LIMITING FRAGMENTATION IN HADRONIC COLLISIONS*

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Limiting fragmentation in hadronic collisions is analyzed in the framework of the $k_\perp$ factorization and the nonlinear Balitsky–Kovchegov equation. Reasonable agreement with the experimental data is obtained. It is concluded that in order to obtain the limiting fragmentation, the factorization in the parton distributions in the target and the projectile is necessary as well as the independence of the parton distributions in the target of the scales in the process.

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In this talk I discuss the phenomenon of limiting fragmentation. The hypothesis of this phenomenon was suggested long time ago [1]. The main statements are:

- For very high energy particle collisions, in the frame where a target or a projectile is at rest, some of the outgoing particles approach limiting distributions.
- These distributions represent the broken-up fragments of the target. The fragments of the projectile move with the increasing velocity as the energy increases (in the lab frame) and do not contribute to the limiting fragmentation in this frame. To study these fragments it is necessary to change a frame to one in which the projectile is at rest.
- In the laboratory frame, the projectile is a highly contracted thin system which passes through the target causing excitation and a possible decay of the target particle.

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- In order to have a limiting distribution one has to assume that the cross section is approximately constant. More precisely, one has to assume that the probability of the interaction does not change rapidly as the energy is further increased.

The limiting fragmentation is confirmed experimentally in a wide range of processes: proton–proton collisions, proton–nucleus and nucleus-nucleus collisions [2–5]. Recently, BRAHMS and PHOBOS experiments at Brookhaven National Laboratory performed precise measurements of the multiplicity distributions in a wide range of the pseudorapidities. The limiting fragmentation in that case means that, the pseudorapidity distribution of the produced particles

$$\frac{dN}{d\eta}\,,$$

when shifted to a laboratory frame of one of the colliding particles $\eta' \equiv \eta - Y_{\rm beam}$ ($Y_{\rm beam} = \ln\sqrt{s}/m$, m is a mass of the particle) is independent of the centre-of-mass energy $\sqrt{s}$ *i.e.*

$$\frac{dN}{d\eta'}(\eta',\sqrt{s},b) = \frac{dN}{d\eta'}(\eta',b)\,,$$

around $\eta' \simeq 0$ and where b is an impact parameter of the collision.

In this talk, I present the calculation of the rapidity distributions using a formalism of the $k_\perp$ factorization of the unintegrated parton distributions and the nonlinear evolution equation to calculate the latter. The $k_\perp$ factorization formula for the single inclusive gluon production reads

$$\frac{dN_{\rm g}}{dyd^2p_\perp} = \frac{\alpha_s S_{AB}}{2\pi^4 C_F S_A S_B}\,\frac{1}{p_\perp^2}\int \frac{d^2k_\perp}{(2\pi)^2}\,\phi_A(x_1,k_\perp)\,\phi_B(x_2,|p_\perp - k_\perp|)\,, \tag{1}$$

where $S_{A,B}$ is a total transverse area for hadrons (nuclei) A and B, S_{AB} is a transverse area for an overlap region, $p_\perp$ is the transverse momentum of the produced gluon and $x_{1,2}$ are longitudinal momentum fractions of the gluons probed in the target and the projectile, respectively. They are defined as

$$x_1 = \frac{p_\perp}{m}e^{y-Y_{\rm beam}}\,, \qquad x_2 = \frac{p_\perp}{m}e^{-y-Y_{\rm beam}}\,.$$

Functions $\phi(x,k_\perp)$ are the unintegrated gluon distributions in the target and the projectile which depend on the transverse momentum. These objects are related to the standard gluon distribution as follows[1]

$$xg\left(x,Q^2\right) = \int^{Q^2} dk_\perp^2\,\phi(x,k_\perp)\,.$$

[1] As discussed in [7] the precise definition of the unintegrated gluon distribution may actually differ from the one used in this paper.

