LITTLE HIGGS AND SUPERSYMMETRY*

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We discuss a mechanism of double protection of the Higgs potential by supersymmetry and by a global symmetry. This occurs in supersymmetric models in which the Higgs boson is a Goldstone boson of a spontaneously broken approximate global symmetry, as in little Higgs. In such models the parameters of the electroweak sector require no fine-tuning at all.

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1. Introduction

Supersymmetry is a very attractive scenario for physics at the TeV scale. Unfortunately, its simplest implementation, the MSSM, requires fine-tuning of parameters once experimental constraints are imposed. The problem can be summarized as follows. The electroweak scale in the MSSM depends on the Higgs soft mass term m_H^2, which receives radiative corrections when supersymmetry is broken. The largest correction is due to top/stop loops

$$\delta m_H^2 \simeq -\frac{3y_{\rm t}^2}{8\pi^2}\, m_{\tilde{\rm t}}^2 \,\log\left(\frac{\Lambda_{\rm UV}^2}{m_{\tilde{\rm t}}^2}\right) , \tag{1}$$

where $y_{\rm t}$ is the top Yukawa coupling, $m_{\tilde{\rm t}}$ is the stop mass (for simplicity, we consider degenerate left and right-handed stops and ignore stop mixing), and $\Lambda_{\rm UV}$ is the high energy scale at which the soft masses are generated. For large values of $\Lambda_{\rm UV}$ (for example $\Lambda_{\rm UV} \sim M_{\rm Planck}$ as in gravity mediated supersymmetry breaking), the suppression due to the loop factor is canceled by the large logarithm, and we obtain $\delta m_H^2 \sim m_{\tilde{\rm t}}^2$. Therefore, for the MSSM

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to be a natural theory the superpartner mass scale $M_{\rm SUSY}$ should be of the order of the Higgs mass and the electroweak scale, $M_{\rm EW} \sim 100\,{\rm GeV}$. Direct searches, limits from precision electroweak and flavor constraints do not confirm this expectation. Most importantly, in the MSSM the lower bound on the mass of the Higgs boson implies the limit on the stop mass, $m_{\tilde{\rm t}}^2 \gtrsim (1\,{\rm TeV})^2$. This means that radiative corrections to $M_{\rm EW}$ are at least hundred times larger than $M_{\rm EW}$ itself. In other words, fine-tuning in the MSSM is 1% or worse.

We conclude that the MSSM is not a natural theory. This is a serious problem, as naturalness was the main motivation to introduce the whole theoretical framework of supersymmetry! If we believe that supersymmetry is a solution to the gauge hierarchy problem we need to search for extensions of the MSSM that could accommodate the modest hierarchy of scales, $M_{\rm SUSY} \gtrsim 4\pi\, M_{\rm EW}$, without fine-tuning.

Improving naturalness of supersymmetric theories with heavy superpartners therefore requires removing the large logarithm. Two possibilities suggest themselves. One is to lower the scale $\Lambda_{\rm UV}$ down to 1–100 TeV range. This can be achieved in scenarios in which supersymmetry breaking is mediated at a low scale, such as gauge mediation or theories with large extra dimensions. The other possibility [1–7] is to treat the Higgs differently from the superpartners by making it a pseudo-Goldstone boson. In this approach, radiative corrections to the Higgs soft mass are finite because they are "doubly protected" by softly broken supersymmetry and by a global symmetry. At tree-level the soft mass of a doubly protected Higgs vanishes, while the dominant radiative correction has the form

$$\delta m_H^2 \approx -\frac{3y_{\rm t}^2}{8\pi^2}\left[(m_{\tilde{\rm t}}^2 + m_{\rm T}^2)\log(m_{\tilde{\rm t}}^2 + m_{\rm T}^2) - m_{\tilde{\rm t}}^2\log(m_{\tilde{\rm t}}^2) - m_{\rm T}^2\log(m_{\rm T}^2)\right], \quad (2)$$

where $m_{\rm T}$ is the mass of the top quark's heavy partner whose presence is required by the global symmetry.

