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EMPIRICS VERSUS RMT IN FINANCIAL CROSS-CORRELATIONS*

S. Drożdż[a,b], J. Kwapień[a], P. Oświęcimka[a]

[a]H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland
[b]Institute of Physics, University of Rzeszów
35-310 Rzeszów, Poland

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In order to pursue the issue of the relation between the financial cross-correlations and the conventional Random Matrix Theory we analyse several characteristics of the stock market correlation matrices like the distribution of eigenvalues, the cross-correlations among signs of the returns, the volatility cross-correlations, and the multifractal characteristics of the principal values. The results indicate that the stock market dynamics is not simply decomposable into 'market', 'sectors', and the Wishart random bulk. This clearly is seen when the time series used to construct the correlation matrices are sufficiently long and thus the measurement noise suppressed. Instead, a hierarchically convoluted and highly nonlinear organization of the market emerges and indicates that the relevant information about the whole market is encoded already in its constituents.

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1. Introduction

The financial markets represent probably the most complex structure that is associated with the contemporary civilization. They involve extremely many constituents, many different space and time scales and an uncountable number of convoluted factors that drive the financial dynamics towards a real complexity. Its most relevant feature is a permanent competitive coexistence of collectivity and noise. The related quantitative characteristics can be studied using multivariate ensembles of parameters that represent dynamics of various financial assets. Due to this multi-dimensionality the most natural and efficient formal frame to quantify the whole variety of

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effects connected with complexity is in terms of matrices [1]. Since the dynamics of complexity is inherently embedded in noise, the Random Matrix Theory (RMT) [2,3] offers an appropriate reference. Deviations from RMT help to detect real information and to potentially extract it from what is universal in the RMT sense and thus practically not very informative. An extremely useful matrix approach to the financial dynamics is based on using the correlation matrices formed from *(i)* the time series representing the price changes of a certain basket of different assets over the same period of time or *(ii)* from the time series representing different disconnected periods of time (days or weeks) for either a single asset or an index. The simplest commonly used variant of RMT to serve as a null hypothesis in these cases corresponds to the ensemble of Wishart matrices [4]. The resulting eigenvalue distribution $\rho(\lambda)$ is then described by the Marchenko–Pastur formula [5,6] which confines this distribution within the bounds

$$\lambda_{\min}^{\max} = 1 + \frac{1}{Q} \pm \frac{2}{\sqrt{Q}}\,. \tag{1}$$

Here $Q = T/N$ where N is the number of time series of length T. Relating eigenspectra of the empirical $N \otimes N$ stock market correlation matrices to this formula shows that typically only a few of the eigenvalues, representing a global or some more local collective moves within the market, are located sizeably above $\lambda_{\max}$ while the bulk of the empirical eigenvalue distribution satisfactorily falls within the lower and the upper bound. This is interpreted as an indication that eigenvectors associated with the bulk are undistinguishable from noise and thus carry no information. Such a situation is quite convincing in the case denoted above as *(ii)* [7,8]. The results of the original study of cross-correlations among the stock market companies were interpreted analogously [9,10]. A more systematic analysis of this kind of correlations (case *(i)* above) shows however [11] that they are much more subtle, the overlap of the bulk with the bounds prescribed by RMT dissolves as T increases and eigenvectors even from the middle of the spectrum carry significant information. Below we recapitulate the most relevant results and provide some further arguments in favor of the statement that there is nontrivial information encoded also in the bulk of the eigenvalue spectrum of the stock market correlation matrix (see also [12]). These results should be taken care of also in the context of the Markowitz optimal portfolio theory [13] and for denoising of the empirical correlation matrices [14,15].

2. Notation

In the financial context one considers a portfolio P consisting of a number of securities $X_s, s = 1, \dots, N$ associated with weights w_s that reflect the fraction of the total capital invested in a particular security. On the time scale Δt the return of such a portfolio at the t_j instant of time is the weighted sum

$$G_P(j, \Delta t) = \sum_{s=1}^{N} w_s g_s(j, \Delta t) \tag{2}$$

of logarithmic price increments

$$g_s(j, \Delta t) = \ln p_s(t_j + \Delta t) - \ln p_s(t_j) \quad s = 1, \dots N\,; \; j = 1, \dots, T \tag{3}$$

of individual securities X_s. Each such return can be considered a product

$$g_s(j) = \mathrm{sign}_s(j) \times v_s(j) \tag{4}$$

of its sign and of its magnitude $v_s(j)$ which measures the volatility.

