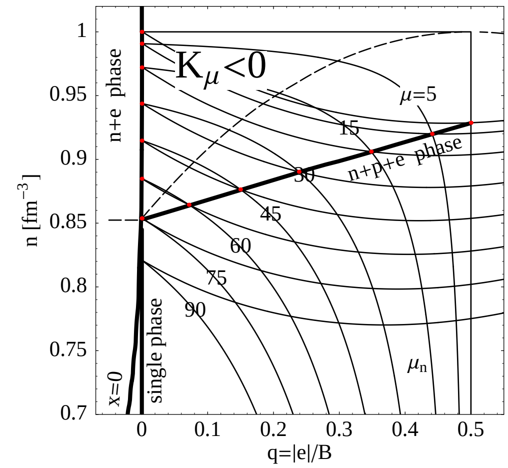


Fig. 5. Phase diagram for the model stiff, A.

The neutron phase (see its volume fraction V^N/V in the Fig. 6) quickly absorbs the proton–neutron phase. The proton–neutron phase has positive charge and becomes more and more symmetric with increasing density. It disappears completely at the point where its proton fraction is $1/2$. Disappearance the proton phase occurs at the density of ~ 1 fm^{-3}. Above that point the system again becomes homogeneous, consisting of pure neutron matter.

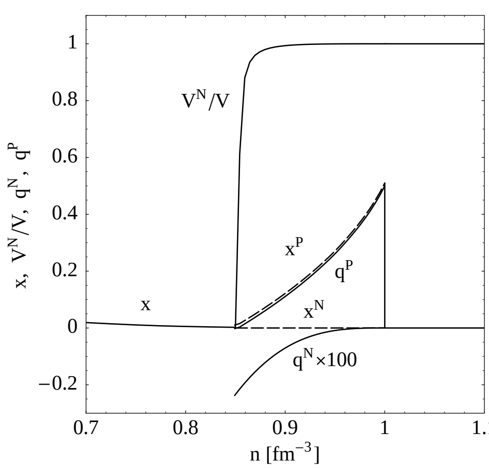
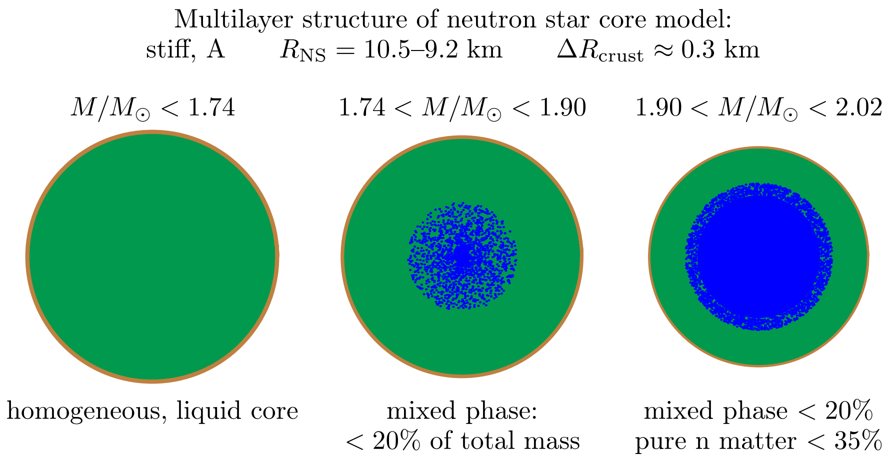


Fig. 6. The basic properties of two phase system.

Above discussion leads to an interesting prediction for a possible neutron star structure which depends on the mass of a star. Figure below shows various types of neutron stars including different layers in its core.



It is an open question about the properties of matter in the region where the phases are separated. One may suspect formation of mixed phase with liquid properties or solidification of the central part of stellar core. To answer the question, what actually happens above the critical density, requires more detailed analysis including the finite-size effects like surface and Coulomb energy. The presence of such multilayer structure should have consequences for pulsar observations like precession, glitching and their overall evolution.

6. Summary and discussion

In this report we present the simple connection between the symmetry energy $E_{\rm s}$ and the phase stability of dense matter filling the neutron star interior. It was shown that relevant quantity in such considerations is K_μ — the compressibility under constant chemical potential, rather than K_q — the compressibility under constant charge. The instability of matter under low density, below n_0 leads to phase separation and corresponds to the transition from the liquid core to the solid crust. Pulsar glitching phenomenon allows to estimate the size of neutron star crust [16] so in this way one may get constraint on $E_{\rm s}$ behavior at low densities coming from pulsar observations.

The stability considerations were also carried out at very high density. It was shown that for nuclear models with small values of $E_{\rm s}$ the instability does occur and leads to phase coexistence. The value of critical density depends mainly on $E_{\rm s}$ but also the stiffness of EOS influences the onset of instability. The phase separation leads to the formation of unexpected structures in the core of a star what seems to be especially interesting in connection to rotational and magnetic properties of pulsars.

This work was supported by the grant of the Polish Ministry of Science and Higher Education No. NN 203 3866 33.

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