In the remainder of this article we explore the mechanism of double protection, that occurs in models combining supersymmetry with a global symmetry. The global symmetry is broken both spontaneously and explicitly, in a similar fashion as in little Higgs theories [8]. The marriage between supersymmetry and little Higgs yields a framework referred to as Little Susy.

2. Doubly protected electroweak breaking

For double protection to be implemented a model at the TeV scale should have the following features:

1. Supersymmetry softly broken at the scale $M_{\rm SUSY}$ of order TeV.

2. An approximate global symmetry of the Higgs potential.

3. Spontaneous breaking of the global symmetry at a scale of order TeV.
4. Interactions between the Higgs and the top sectors explicitly and softly breaking the global symmetry.

In the present context, soft breaking of a global symmetry means that the mass parameter $M_{\rm G}$ breaking explicitly the global symmetry does not break supersymmetry and, simultaneously, the soft supersymmetry breaking terms preserve the global symmetry. Models with the features 1–4 exhibit *no one-loop divergent corrections to the Higgs potential from the top sector.* This surprising result follows from a simple dimensional analysis. Quadratic divergences are of course forbidden by supersymmetry. Typically, we would expect the Higgs mass parameter to be logarithmically divergent, $\delta m_H^2 \sim M_{\rm SUSY}^2 \log \Lambda_{\rm UV}$. However, in Little Susy this is not allowed as the corrections to the Higgs mass must be proportional to a parameter that breaks the global symmetry (if the global symmetry were exact the Higgs would be exactly massless to all orders). At the same time $\delta m_H^2 \sim M_{\rm G}^2 \log \Lambda_{\rm UV}$ also cannot occur, as the Higgs mass must be proportional to a parameter that breaks supersymmetry.

Let us now be more explicit about the global symmetry we need. The two MSSM Higgs $\mathrm{SU}(2)_{\rm weak}$ doublets H_d, H_u are combined with SM singlet fields S_u and S_d into multiplets ϕ_u, ϕ_d of a global symmetry whose $\mathrm{SU}(2)_{\rm weak}$ is a subgroup.

$$H_d \to \phi_d = (H_d, S_d)\,, \qquad H_u \to \phi_u = (H_u, S_u)^T\,. \tag{3}$$

We assume that $|\phi_u|^2$, $|\phi_d|^2$ and $\phi_d\phi_u$ are invariants of the global symmetry.

Next, the global symmetry is spontaneously broken by vevs of the SM singlet fields $\langle S_u\rangle = f_u$, $\langle S_d\rangle = f_d$. It is well known that spontaneous breaking of a global symmetry implies a presence of a multiplet of massless scalars H^a called the Goldstone bosons. These are excitations along the spontaneously broken directions in the group space and can be parameterized as

$$\phi_u \to e^{iH^aT^a}(0, \langle S_u\rangle)^T\,, \qquad \phi_d \to (0, \langle S_d\rangle)e^{-iH^aT^a}\,, \tag{4}$$

where T^a are the broken generators of the global symmetry. The point about this parametrization is that the global symmetry group invariants $|\phi_u|^2$, $|\phi_d|^2$ and $\phi_d\phi_u$, do not depend on H^a. We assume that the Higgs potential respects the global symmetry, that is, it depends only on these invariants. This ensures that the Goldstone bosons are massless at tree level (in fact, it implies that the potential for the Goldstones is exactly flat). A subset of the Goldstone bosons is identified with the SM Higgs field H.

We move to discussing the radiatively generated Higgs potential. We assume here that the loop corrections to the Higgs potential come mostly from the top sector, which consists here of the SM top t and its heavy partner T. It turns out that the global symmetry and its soft breaking imply a relation between the masses of the two quarks in the presence of the electroweak breaking vev H:

$$m_{\rm t}^2(H) + m_{\rm T}^2(H) = m_{\rm T}^2\,, \tag{5}$$

where the right hand side does not depend on H. Thus, we can parameterize the top quark masses squared as:

$$m_{\rm t}^2(H) \approx y_{\rm t}^2|H|^2 + \mathcal{O}\left(\frac{H^4}{f^2}\right)\,, \qquad m_{\rm T}^2(H) = m_{\rm T}^2 - m_{\rm t}^2(H)\,. \tag{6}$$