These time series can be used to create an $N \times T$ data matrix $\boldsymbol{M}$ and then a correlation matrix $\boldsymbol{C}$ according to the formula

$$\boldsymbol{C} = (1/T)\boldsymbol{M}\boldsymbol{M}^{\mathrm{T}}\,. \tag{5}$$

Each element of $\boldsymbol{C}$ is thus the Pearson correlation coefficient C_{mn} between a pair of signals m and n. By solving the eigenvalue problem

$$\boldsymbol{C}\boldsymbol{x}_i = \lambda_i \boldsymbol{x}_i\,, \quad i = 1, \dots, N\,, \tag{6}$$

this matrix can be transformed to the diagonal form. From the point of view of investment theories, each eigenvector $\boldsymbol{x}_i$ can be considered as a realization of an N-security portfolio P_i with the weights equal to the eigenvector components $x_i^{(k)}, k = 1, \dots, N$. For a non-degenerate matrix $\boldsymbol{C}$, P_i and P_j are independent for each pair of their indices, which allows one to choose such a portfolio, whose risk is independent of others. According to the classical theory [13], the risk $R(P) = \sigma^2(P) = \mathrm{var}\{G_P(j)\}_{j=1}^{T}$ for the relevant group of securities can be related to correlations (or covariances) between the time series of individual security returns $g_s(j), j = 1, \dots, T$.

Each eigenvector determined by Eq. (6) (and thus portfolio) can be associated with the corresponding time series of the portfolio's returns by the expression

$$z_i(j, \Delta t) = \sum_{k=1}^{N} x_i^{(k)} g_k(j, \Delta t)\,, \qquad i = 1, \dots, N; \qquad j = 1, \dots, T\,, \tag{7}$$

which is analogous to Eq. (2). These principal value time series we shall call the eigensignals Z_i (see also [7, 16] for some alternative realizations). The risk associated with such eigensignals is related with the corresponding eigenvalues:

$$R(P_i) = \sigma^2(Z_i) = \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{C} \boldsymbol{x}_i = \lambda_i \, . \tag{8}$$

Thus, the eigenvalue size is a risk measure and, in consequence, the larger λ_i, the larger variance of Z_i and also the larger risk of the corresponding portfolio P_i.

3. Data specification

This study of inter-stock correlations is based on high-frequency data from the American stock market [17] in the period 1 Dec 1997–Dec 1999. We chose a set of stocks of $N = 100$ highly capitalized companies listed in NYSE or NASDAQ (capitalization $> \$10^{10}$ in each case). These stocks are sufficiently frequently traded (0.01–1 transactions/s) so that the time scale of $\Delta t = 5$ min allows to perform a statistically significant analysis. For such a time scale the length of the time series exceeds 40,000 data points. From the perspective of our present purpose this time scale and the corresponding length of the series turn out to constitute a reasonable compromise. Thinking in terms of the Epps effect [18], in the liquid contemporary markets the time horizon of $\Delta t = 5$ min is long enough so that the cross-correlations get sufficiently expressed beyond the noise level [19, 20]. In case of the data considered here the time horizon at which λ_1 saturates at its maximum corresponds to about 30 min, while for $\Delta t = 5$ min λ_1 assumes approx. 2/3 of its saturation level. The length of the time series, on the other hand, allows one to study the T dependence of cross-correlations in a relatively broad range of time intervals up to the maximum which corresponds to $Q = 406$.

4. Data analysis

One natural characteristics that may offer some introductory insight when relating a given correlation matrix to the RMT is the distribution of matrix elements. For our correlation matrices two such distributions corresponding to the full $Q = 406$ and to $Q = 3$, which in this latter case is obtained by properly windowing the same time series and averaging over the windows, versus the best Gaussian fits, are shown in figure 1. Both these distributions are shifted more towards positive values. In the case of $Q = 406$ the distribution is naturally much narrower than for $Q = 3$ and shows essentially no presence of negative matrix elements. This signals that the real correlations are less contaminated by the measurement noise for $Q = 406$. Also the Gaussian fit in this case is less satisfactory, especially in the region of larger positive values of C_{mn}. Here, on the level of 1% probability one finds deviations of about 8σ (mean standard deviation) while for $Q = 3$ analogous deviations reach at most 3σ.

