

MASS SPECTROSCOPY OF SCALAR AND AXIAL
VECTOR MESONS IN LATTICE QCD*

T. KUNIHIRO

YITP, Kyoto University, Kyoto 606-8502, Japan

S. MUROYA

Matsumoto University, Nagano 390-1295, Japan

A. NAKAMURA

RIISE, Hiroshima University, Higshi-Hiroshima 739-8521, Japan

C. NONAKA

Department of Physics, Nagoya University, Nagoya 464-8602, Japan

M. SEKIGUCHI

Faculty of Engineering, Kokushikan University, Tokyo 154-8515, Japan

H. WADA

Faculty of Political Science and Economics, Kokushikan University
Tokyo 154-8515, Japan*(Received November 15, 2006)*

We present our recent study of $I = 1/2$ scalar (κ) and axial vector mesons (K_1) in the quenched approximation. Our results show that κ meson does not agree with recent experimental values.

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1. Introduction

The nature of mass spectroscopy of hadrons continues to be an intriguing problem in QCD. Experimentally, many exotic hadrons have been observed with the mass above 2 GeV containing heavy quarks [1]. This experimental development leads us to theoretical study of structure and dynamics of

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exotic hadrons. Actually, such a controversy on the structure of hadrons is also the case for the scalar mesons below 1 GeV: Especially, the evidence of the σ meson ($I = 0, J^{\text{PC}}=0^{++}$) was obscure problem for many years. But, its evidence has been reported not only in $\pi\pi$ scattering but also in various production processes. Now, the σ meson with a low mass 500 – 600 MeV and a broad width is widely accepted [2]. In the non-relativistic constituent quark model, $J^{\text{PC}}=0^{++}$ meson is realized as 3P_0 state, which implies that the mass of the σ meson should be in 1.2 – 1.6 GeV region. Therefore, some mechanism is needed to down the mass for the σ meson. Many suggestions are discussed in the literature [3,4,7–9]. We have observed that the disconnected diagram makes the σ meson mass very light by full lattice QCD simulation on the $8^3 \times 16$ lattice using the plaquette action and Wilson fermions [10–13].

If the σ meson certainly exists, it is natural to consider the existence of κ meson of the nonet scalar states chiral $\text{SU}(3) \times \text{SU}(3)$ symmetry. The κ meson has been observed in analyses on $K\pi$ scattering phase shifts by several groups. However, some group have not found evidence for the κ . Recently, evidence for the κ meson $I = 1/2, J^{\text{PC}} = 0^{++}$ was reported in production process by E791 experiment at Fermilab in analysis of $D^+ \rightarrow K^- \pi^+ \pi^+$ [5] and by BES experiment in analysis $J/\psi \rightarrow K^+ K^- \pi^+ \pi^-$ [6]. The observed mass of the κ is 800 – 900 MeV and the width is large.

It is very important to investigate the κ meson by lattice QCD in order to establish the mass spectroscopy of the scalar mesons. Lattice QCD provides a first principal approach of hadron physics and allows us to study non-perturbative aspects of quark–gluon dynamics. In the previous paper [13, 15], we have treated the s quark as a valence quark, while u and d quarks dynamically. Prelovsek *et al.* [16] have presented a rough estimate by extrapolating the mass of κ obtained from the dynamical simulation. They assumed that the u , d and s quarks are degenerate. The UKQCD Collaboration studied the κ by the dynamical simulation with $N_f = 2$ sea quarks and a strange quark treated as a valence quark [17]. In our previous simulation, the cutoff was not sufficiently high to accommodate large masses $m_\kappa a > 1$, where a is the lattice spacing. Hence, we present a quench simulation at weaker couplings on a larger lattice. And we shall also mention that our simulation suggests the existence of $I = 1/2, J^{\text{PC}} = 1^{++}$ axial vector meson (K_1). The K_1 comes from the mixing between $I = 1/2, J^{\text{PC}}=1^{++}$ and 1^{+-} . In fact, the axial vector meson K_{1A} and K_{1B} are mixtures of the $K_1(1270)$ and $K_1(1400)$. Hence, our simulation for the axial vector meson is an ideal case. Moreover, we shall present the valence σ meson and a_1 meson.

