

NATURAL DARK MATTER*

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In this talk we analyse the claim that supersymmetry (SUSY) naturally accounts for the observed dark matter density. In many cases, it is necessary to tune the parameters of a SUSY model to fit the WMAP data. We provide a quantitative analysis of the degree of tuning required for different annihilation channels. Some regions are natural, requiring no tuning at all, whereas others require tuning at the 0.1% level.

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1. Introduction

A key motivation for TeV scale supersymmetry (SUSY) is that it provides a natural dark matter candidate if the lightest neutralino is the LSP. However, the regions of parameter space that yield neutralino dark matter in agreement with WMAP look very restricted. We recently studied the naturalness of dark matter in [1].

Questions of fine-tuning have long been considered in the case of electroweak symmetry breaking. In many of these studies the degree of fine-tuning required was quantified through a measure of the sensitivity of $m_{\frac{Z}{2}}^2$ to the input parameters of the MSSM a_{MSSM} . We use a similar measure [2] to quantify the degree of fine-tuning required of the MSSM parameters to produce an LSP that reproduces the observed dark matter relic density:

$$\Delta_{a_{\text{MSSM}}}^{\Omega} = \left| \frac{\partial \ln (\Omega_{\text{CDM}} h^2)}{\partial \ln (a_{\text{MSSM}})} \right|. \quad (1)$$

We take the total tuning of a point to be $\Delta^{\Omega} = \max (\Delta_a^{\Omega})$.

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The calculation of $\Omega_{\text{CDM}}h^2$ primarily depends on a_{MSSM} through their effect on the annihilation cross-section of the lightest neutralino $\tilde{\chi}_1^0$. This is primarily determined by the mass and composition of $\tilde{\chi}_1^0$. This in turn, is determined by diagonalising the neutralino mass matrix at low energy. The matrix depends upon M_1 , M_2 , and μ . If one of these is much lighter than the others $\tilde{\chi}_1^0$ will be primarily of that form. This allows us to divide up the MSSM parameter space depending on the composition of $\tilde{\chi}_1^0$. A wino LSP ($M_2 \ll M_1, \mu$) or higgsino LSP ($\mu \ll M_1, M_2$) annihilates very efficiently, resulting in too little dark matter, $\Omega_{\text{CDM}}h^2 \ll \Omega_{\text{CDM}}^{\text{WMAP}}h^2$. A bino LSP ($M_2 \ll M_1, \mu$) annihilates primarily via t -channel slepton exchange. This process is only efficient for light sleptons and so bino LSPs generally result in too large a relic $\Omega_{\text{CDM}}h^2 \gg \Omega_{\text{CDM}}^{\text{WMAP}}h^2$.

Therefore to fit the observed dark matter density we are required to move to unusual regions of the parameter space. One possibility is to consider a mixed LSP. If we have a bino LSP with just enough of either wino or higgsino mixed in, we can fit $\Omega_{\text{CDM}}^{\text{WMAP}}h^2$. This is the so called ‘‘well-tempered’’ neutralino championed in [3]. Alternatively we can consider a bino LSP in which the annihilation cross-section is enhanced via some means. This can occur in a few different ways. Firstly, if there are light sfermions, t -channel sfermion is enhanced. Secondly if $2m_{\tilde{\chi}_1^0} = m_{Z,A,h}$ the neutralinos can annihilate to an on-shell boson. Finally if the NLSP is quasi-degenerate in mass with the LSP, there will be a significant NLSP number density at freeze out and we must factor in annihilations of the NLSP into our calculations of the SUSY relic density. All of these effects can enhance annihilation of a bino LSP to the extent that we fit the observed dark matter relic density. We would expect each region to exhibit a different sensitivity to the MSSM input parameters. To study these regions we take 4 different sets of boundary conditions on the MSSM input parameters a_{MSSM} at m_{GUT} , beginning with the familiar case of the constrained minimal supersymmetric standard model (CMSSM).

The rest of this talk is set out as follows. In Sec. 2 we consider the CMSSM, which provides us with a useful reference point against which the subsequent non-universal cases may be compared. In Sec. 3 we allow the third family soft sfermion and Higgs mass squared to vary independently. In Sec. 4 we consider neutralino dark matter with non-universal gaugino masses, but with a universal soft scalar mass. In Sec. 5 we consider *both* the effects of including an independent third family sfermion mass squared *and* non-universal soft gaugino masses. Sec. 6 concludes the talk.