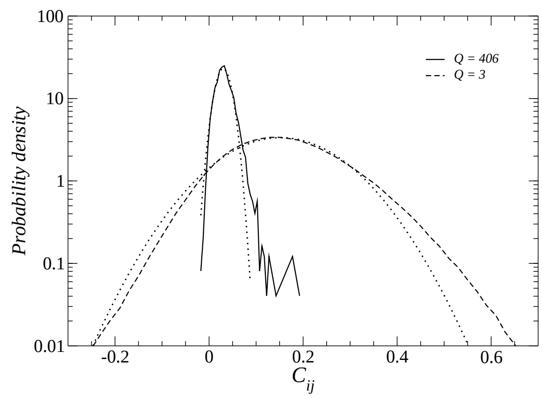

Fig. 1. Probability density distribution of entries of the empirical correlation matrices $\boldsymbol{C}$ calculated for 100 highly capitalized American companies over the period 1998–1999; the solid line corresponds to $Q = 406$ and the dashed line to $Q = 3$. The corresponding best Gaussian fits are indicated by the dotted lines.

5. Eigenvalue distribution

A complementary and an even more informative characteristics of the matrix is its eigenvalue distribution. Figures 2(a) and 2(b) show all 100 eigenvalues distributed along the horizontal axis, denoted by vertical lines, for the above presented cases of $Q = 406$ and $Q = 3$, respectively. The largest eigenvalue λ_1, assuming very similar values ($\sim$ 18–19), repelled from the rest of the spectrum, is seen in both cases and describes the collective eigenstate which can be identified with the market. The rest of the spectra develop however a significantly different structure. For $Q = 3$ this rest covers a much wider range of values but at the same time its overlap with the corresponding random Wishart matrices region (shaded vertical), whose bounds are prescribed by the Eq. (1), is very substantial (87%) while for $Q = 406$ it is rudimentary and looks pure coincidence. Of course, concerning agreement of the empirical spectra with the RMT this case of $Q = 406$ is much more meaningful as compared to $Q = 3$. One more interesting, and probably related effect, is that for $Q = 406$ one sees (Fig. 2(a)) the second λ_2, and even the third λ_3, eigenvalues that also are clearly separated from the bulk. These eigenvalues can be related to some branch-specific factors. No such factors can directly be seen for $Q = 3$ (Fig. 2(b)).