2. Simulation

We adopt the following interpolation operator for creating the κ^+ meson with $I = 1/2$ and $J^{\text{PC}} = 0^{++}$ as

$$\hat{\kappa}(x) \equiv \sum_{c=1}^3 \sum_{\alpha=1}^4 \bar{s}_\alpha^c(x) u_\alpha^c(x), \quad (1)$$

where s and u indicate the corresponding quark spinors. Indices c and α stand for color and Dirac spinor indices, respectively. The κ meson propagator is written as

$$\begin{aligned} G(y, x) = \langle \text{Tr} \hat{\kappa}(y) \hat{\kappa}(x)^\dagger \rangle &= \frac{1}{Z} \int DU D\bar{u} Du D\bar{s} Ds \sum_{a,b=1}^3 \sum_{\alpha,\beta=1}^4 \bar{s}_\beta^b(y) u_\beta^b(y) \\ &\times \left(\bar{s}_\alpha^a(x) u_\alpha^a(x) \right)^\dagger e^{-S_G - \bar{u} W_u u - \bar{s} W_s s}. \end{aligned} \quad (2)$$

In Eq. (2), W_u^{-1} (W_s^{-1})'s is u (s) quark propagator, U 's are link variables of gluon, and S_G is the gauge action. By integrating over u , $\bar{u}$, s and $\bar{s}$ fields, the κ meson propagator is given by

$$G(y, x) = -\langle \text{Tr} W_s^{-1}(x, y) W_u^{-1}(y, x) \rangle, \quad (3)$$

where “Tr” represents summation over color and Dirac spinor indices. From Eq. (3), we can see that κ propagator is composed of a connected diagram and contains no disconnected part, the latter of which was the origin of the light mass of the σ meson. For K_1 , the operator is $K_1^+ = \bar{s} \gamma^\mu \gamma_5 u$ and also the K_1 meson propagator is composed of a connected diagram. We employ Wilson fermions and the plaquette gauge action.

As for the simulation parameters, we first note that CP-PACS performed the quenched approximation calculation of the light meson spectroscopy with great success [18]. Therefore, we use the same values of the simulation parameters as those used by CP-PACS, *i.e.*, $\beta = 5.9$, and the following three values for the u and d quark hopping parameter $h_{u/d} = 0.1589, 0.1583$ and 0.1574 , and the two values for the s quark hopping parameter $h_s = 0.1566$ and 0.1557 , except for the lattice size; our lattice size is $20^3 \times 24$. The meson propagators are calculated on 80 configurations at interval of 2000 configurations generated by the heat bath method. We shall use the point source and sink, which together with the smaller lattice size, leads to large masses due to mixture of higher mass states. In other words, the masses to be obtained in our simulation should be considered as upper limits. The mass ratios of m_π/m_ρ are summarized in Table I. Our results for the ρ meson mass is only slightly ($< 5\%$) larger than the CP-PACS's result.

We emphasize that the deviation between our results and the larger lattice result (CP-PACS) is so small in spite of the large difference in the lattice size.