2. CMSSM

The CMSSM has 4 free inputs:

$$a_{\text{CMSSM}} \in \{m_0, m_{1/2}, A_0, \tan\beta \text{ and sign}(\mu)\}. \tag{2}$$

m_0 is a common scalar mass that sets the soft masses of the sfermion and Higgs sectors. $m_{1/2}$ is a common gaugino mass. A_0 sets the soft SUSY breaking trilinear coupling. $\tan\beta$ is the ratio of the Higgs VEVs. Finally the requirement that the model provides radiative electroweak symmetry breaking determines the magnitude of the SUSY conserving Higgs mass μ but leaves the sign as a free parameter.

The mass and composition of the lightest neutralino is determined by diagonalising a mass matrix that depends upon the parameters M_1 , M_2 and μ at the electroweak scale. In the CMSSM $M_1 = M_2 = m_{1/2}$ at the GUT scale. As running effects mean that $M_1(m_Z) \approx 0.4M_1(m_{\text{GUT}})$ and $M_2(m_Z) \approx 0.8M_2(m_{\text{GUT}})$, gaugino mass unification sets $M_1(m_Z) \approx 0.5M_2(m_Z)$. Therefore, unless $\mu < M_1$, the lightest neutralino will be dominantly bino.

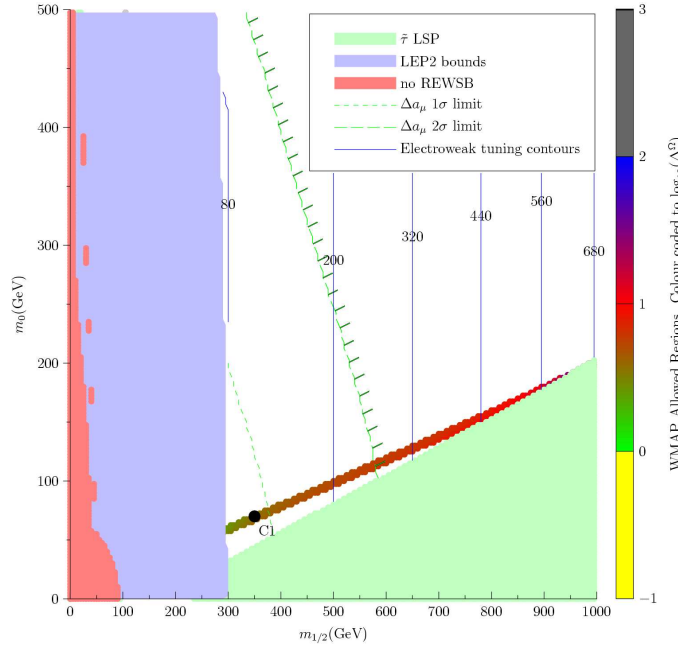


Fig. 1. The $(m_{1/2}, m_0)$ plane for the CMSSM with $A_0 = 0$, $\tan\beta = 10$.

In Fig. 1 we consider the $(m_0, m_{1/2})$ plane of the CMSSM parameter space with $\tan\beta = 10$, $A_0 = 0$. Across this parameter space $\tilde{\chi}_1^0$ is bino. This generally results in $\Omega_{\text{CDM}} h^2 \gg \Omega_{\text{CDM}}^{\text{WMAP}} h^2$. However, at low m_0 the $\tilde{\tau}$

becomes light. In the light (green) region $m_{\tilde{\tau}} < m_{\tilde{\chi}_1^0}$ and the region is ruled out as this would result in a charged LSP. Along the edge of this region $m_{\tilde{\tau}} \approx m_{\tilde{\chi}_1^0}$. This means that at the time of freeze out there would have been a large number density of $\tilde{\tau}$ alongside the $\tilde{\chi}_1^0$, allowing many more annihilation channels than are open for neutralinos alone. This results in a significant decrease of the neutralino relic density. In the multicoloured strip that lies alongside the $\tilde{\tau}$ LSP region, this coannihilation process results in $\Omega_{\text{CDM}} h^2 = \Omega_{\text{CDM}}^{\text{WMAP}} h^2$. The varying colours of this strip represent the value of Δ^Ω , defined by the colour legend on the right.