We calculate the unintegrated distribution using the nonlinear Balitsky–Kovchegov equation (BK) [8] for the dipole–hadron scattering amplitude T. The equation in the momentum space takes the following form

$$\frac{\partial\tilde{T}(k_\perp,Y)}{\partial Y} = \bar{\alpha}_s(K\otimes\tilde{T})(k_\perp,Y) - \bar{\alpha}_s\,\tilde{T}^2(k_\perp,Y)\,,$$

with $\bar{\alpha}_s = \alpha_s N_c/\pi$ and the amplitude $\tilde{T}$ is related to the unintegrated gluon density

$$\phi(x,k_\perp) = \frac{\pi S_A N_c k_\perp^2}{2\alpha_s}\int\limits_0^\infty r_\perp dr_\perp J_0(k_\perp r_\perp)[1-T(r_\perp, Y=\ln 1/x)]^2\,,$$

together with the Fourier–Bessel transform

$$\tilde{T}(k_\perp,Y) = \int\limits_0^\infty \frac{dr_\perp}{r_\perp}J_0(k_\perp r_\perp)T(r_\perp,Y)\,.$$

The solution to the BK equation is shown in Fig. 1 as a function of the transverse momentum squared for different values of the rapidity.

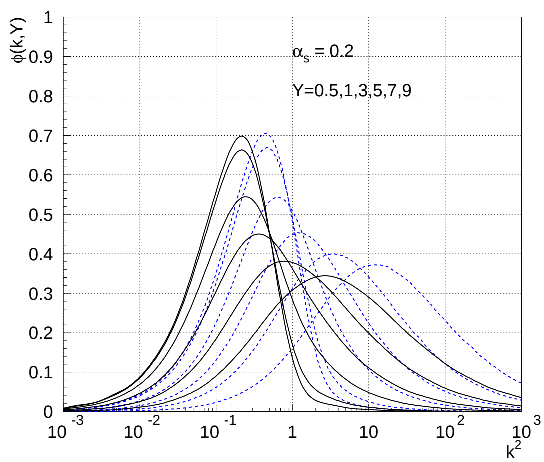

Fig. 1. Solution to the BK equation in the form of the unintegrated distribution $\phi(x,k_\perp)$ for different values of the rapidity.

The peak of the distribution is positioned at the saturation scale $Q_s(Y = \ln 1/x)$ and the distribution shifts to the higher values of transverse momenta as the typical x becomes smaller. The distribution $\phi(x,k_\perp)$ satisfies the following constraint

$$\int \frac{d^2k_\perp}{k_\perp}\,\phi(x,k_\perp) = \text{const.}$$

In order to obtain the solution to the BK equation we need to specify the initial conditions at initial value of the rapidity $Y_0 = \ln 1/x_0$. We choose the McLerran–Venugopalan (MV) [9] and Golec-Biernat–Wuesthoff (GBW) [10] as our models with fixed initial value for the saturation scale $Q_{\rm s}(x)$. The BK evolution should be correct at small values of x where the gluon density is expected to be very high. At large x we used a phenomenological extrapolation

$$\phi(x,k_\perp) = \left(\frac{1-x}{1-x_0}\right)^\beta \left(\frac{x_0}{x}\right)^{\lambda_0} \phi(x_0,k_\perp)\,,$$

with $\phi(x_0,k_\perp)$ being the initial condition. We varied the parameters β in the range 4–5 and λ_0 in the range 0.0–0.1, respectively.

How the limiting fragmentation can be understood within this framework? In the situation when $x_1 \sim x_2$, that is in the mid-rapidity regime the typical scales of two distributions are also comparable $Q_{\rm s}^A(x_1) \sim Q_{\rm s}^B(x_2)$, and there is an entanglement in the transverse momenta. In the fragmentation region of one of the particles we have asymmetric configuration $x_1 \gg x_2$ and in this case $Q_{\rm s}^A(x_1) \ll Q_{\rm s}^B(x_2)$. The two distributions $\phi_A(x_1)$ and $\phi_B(x_2)$ are separated. Therefore, we can approximate the gluon production formula with

$$\frac{dN}{dy} \sim \int \frac{d^2p_\perp}{p_\perp^2} \int d^2k_\perp\,\phi_A(x_1,k_\perp)\phi_B(x_2,p_\perp)\,,$$

consequently the two integrals become factorized. The integral over the projectile density gives an overall normalization $\int d^2p_\perp/p_\perp^2\,\phi_B(x_2,p_\perp) = \text{const.}$, on the other hand the integral over the target density gives the integrated parton distribution

$$\int^{Q_{\rm s}(x_2)} d^2k_\perp \phi_A(x_1,|k_\perp|) = x_1\, g(x_1,Q_{\rm s}(x_2))\,. \tag{2}$$

This distribution is evaluated at large values of x_1 where we know the Bjorken scaling holds

$$x_1 g(x_1,Q_{\rm s}(x_2)) = x_1 g(x_1)\,. \tag{3}$$

From this we can conclude that

$$\frac{dN}{dY} \simeq N x_1 g(x_1) = F(y - Y_{\rm beam}), \qquad x_1 \gg x_2$$

the rapidity distribution of the produced partons scales with $Y - Y_{\rm beam}$ (recall that $x_1 \sim \exp(Y - Y_{\rm beam})$).