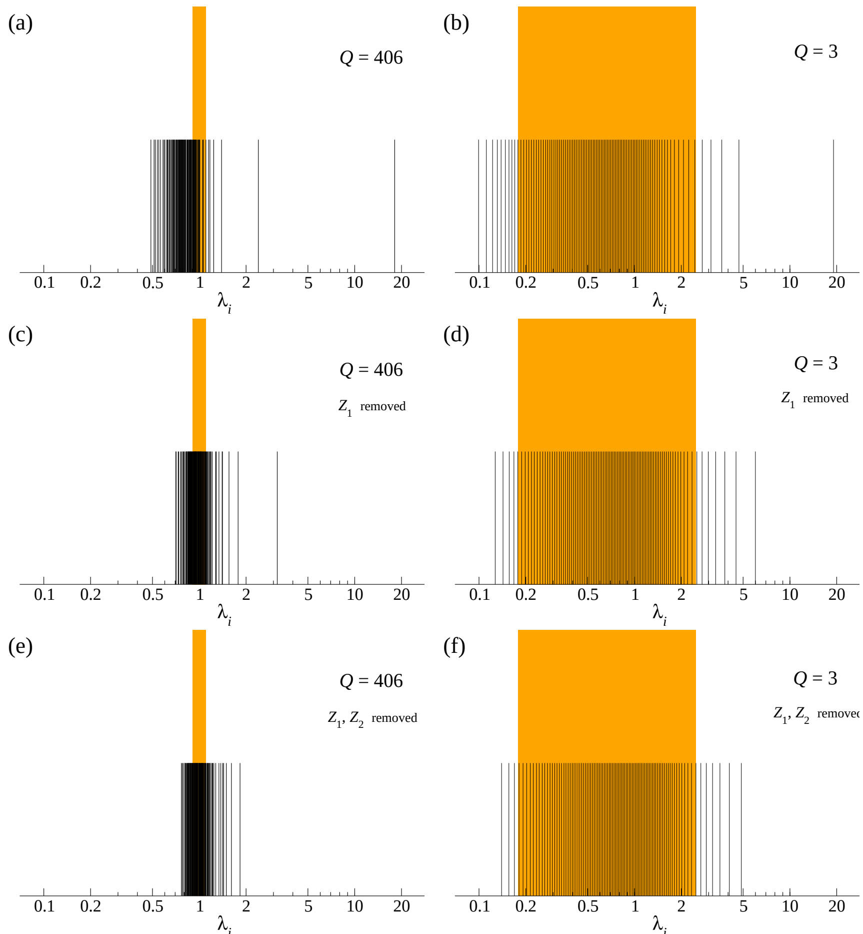

Fig. 2. Empirical eigenvalue spectrum of the correlation matrix $\boldsymbol{C}$ (vertical lines), calculated for 100 highly capitalized American companies over the period 1998–1999 for $Q = 406$ (a) and for $Q = 3$ (b); the eigenvalues of a random Wishart matrix with the same Q may lie only within the shaded vertical region. Eigenvalue spectrum after effective rank reduction of $\boldsymbol{C}$, *i.e.* after subtracting the contribution of the most collective eigensignal Z_1 for $Q = 406$ (c) and for $Q = 3$ (d), and of the two most collective ones Z_1 and Z_2 (c) for $Q = 406$ (e) and $Q = 3$ (f).

Due to the matrix trace conservation (here $\mathrm{Tr}\,\boldsymbol{C} = 100$), the existence of strong collective components can effectively supress the noisy part of the $\boldsymbol{C}$ eigenspectrum, shifting smaller eigenvalues towards zero and thus may distort their relation to the RMT case. In order to correct these effects, which are more affecting the case of figure 2(a), it is recommended to remove the market factor Z_1 from the data [21]. One way to do this is by means of the least square fitting of this factor represented by $z_1(j)$ to each of the original stock signals $g_k(j)$:

$$g_k(j) = \alpha_k + \beta_k z_1(j) + \epsilon_k^{(1)}(j)\,, \tag{9}$$

where α_i, β_i are parameters, and then one can construct a new correlation matrix $\boldsymbol{C}^{(1)}$ from the residuals $\epsilon_k^{(1)}(j)$ (*e.g.* Ref. [9, 21]). After this is performed significantly more eigenvalues for $Q = 406$ fall within the shaded RMT region as figure 2(c) illustrates. For $Q = 3$ such a removal does not affect so much the bulk of the original spectrum as figure 2(d) compared to 2(b) shows. Such a removal can be executed once again and the λ_2 components can also be removed leading to the eigenspectra presented in Figs. 2(e) and 2(f). The effect of this second removal is already much smaller but is more noticeable in the former case. In the corresponding figure 2(e) one still finds only ($\gamma = 49\%$) eigenvalues overlapping with the RMT interval $\langle\lambda_{\min}, \lambda_{\max}\rangle$. This is almost a factor of two less than the case of $Q = 3$ in figure 2(f) and only this latter result remains in agreement with results presented earlier in [9, 21] (based on the daily data but with similar small values of Q) where a vast majority of the eigenvalues was within the RMT bounds.

6. Auxiliary tests

There is potentially one effect that may partly be responsible for such a sizeable disagreement between the $Q = 406$ empirical and the corresponding RMT results. There namely exists some time correlations — especially the volatility correlations — in the individual empirical time series that may effectively reduce the number of independent events in each series. If this is the case then the parameter Q used in the reference RMT formula should be proportionally smaller, the RMT bounds wider and thus an agreement improved. In order to verify to what extent such an effect may here be present we perform the following exercise. Imagine all the time series are progressing along the independent circles each, such that the end of the series is connected to its starting point. The circles are then rotated against each other by a random angle. This procedure preserves the internal correlations within each series but destroys the cross-correlations. The spectrum of eigenvalues of the so-randomized empirical correlation matrix is shown in figure 3(a). The perfect coincidence between this empirical and the RMT result can be seen. This provides a strong evidence that the corresponding disagreement in Figs. 2 is entirely due to the real cross-correlations and is fully informative.