Coannihilation occurs when the LSP and NLSP are close in mass. As a result, the efficiency of coannihilation processes is highly sensitive to the mass difference $\delta m = m_{\text{NLSP}} - m_{\text{LSP}}$. If these masses are determined by separate parameters we would expect that a large degree of tuning would be required to fit the observed dark matter density. In the coannihilation strip of Fig. 1 the NLSP is the stau. The stau mass is set at the GUT scale by m_0 and the neutralino mass is set by $m_{1/2}$. As these are independent parameters, we would expect the $\tilde{\tau} - \tilde{\chi}_1^0$ coannihilation region to exhibit considerable fine-tuning.

The colour coding of the coannihilation strip shows a tuning of 3–15, considerably lower than would be expected if $m_{\tilde{\tau}}$ and $m_{\tilde{\chi}_1^0}$ were unrelated. The smallness of the tuning comes from the fact that along the coannihilation strip $m_0 < m_{1/2}$. The running of the right handed slepton masses are strongly dependent on M_1 . When m_0 is small, the dominant contribution to the low energy $\tilde{\tau}$ mass is via this running contribution from M_1 . Thus in this region of the CMSSM $m_{\tilde{\tau}}$ depends strongly on $m_{1/2}$, resulting in a correlation of the masses of the neutralino and the stau at low energy. It is this correlation of the masses that results in the low tuning observed.

Though we do not show it here, we have also investigated the other regions of the CMSSM that fit $\Omega_{\text{CDM}}^{\text{WMAP}} h^2$. For large m_0 μ becomes small and we have a bino/higgsino LSP. We find that such regions exhibit a tuning $\Delta^\Omega = 30\text{--}60$, less natural than the coannihilation strip. For large $\tan\beta$ we can also access a region in which $m_A \approx 2m_{\tilde{\chi}_1^0}$. This allows for neutralino annihilation via the production of an on-shell pseudoscalar Higgs boson. We find such an annihilation channel to require a tuning $\Delta^\Omega = 80\text{--}300$. Finally at large $\tan\beta$ the running of the $\tilde{\tau}$ mass is no longer dominated by M_1 . This breaks the correlation between $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\tau}}$ at low energies and results in the coannihilation strip that requires a tuning $\Delta^\Omega \approx 50$. This bears out our expectations that coannihilation should require significant tuning to achieve in normal circumstances.

3. Non-universal scalars

Our first move away from the universality of the CMSSM is to relax the universality between the generations of sfermions, setting the soft masses to be:

$$m_{\tilde{Q}}^2, m_{\tilde{L}}^2, m_{\tilde{u}}^2, m_{\tilde{d}}^2, m_{\tilde{e}}^2 = \begin{pmatrix} m_0^2 & 0 & 0 \\ 0 & m_0^2 & 0 \\ 0 & 0 & m_{0,3}^2 \end{pmatrix},$$

$$m_{H_u}^2 = m_{H_d}^2 = m_{0,3}^2,$$

$$M_\alpha = m_{1/2}.$$

This allows us to vary the 3rd family sfermion and Higgs mass squared separately from the 1st and 2nd families. This allows us to have light 1st and 2nd family sfermions without violating LEP bounds on the lightest Higgs. In such a region we will also have a normal mass hierarchy (NMH) in which the 1st family sfermions are the lightest and the 3rd family sfermions are the heaviest, in contrast to the inverted mass hierarchy found in the case of universal soft scalar masses.

In Fig. 2 we display the $(m_0, m_{1/2})$ plane for $m_{0,3} = 1$ TeV, $A_0 = 0$, $\tan\beta = 10$. By increasing the soft mass of the Higgs bosons to 1 TeV the LEP bound has moved down to 200 GeV. As before we have a coannihilation strip but as $m_{\tilde{\tau}} > m_{\tilde{e}_R, \tilde{\mu}_R}$, the coannihilation here is with selectrons and smuons rather than the stau.

As before the coannihilation strip exhibits a tuning $\Delta^\Omega \approx 10$ across much of its length. This drops to $\Delta^\Omega \approx 2$ for $m_{1/2} < 260$ GeV. This decrease has two causes. Firstly, for $m_{1/2} < 370$ GeV we can access $m_0 = 0$. For $m_0 \approx 0$, the mass of \tilde{e}_R and $\tilde{\mu}_R$ are almost entirely determined by the running effects from M_1 resulting in a strong correlation between $m_{\tilde{e}_R, \tilde{\mu}_R}$ and $m_{\tilde{\chi}_1^0}$. This decreases the tuning required to provide coannihilation. Secondly, as we move to low m_0 and $m_{1/2}$, we decrease the mass of the sleptons themselves, enhancing neutralino annihilation via t -channel slepton exchange. Indeed at point S2 t -channel slepton exchange accounts for 60% of the annihilation. The cross-section for t -channel slepton exchange varies slowly with the mass of the exchanged slepton and is relatively insensitive to other parameters. Therefore it requires little or no tuning to achieve. By maximising annihilation via t -channel slepton exchange we minimise the required tuning.