Of course, the mass of the light top is proportional to the Higgs vev. The masses of the supersymmetric partners of the top quarks are in addition sensitive to the supersymmetry breaking terms. Assuming universality of stop soft masses and negligible stop mixing[1] the stop masses can be written as

$$m_{\tilde{\rm t}}^2(H) \approx m_{\tilde{\rm t}}^2 + m_{\rm t}^2(H)\,, \qquad m_{\tilde{\rm T}}^2(H) = m_{\tilde{\rm t}}^2 + m_{\rm T}^2(H)\,. \tag{7}$$

Inserting the top and stop masses into the Coleman–Weinberg formula

$$V_{\rm CW} = \frac{1}{64\pi^2}\,{\rm STr}\Big[\mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2} - \frac{3}{2}\mathcal{M}^4\Big] \tag{8}$$

we obtain

$$\begin{aligned} V(H) \;=\; -\frac{3}{16\pi^2}\Big[& m_{\rm t}^4(H)\log(m_{\rm t}^2(H)) + m_{\rm T}^4(H)\log(m_{\rm T}^2(H)) \\ & - m_{\tilde{\rm t}}^4(H)\log\left(m_{\tilde{\rm t}}^2(H)\right) - m_{\tilde{\rm T}}^4(H)\log\left(m_{\tilde{\rm T}}^2(H)\right)\Big]\,. \end{aligned} \tag{9}$$

Expanding in powers of the Higgs field we easily find

$$\begin{aligned} V(H) \;&=\; \delta m_H^2|H|^2 + \delta\lambda|H|^4 + \ldots \\ \delta m_h^2 \;&\approx\; -\frac{3}{8\pi^2}y_{\rm t}^2\Big[(m_{\tilde{\rm t}}^2 + m_{\rm T}^2)\log(m_{\tilde{\rm t}}^2 + m_{\rm T}^2) - m_{\tilde{\rm t}}^2\log(m_{\tilde{\rm t}}^2) - m_{\rm T}^2\log(m_{\rm T}^2)\Big]\,, \\ \delta\lambda \;&\approx\; \frac{3}{16\pi^2}y_{\rm t}^4\left[\log\left(\frac{m_{\rm T}^2 m_{\tilde{\rm t}}^2}{(m_{\tilde{\rm t}}^2 + m_{\rm T}^2)m_{\rm t}^2}\right) + \frac{3}{2} - 2\frac{m_{\tilde{\rm t}}^2}{m_{\rm T}^2}\log\left(\frac{m_{\tilde{\rm t}}^2 + m_{\rm T}^2}{m_{\tilde{\rm t}}^2}\right)\right]. \end{aligned} \tag{10}$$

[1] These assumptions facilitate the computation of the effective potential but are not crucial for implementing double protection. The sufficient condition is that the soft terms respect the global symmetry.

As in the MSSM the mass correction from the top sector is negative and may trigger the electroweak breaking. Little Susy renders it with the double protection structure: it vanishes in the supersymmetric limit $m_{\tilde{t}} \to 0$ as well as in the globally symmetric limit $m_{\rm T} \to 0$. For $m_{\rm T} \gg m_{\tilde{t}}$ we find $\delta m_h^2 \approx -3/(8\pi^2) y_t^2 \log[(m_{\rm T}^2)/(m_{\tilde{t}}^2)]$ which shows that the additional top and stops cut-off the log divergence of the MSSM. For $m_{\rm T} \sim m_{\tilde{t}} \sim 1\,\text{TeV}$ we find

$$\delta m_H^2 \sim \frac{m_{\tilde{t}}^2}{4\pi}\,, \tag{11}$$

which removes the fine-tuning problem.

The correction to the quartic Higgs terms is qualitatively similar to that in the MSSM: it behaves as $\delta\lambda \sim 3/(16\pi^2)\, y_t^4 \log\,[\,(\text{Min}(m_{\tilde{t}}^2, m_{\rm T}^2))/(m_t^2)\,]$. By itself, this term is not enough to push the Higgs boson mass above the experimental limit. To make the theory phenomenologically viable we need additional tree-level contributions to the Higgs quartic term.