For comparison with the data one needs to model the parton distribution $\phi_A(x_1, k_\perp)$ at large values of x_1. Since the distribution $x_1 g(x_1)$ should obey the x_1 scaling:

$$x_1 g(x_1) = x_1 g(x_1, Q_{\rm s}^2(x_2)) = \int^{Q_{\rm s}^2} dk^2 \phi_A(x_1, k_\perp)\,, \tag{4}$$

the distribution ϕ_A must be very peaked at low $k_\perp$ and have a sharp fall off at large values of $k_\perp$. Since there is practically no information about the parton distribution at low $k_\perp$ and large x_1 the distribution, ϕ_A is the source of the largest uncertainty when comparing with the data.

To compare with the experimental data one needs to calculate the pseudorapidity distributions $dN/d\eta$ whereas the $k_\perp$ factorized expression (1) gives rapidity distribution dN/dy. In order to change from the pseudorapidity to the rapidity we need to assume some value for the mass of the produced particles which we choose to be $m \simeq 150$–$300\,$MeV. In addition we need to introduce an infrared regulator for the integration over $p_\perp$. We choose it to be the same value of mass m used in the Jacobian transformation from η to y.

In Fig. 2 we present the pseudorapidity distributions of the charged particles which are produced in nucleon–nucleon collisions at UA5 at energies $\sqrt{s} = 53, 200, 546, 900\,$GeV and at PHOBOS with energy $\sqrt{s} = 200\,$GeV. The distributions are presented as a function of the shifted pseudorapidity variable $\eta - Y_{\rm beam}$. Apart from the parameter λ_0 we also have the parameter $\lambda_{\rm s} \simeq 4.88\bar{\alpha}_{\rm s}$ which controls the growth of the gluon density in the BK equation. By changing $\alpha_{\rm s}$ we effectively change the intercept of the hard Pomeron. Since the BK equation was derived within the LLx approximation by reducing the value of the $\alpha_{\rm s}$ we effectively take into account subleading corrections. Plots are shown for different values of parameters $\lambda_0, \lambda_{\rm s}$ as well as different models for the input distribution at large x. We clearly see that the GBW model is favored over the MV model, which can be easily understood since the ϕ_A distribution from the MV model has a large $k_\perp$ tail which should not be present at large values of x. This means that effectively the MV model does not satisfy condition (4) at large x required to get the observed limiting fragmentation. The models describe the data in the fragmentation region quite well, but there is a discrepancy at the mid-rapidity. This can be due to the fact that the formalism of the $k_\perp$ factorisation is not applicable at this region, and other contributions which violate the factorization should become important at the mid-rapidity. In Fig. 3 we show the comparison of the calculation with the nucleus–nucleus data from PHOBOS. We observe a good agreement of the calculations in the fragmentation region and a discrepancy at the mid-rapidity as already observed in the proton–proton case.

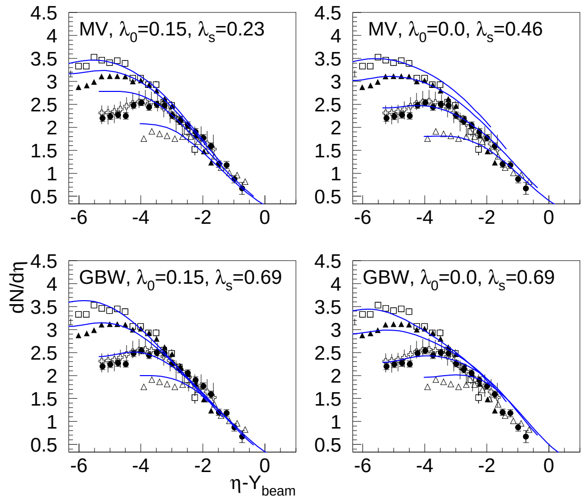

Fig. 2. Pseudorapidity distributions $dN/d\eta$ for charged particles from nucleon-nucleon collisions at energies $\sqrt{s} = 53, 200, 546, 900\,\mathrm{GeV}$. Data are from UA5 [2] and PHOBOS [4]. Initial distributions for the BK equation are from the MV model [9] (upper plots) and GBW the model [10] (lower plots).