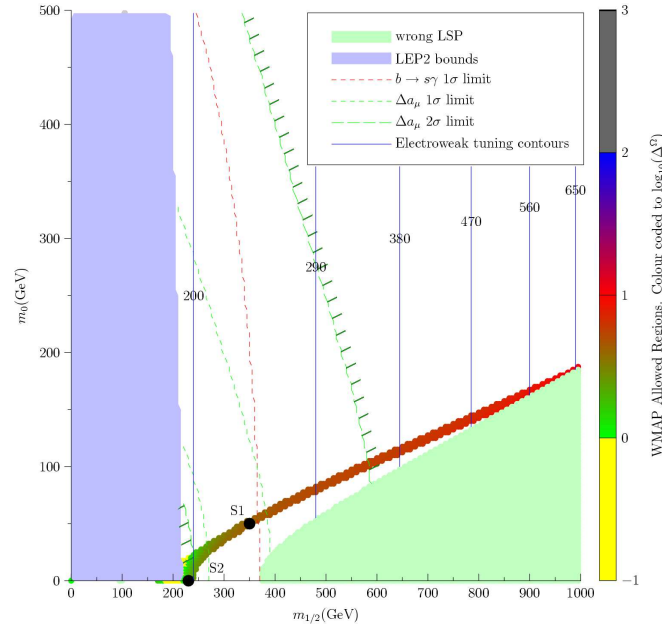


Fig. 2. The $(m_{1/2}, m_0)$ plane for non-universal sfermion masses with $m_{0,3} = 1$ TeV, $A_0 = 0$, $\tan\beta = 10$.

4. Non-universal gauginos

We now relax the constraint of universal gaugino masses. By allowing M_1 , M_2 and M_3 to vary independently we can control the bino/wino mixture of $\tilde{\chi}_1^0$. M_3 also has a strong effect on the running of the Higgs masses so by keeping M_3 large we can avoid the LEP bound on the lightest Higgs.

In Fig. 3 we show the (m_0, M_1) plane for $M_2 = M_3 = 350$ GeV, $A_0 = 0$ and $\tan\beta = 10$. In contrast to previous figures there are a large number of strips that agree with $\Omega_{\text{CDM}}^{\text{WMAP}} h^2$. The strip running from G2 to G3 is the stau coannihilation strip we have seen before. The tuning of this strip agrees with our previous findings in the case of the CMSSM. The strip that runs from G3 through G4 corresponds to a well-tempered bino/wino neutralino. As this strip exhibits a tuning of order 30 we once again find that such “well-tempered” regions are less natural than coannihilation strips. At low M_1 there are two broken vertical lines. These correspond to neutralino annihilation via the production of an on-shell Z or h^0 . The h^0 resonance stretches to $m_0 > 500$ GeV but is too thin for this plot to resolve. This channel requires tunings $\Delta^\Omega = 10$ –1000 and so cannot be considered natural. Finally, the yellow region that incorporates the point G1 represents annihilation solely via t -channel slepton exchange. This region requires a tuning $\Delta^\Omega < 1$ and represents supernatural dark matter.

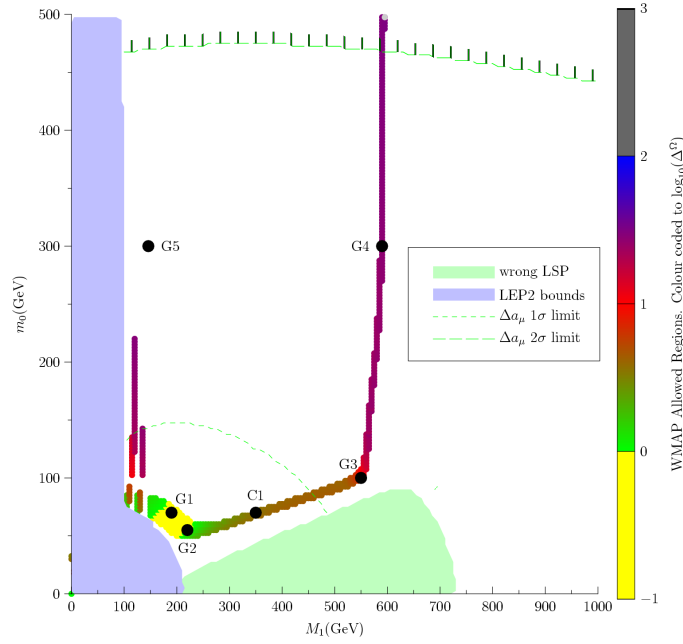


Fig. 3. The (M_1, m_0) plane for non-universal gauginos with $M_2 = M_3 = 350$ GeV, $A_0 = 0$, $\tan\beta = 10$.