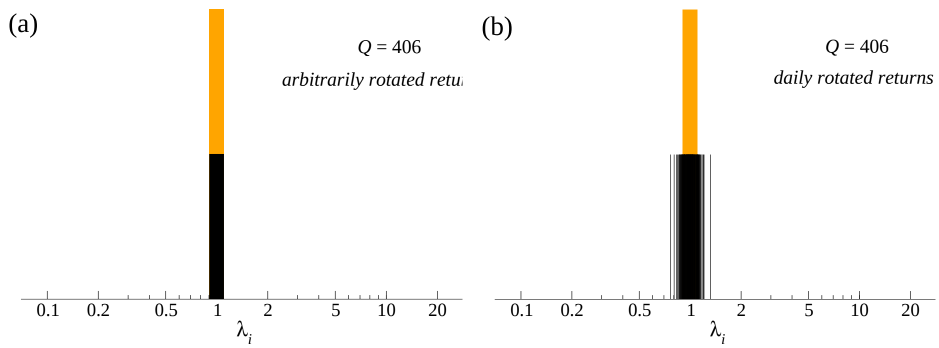

Fig. 3. Eigenvalue spectrum of the test correlation matrices for $Q = 406$ obtained after an unrestricted random shift (see text) of all the original empirical time series against each other (a) and after restricting this random shift to the multiples of one full trading day (b). Shaded regions correspond to RMT predictions for the same value of Q.

As another related test the above circles are randomly rotated but this time the rotation angles are restricted to the multiples of one full trading day (daily rotated). Now, as is shown in figure 3(b), the empirical spectrum broadens more than a factor of two relative to the previous case and of course by the same factor relative to the RMT bounds. This result may reflect the presence of day-to-day repeatable intraday patterns of activity that affect various different securities at similar instants of time during the day.

As a further examination of the character of the stock market cross-correlations two more types of artificial series based on the original empirical data are created using the decomposition as in Eq. (4). Before the correlation matrix is calculated either (a) the signs ($\mathrm{sign}_s(j)$) are randomly reshuffled but the return magnitudes $v_s(j)$ left at their original places or vice versa (b), the signs are left original but $v_s(j)$ reshuffled for each series independently. The resulting spectra are shown in figure 4(a) and 4(b) correspondingly. From this perspective the signs turn out responsible much more for the cross-correlations than the corresponding magnitudes of the returns. As is clearly seen, randomizing signs washes out the cross-correlations almost completely (though deviations relative to the RMT still remain) while randomizing $v_s(j)$ with the signs unaltered largely preserves the original (Fig. 2(a)) structure of the spectrum. To a good approximation the spectrum of Fig. 4(b) looks compressed by a factor of about two relative to that in Fig. 2(a).

As a supplementary material to this kind of the test analysis in figure 5 we show the spectra (a) of the correlations matrices calculated from the time series of $\mathrm{sign}_s(j)$ and (b) from the time series of $v_s(j)$, independently. Consistently with the observation made in figure 4(b) the time series of the

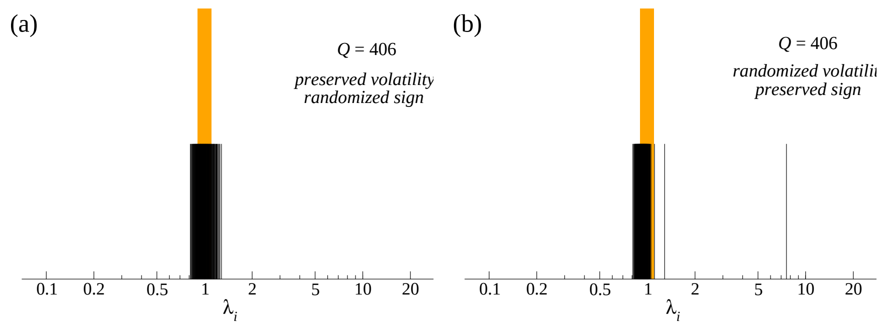

Fig. 4. Eigenvalue spectrum of the test correlation matrices for $Q = 406$ obtained after the signs (Eq. (4)) are randomly reshuffled, independently in each empirical time series and the magnitudes are left unchanged (a), and after the magnitudes are randomly reshuffled and the signs unchanged (b). Shaded regions correspond to RMT predictions for the same value of Q.