3. Finding Little Susy

We have argued that supersymmetry combined with a softly broken approximate global symmetry at the TeV scale ameliorates the fine-tuning problem. However, constructing an explicit model that realizes this scenario and is consistent with all experimental constraints is not so easy. First of all, a global symmetry with desired features is not radiatively stable. Indeed, the MSSM Higgs doublets are charged under the SM gauge interactions, while their partners that acquire the global symmetry breaking vevs must be SM singlets (otherwise their vevs would contribute to W and Z masses). Renormalization effects split the soft masses in the global multiplets and lead to tree-level contributions to the would-be Goldstone masses. Therefore, we need a mechanism to protect the global symmetry. The second problem is that in order to satisfy the experimental constraints on the Higgs boson mass we need a tree-level Higgs quartic term. Such term also breaks the global symmetry and we must find a way of introducing it without generating large contributions to the tree-level Higgs mass parameter at the same time.

How can we protect the global symmetry? This symmetry should relate an $\text{SU}(2)_{\text{weak}}$ doublet H to $\text{SU}(2)_{\text{weak}}$ singlet field(s) S. Two possibilities suggest themselves:

1. There exists a gauge symmetry G which is broken to $\text{SU}(2)_{\text{weak}}$ at a higher scale. H and S could then be unified into one multiplet of G.
2. There exists a discrete symmetry, that acts as $H \longleftrightarrow S$. If this is the case there must be another SU(2) gauge symmetry acting on S, with Z_2 interchanging the two SU(2)'s.

Both cases require extending the SM gauge symmetry, though for different reasons.

The first possibility was explored in Refs. [3–5]. In those papers the gauged $\mathrm{SU}(2)_{\mathrm{weak}}$ is enlarged to a gauged $\mathrm{SU}(3)_{\mathrm{weak}}$ as in the simplest little Higgs [9]. The Goldstone bosons then arise because the $\mathrm{SU}(3)_{\mathrm{weak}}$ gauge symmetry is broken spontaneously to SU(2) by two different sets of fields. If the coupling between these two sets of fields is sufficiently weak, then the theory has an approximate $\mathrm{SU}(3)^2$ symmetry which is spontaneously broken to $\mathrm{SU}(2)^2$, yielding two sets of Goldstones, one linear combination is eaten by the heavy $\mathrm{SU}(3)_{\mathrm{weak}}$ gauge bosons, the other remains light. A general problem with this approach is that the $\mathrm{SU}(3)_{\mathrm{weak}}$ D-terms strongly couple the two sectors and explicitly break the two SU(3)'s of the Higgs sector to a single SU(3).

The second possibility, explored in Refs. [6, 7], is based on the idea of Twin Higgs [10, 11]. The model of Ref. [6] is left–right symmetric with the gauge group $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_X$ as in [11]. The Z_2 symmetry interchanges the left and right SU(2) gauge bosons. Furthermore, every MSSM field has its Z_2 partner. In particular, the Higgs sector consist of four multiplets: two "left" doublets H_u and H_d and two "right" doublets $\tilde{H}_u$ and $\tilde{H}_d$. The Z_2 symmetry imposed on the Higgs sector is sufficient to guarantee an accidental SU(4) symmetry of the dimension 2 terms in the Higgs potential. Thus even though the Yukawa interactions which renormalize the Higgs mass terms do not respect the full SU(4) (even after imposing the Z_2 symmetry), the resulting corrections to the Higgs masses are automatically SU(4) symmetric. This is how double protection is realized in this model. Divergent radiative corrections to soft masses do not lead to masses for the Goldstones because they respect the full global symmetry. The minimal twin supersymmetric model shares a problem with the models based on $\mathrm{SU}(3)_{\mathrm{weak}}$ group discussed above: some of the quartic couplings in the Higgs sector, in particular the $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}} \times \mathrm{U}(1)_X$ D-terms explicitly break the SU(4) symmetry and lead to large tree-level masses for the would-be Goldstones. However, this problem can be solved by setting $\tan\beta = 1$ (which, effectively, removes the dangerous D-terms) and introducing singlet fields that generate tree-level quartic terms, as in the NMSSM.

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