5. Non-universal scalars and gauginos

Finally we relax both the universality of the gaugino masses and the universality between sfermion generations at the same time. This allows us to test the robustness of our findings in each case against further non-universality. It also allows us to study a region in which $M_2(m_Z) \approx \mu$ and the lightest neutralino is a “maximally-tempered” bino/wino/higgsino.

In Fig. 4 we show the (m_0, M_1) plane with $M_2 = M_3 = 350$ GeV, $m_{0,3} = 2250$ GeV, $A_0 = 0$ and $\tan\beta = 10$. At low M_1 we find the h^0 and Z resonances as before. The new feature is the line at $M_1 = 400$ GeV that incorporates the point SG3. Throughout this region $\tilde{\chi}_1^0$ is a mix of bino, wino and higgsino. At point SG3 the wino and higgsino components are roughly equal resulting in a maximally-tempered neutralino. This region exhibits a low tuning $\Delta^\Omega = 4$, considerably below that required for either bino/wino or bino/higgsino neutralinos.

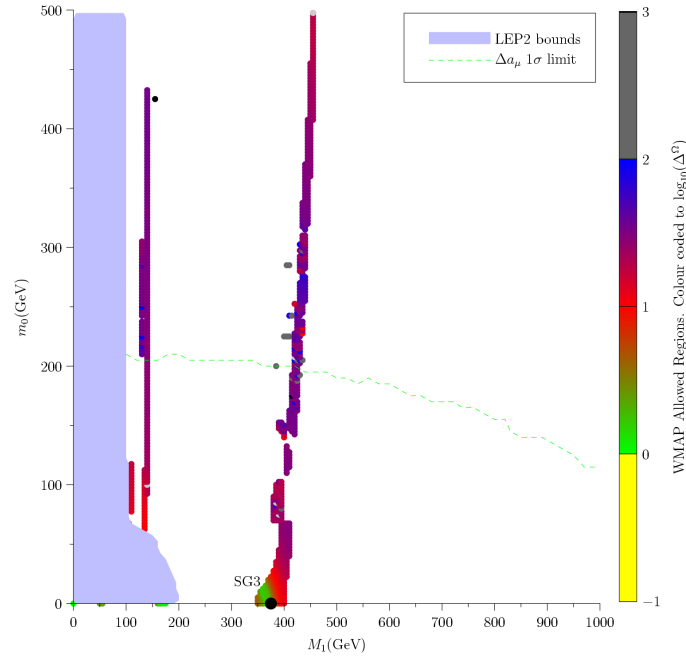


Fig. 4. The (M_1, m_0) plane for non-universal gaugino and sfermion masses with $M_2 = M_3 = 350$ GeV, $m_{0,3} = 2250$ GeV, $A_0 = 0$, $\tan\beta = 10$.

6. Conclusions

We have studied the naturalness of the annihilation channels that allow MSSM neutralinos to account for the observed dark matter density. Within the four different sets of GUT scale boundary conditions considered, these annihilation channels each display characteristic degrees of fine-tuning. The largest tunings ($\Delta^{\mathcal{O}}$ up to 1000) appear for annihilation via on-shell production of Higgs bosons. Moderate tunings $\mathcal{O}(30\text{--}60)$ are required for “well-tempered” neutralinos or slepton coannihilation with uncorrelated masses. The most natural annihilation channel is annihilation via t -channel slepton exchange. We have also found that certain RGE effects can result in surprising drops in the tuning for different channels. This is clearest in the case of slepton coannihilation for low m_0 and $\tan\beta$. In this case the mass of both particles is dominated by M_1 and the low energy masses are correlated. This results in almost natural coannihilation, which refutes the conclusions of [3].

These results have recently been extended to the case of a type I string inspired model [4]. In such a model the input parameters differ from those of the MSSM and the characteristic tunings of different annihilation channels can vary.

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