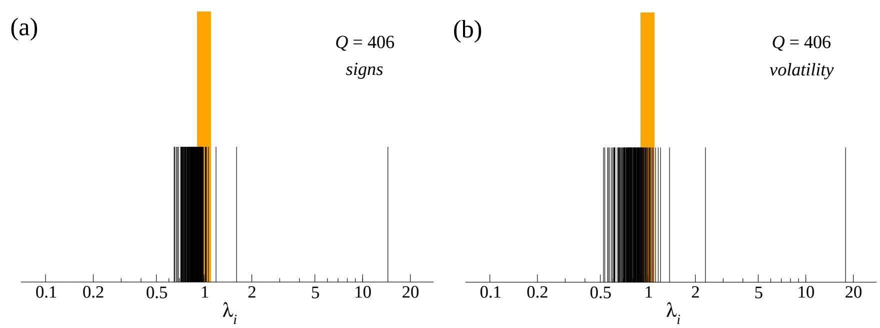

Fig. 5. Eigenvalue spectrum of the test correlation matrices for $Q = 406$ obtained from the time series of signs (a) and from the time series of moduli of the empirical returns (b) as decomposed by Eq. (4). Shaded regions correspond to RMT predictions for the same value of Q.

empirical return's signs show very similar structure of cross-correlations as the full original result (Fig. 2(a)). In view of the result presented in figure 4(a) somewhat surprising may however be considered the fact that the top eigenvalues appear (Fig. 5(b)) even bit larger in the second case of $v_s(j)$ time series. Relevant here is that these volatility related cross-correlations manifest their presence only when the return's signs are entirely discarded, *i.e.*, their moduli are taken, which is a nonlinear operation. The correlation matrix detects the linear (cross-) correlations, but detecting linear correlations in volatility means detecting the nonlinear correlations in returns. Thus the above results taken together also point to the complex nonlinear character of the financial cross-correlations.

7. Eigensignal properties

A deeper exploration of the relation between the characteristics of the empirical financial cross-correlations and those of the conventional RMT needs to involve also the eigensignals since they directly reflect the dynamics of the corresponding portfolio. Figure 6 presents the time series of the eigensignal returns $z_1(j)$ calculated according to Eq. (7) for the most collective eigenstate associated with λ_1 and for another one associated with λ_{52}. Even though this latter case corresponds to the middle of the empirical spectrum, strongly overlapping with the RMT region, it appears difficult to detect any significant differences, if one compares both series visually, ignoring different scales in vertical axis. Both eigensignals are nonstationary with likely extreme fluctuations and both of them also exhibit volatility clustering. A compact form to quantify the related effects is in terms of the multifractal spectra. It is a well established fact that stock returns form signals which are multifractal both on daily and on high-frequency time scales [22–25].

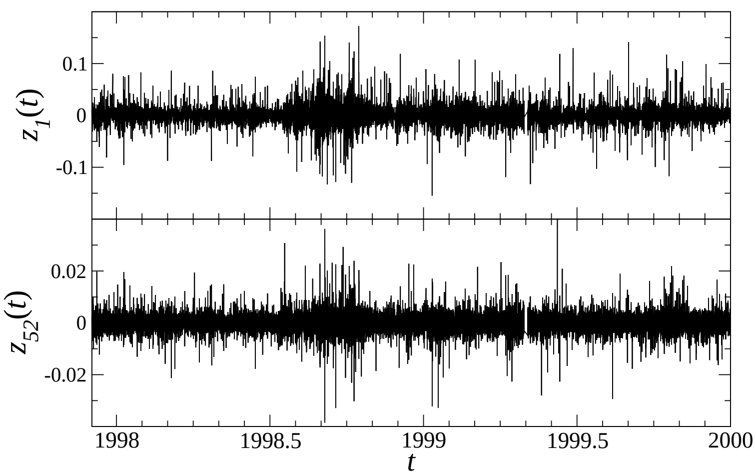

Fig. 6. Time series of the eigensignals for λ_1 (top) and λ_{52} (bottom). Note different scales in vertical axes of both panels.

In order to evaluate the singularity spectra $f(\alpha)$ we use the Multifractal Detrended Fluctuation Analysis (MFDFA) [26] method which for the present purpose appears [27] more stable than the Wavelet Transform Modulus Maxima (WTMM) method [28]. Accordingly, we start from our eigensignal i represented by the time series $z_i(j)$ of length $N_{\rm es}$ and evaluate the signal profile

$$Y(j) = \sum_{k=1}^{j} \left(z_i(k) - \langle z_i \rangle \right), \qquad j = 1, \ldots, N_{\rm es}, \tag{10}$$

where $\langle \ldots \rangle$ denotes averaging over $z_i(k)$. In the next step Y is divided into $M_{\rm es}$ segments of length n ($n < N_{\rm es}$) starting from both the beginning and the end of the time series so that eventually there are $2M_{\rm es}$ segments. In each segment ν a local trend is removed by fitting an l-th order polynomial $P_\nu^{(l)}$ to the data. Then, after calculating the variance