TABLE I

The summary results for $\bar{q}q$ type mesons.

h_u/d	0.1589	0.1583	0.1574	0.1566	0.1557
m_π	0.2064(62)	0.2691(36)	0.3401(29)	0.3935(28)	0.4478(28)
m_ρ	0.442(13)	0.461(06)	0.496(05)	0.527(04)	0.563(03)
m_π/m_ρ	0.467(21)	0.584(10)	0.686(05)	0.746(03)	0.796(03)
m_{σ_v}	1.12(74)	0.84(23)	0.886(98)	0.857(52)	0.897(35)
m_{a_1}	0.862(76)	0.895(48)	0.913(32)	0.932(24)	0.959(20)
CP-PACS [18]					
m_π	0.20827(33)	0.26411(28)	0.33114(26)	0.38255(25)	—
m_ρ	0.42391(132)	0.44514(96)	0.47862(71)	0.50900(60)	—
m_π/m_ρ	0.491(2)	0.593(1)	0.692(1)	0.752(1)	—

In Table I masses of valence σ for each hopping parameter are shown. m_{σ_v}/m_ρ varies from 1.6 to 2.5 which our previous results [13]. In other word, without disconnected part of the propagator the mass of “ σ ” become heavy. We also present a_1 mass in Table I. From our results, we evaluate the critical value of the hopping parameter $h_{\text{crit}} = 0.1598(1)$ and lattice space $a = 0.1038(33)$ fm in the chiral limit ($m_\pi^2 = 0$) from the value $m_\rho a = 0.406(13)$ at the point with the physical ρ meson mass being used for m_ρ .

TABLE II

The summary results for K , K^* , κ and K_1 mesons at $h_s = 0.1566$.

h_u/d	$h_{\text{crit}}^{(*)}$	0.1589	0.1583	0.1574
m_K	0.2829(23)	0.3138(33)	0.3368(30)	0.3677(29)
m_{K^*}	0.4649(69)	0.4821(57)	0.4941(49)	0.5117(42)
m_κ	0.89(29)	0.88(23)	0.81(12)	0.814(81)
m_{K_1}	0.919(43)	0.922(36)	0.915(32)	0.922(27)
CP-PACS [18]				
m_K	—	0.30769(28)	0.32833(26)	—
m_{K^*}	—	0.46724(84)	0.47749(74)	—

$(*)h_{\text{crit}} = 0.1598(1)$.

TABLE III

The summary results for K , K^* , κ and K_1 mesons at $h_s = 0.1557$.

$h_{u/d}$	$h_{\text{crit}}^{(*)}$	0.1589	0.1583	0.1574
m_K	0.3188(25)	0.3474(31)	0.3684(29)	0.3971(28)
m_{K^*}	0.4835(61)	0.5006(52)	0.5126(44)	0.5299(37)
m_κ	0.89(21)	0.88(16)	0.828(96)	0.833(72)
m_{K_1}	0.931(36)	0.935(30)	0.928(26)	0.935(23)

$(^*)h_{\text{crit}} = 0.1598(1)$.

The propagators of the K , K^* , κ and K_1 mesons are calculated with the same configurations using the s -quark hopping parameter, $h_s = 0.1566$ and 0.1557 . The masses of the K , K^* , κ and K_1 which are extracted from the effective mass plots are summarized in Table II and Table III. We find that the effective masses of the K and K^* mesons have only small errors. Table IV gives the mass ratios m_K/m_{K^*} , m_κ/m_{K^*} and m_{K_1}/m_{K^*} at chiral limit.

TABLE IV

The summary results for the mass ratios, m_K/m_{K^*} , m_κ/m_{K^*} and m_{K_1}/m_{K^*} at chiral limit for u/d quarks.

h_s	0.1566	0.1557
$1/h_s$	6.3857	6.4226
m_K/m_{K^*}	0.6086(79)	0.6593(63)
m_κ/m_{K^*}	1.92(61)	1.84(43)
m_{K_1}/m_{K^*}	1.977(86)	1.926(69)

3. Summary

We have reported our results on the κ and K_1 meson propagator based on the quenched QCD. We see that the κ and K_1 mass are almost twice heavy as the K^* meson for all values of the hopping parameter. Our result of κ meson does not agree with recent experimental values. This result suggests that κ meson may have interesting structure like $qq\bar{q}\bar{q}$ and $K\pi$ molecules state. It is necessary to improve the statistical precision of the estimation of κ and K_1 propagators, because error bars are still large.

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