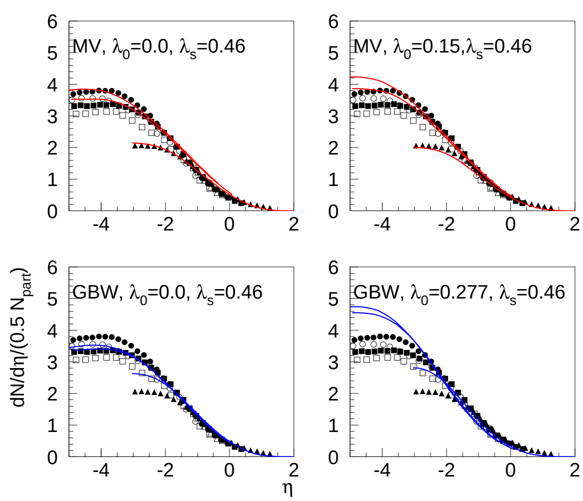

Fig. 3. Pseudorapidity distributions normalised by the number of participants for charged hadrons in gold–gold collisions. The data are from RHIC [4–6]. The c.m.s energies are $\sqrt{s} = 19.6, 62.4, 130, 200\,\mathrm{GeV}$.

We also present the extrapolations of the calculations to the LHC energies. In Fig. 4 we show the curves for the nucleon–nucleon case at $\sqrt{s} = 14\,\text{TeV}$ and in Fig. 5 we show the extrapolation to $\sqrt{s} = 5.5\,\text{TeV}$ for gold–gold collision. The large band on the plot for nucleus–nucleus collision corresponds to the theoretical uncertainty.

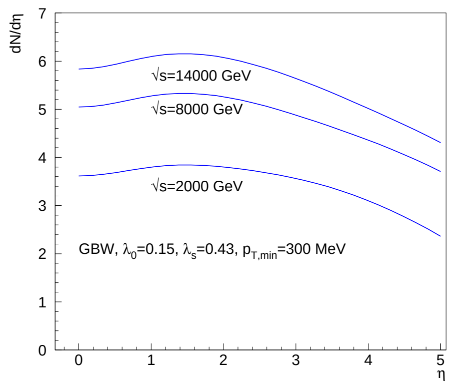

Fig. 4. Predictions for the pseudorapidity distributions in proton–proton collisions at high center-of-mass energies $\sqrt{s} = 2, 8, 14\,\text{TeV}$ for the GBW input model.

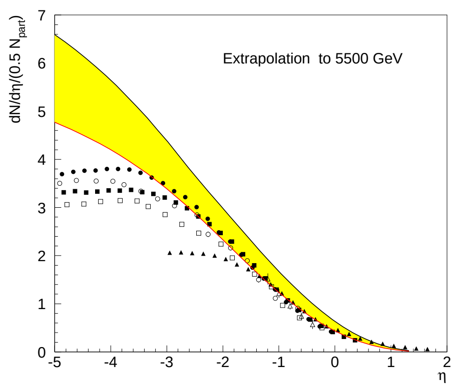

Fig. 5. Extrapolation of the calculations for gold–gold collisions to the LHC energy. The band is an estimate of the theoretical uncertainty in our calculations.

In summary, we have presented the description of the limiting fragmentation distribution using the $k_\perp$ factorization and the gluon density calculated from the nonlinear evolution equation. We conclude that the observed limiting fragmentation is the result of the factorization of the parton distributions in the target and the projectile at large rapidities. The multiplicity distribution at large rapidity seems to be directly proportional to the distributions of the parton denstiy in the target. This parton density is independent of the scales in the process and consequently of the c.m.s energy in the process. The presented analysis implies that the limiting fragmentation arises because the rapidity distribution of the produced particles is determined essentially by the form of the initial hadron states.

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