$$F^2(\nu, n) = \frac{1}{n} \sum_{k=1}^{n} \{Y[(\nu - 1)n + k] - P_\nu^{(l)}(k)\}^2 \tag{11}$$

and averaging it over ν's, we get the qth order fluctuation function

$$F_q(n) = \left\{ \frac{1}{2M_{\rm es}} \sum_{\nu=1}^{2M_{\rm es}} [F^2(\nu, n)]^{q/2} \right\}^{1/q}, \qquad q \in \boldsymbol{R} \tag{12}$$

for all values of n. For a signal of the fractal character $F_q(n)$ obeys a power-law functional dependence on n:

$$F_q(n) \sim n^{h(q)}, \tag{13}$$

at least for some range of n. If this is the case the MF–DFA procedure provides a family of generalized Hurst exponents $h(q)$, which form a decreasing function of q for a multifractal signal or are independent of q for a monofractal one. A compact form to present the result graphically is to calculate the singularity spectrum $f(\alpha)$ through the relations:

$$\alpha = h(q) + qh'(q)\,, \qquad f(\alpha) = q[\alpha - h(q)] + 1\,. \tag{14}$$

Some representative final results of such an analysis are shown in figure 7. Both eigensignals presented in figure 6 (Z_1 and Z_{52}) develop convincing multifractality with the spectrum $f(\alpha)$ of about the same width even though the later one represents the middle of the eigenvalue spectrum. The maxima of these $f(\alpha)$ spectra are however located at different positions, even relative to $\alpha = 0.5$, which may reflect either persistency or antipersistency in the underlying time series. As far as the width of $f(\alpha)$ is concerned they are of comparable magnitude for all other eigensignals. As a global documentation of this fact the average over all ($i = 2, \ldots, 100$) the corresponding singularity spectra is also shown in this figure. This average displays maximum at $\alpha = 0.5$ exactly.

8. Summary

The results presented in this contribution provide further evidence that the financial markets constitute a real complexity. The stock market cross-correlations viewed through the eigenspectrum of the correlation matrix

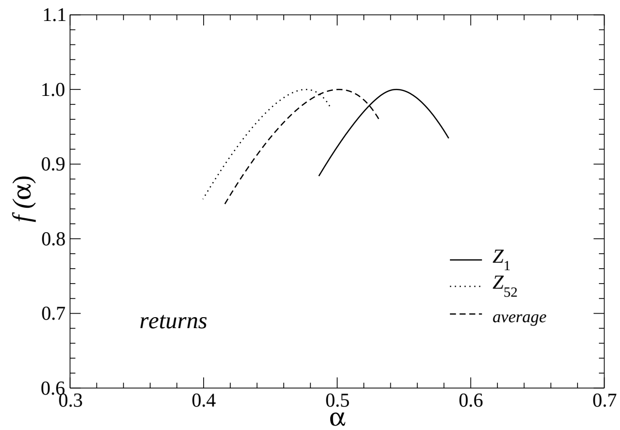

Fig. 7. (a) Singularity spectra $f(\alpha)$ for the eigensignal corresponding to the largest eigenvalue λ_1 (solid line), to the average over all other eigensignals Z_i, $i = 2,\ldots,100$ (dashed line), and to Z_{52} (dotted line) of the empirical correlation matrix.

show existence of the market linear collective component represented by one pronounced eigenvalue which is well separated from the bulk of eigenvalues. This 'bulk' is however not of the Wishart random matrix ensemble type which is especially clearly seen when the time series used to construct the correlation matrix are sufficiently long. The fact that the financial cross-correlations appear not to be simply decomposable into 'market', 'sectors', and an uncorrelated Wishart 'bulk' has to do with their nonlinear character both in space and in time. This profound nonlinearity manifests itself in the multifractal nature of all the principal components (eigensignals) which represent different portfolios and in the volatility cross-correlations. This signals that information about the whole market is encoded already in all its constituents. This does not necessarily mean that the involved whole amount of information is of practical interest or importance. In order however to disentangle — in the spirit of the Random Matrix Theory — what is more relevant from what is less, a more extended variant of random matrix ensemble is called for. In view of the results presented above, when postulating an appropriate RMT variant to be used as a reference in the financial context one definitely needs to redefine the notion of noise such that some of the correlations are already built into.

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