

THE PHYSICS OF BEAUTY (AND CHARM [AND τ])
AT THE LHC AND IN THE ERA OF THE LHC*

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The recent successes of the SM do not weaken the arguments in favor of New Physics residing at the TeV scale. Finding and identifying it represents the prime challenge for a generation of high energy physicists. To differentiate between different scenarios of New Physics we need to analyze their impact on flavor dynamics. A continuing comprehensive program of heavy flavor studies instrumentalizing the high sensitivity of CP analyses is intrinsically connected to LHC's core mission. In B decays we can typically expect no more than moderate deviations from SM predictions. B_s transitions provide an *autonomous* access to New Physics not prejudiced by $\Delta M(B_s)|_{\text{exp}} \simeq \Delta M(B_s)|_{\text{SM}}$. Dedicated studies of charm and τ decays offer unique opportunities to observe New Physics. One challenge is whether LHCb will be able to exploit LHC's huge charm production rate to probe for CP asymmetries. Likewise, to which degree ATLAS/CMS can contribute to B physics and to searches for $\tau \rightarrow 3l$. Yet to saturate the discovery potential for New Physics in beauty, charm and τ decays we will need a comprehensive high quality data base that only a Super-Flavor Factory can provide.

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1. Introduction

Around the turn of the Millennium we have experienced a “quantum jump” in knowledge, though not in understanding:

- The Standard Model (SM) Paradigm of Large CP violation in B decays has been validated.
- ν oscillations have been established experimentally (and the solar model validated as well in the process).
- Evidence for “Dark Energy” has emerged — a concept concisely characterized by the quote: “Who ordered that?”

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Even the first item, a great, unqualified and novel success of the SM, does *not* invalidate the arguments in favor of the SM being incomplete already around the TeV scale.

This is not a surprising statement for the audience at this conference. For the central justification for the LHC is to reveal the dynamics driving the electroweak phase transition. Our foremost goal has to be to make the LHC succeed greatly, even beyond our expectations, and in the process prove Samuel Beckett wrong who said: “Ever tried? Ever failed? No matter. Try again. Fail again. Fail better.” The second and third item above tell us we will not fail forever; furthermore, a “New CP Paradigm” is needed to implement baryogenesis. I am actually confident that we will “succeed” soon.

My central message can be summarized as follows: We must study the impact of that anticipated New Physics on flavor dynamics. The LHCb program is thus *intrinsically connected* to the *core mission* of the LHC. The required comprehensive flavor studies have to include the charm and τ lepton sector. The goal here is not primarily to enlighten us about the flavor mystery, although that could come about, and not to unveil the New CP Paradigm needed for baryogenesis, though it can quite conceivably happen, but to *instrumentalize* the high sensitivity inherent in CP studies to interpret the footprints of New Physics to be revealed in high p_{\perp} studies. Dedicated and comprehensive flavour studies are a necessity, not a luxury, in the Era of the LHC, and I view a Super-Flavour Factory a most desirable component of it.

After an update on the SM’s paradigm of large CP violation in B decays in Sec. 2 I discuss the lifetimes of beauty hadrons in Sec. 3 and sketch future B studies in Sec. 4; D and τ decays are addressed in Sec. 5 before concluding with a plea for a Super-Flavour Factory; some more technical comments on Heavy Quark Theory and Dalitz plots analyses are shifted to Appendices.

2. The SM’s paradigm of large CP violation in B decays — a triple triumph

Three central consequences of CKM theory had been predicted:

1. Some B decay modes like $B_d \rightarrow \psi K_S$, $B_d \rightarrow \pi^+\pi^-$ and $B \rightarrow K^+\pi^-$ have to exhibit truly large CP asymmetries — there is no “plausible deniability”. It is very nontrivial to infer from a CP asymmetry in the $K^0-\bar{K}^0$ system measured to be on the few $\times 10^{-3}$ level that B decays should exhibit CP violation hundred times larger, *i.e.* close to the largest values mathematically possible [1].
2. Large *direct* CP violation has to occur as well [2].

3. The magnitudes of CP insensitive observables — $|V(ub)/V(cb)|$ and $\Delta M(B_d)/\Delta M(B_s)$ — control the existence and strength of CP violation, as expressed through ε_K and $\sin 2\phi_1$.

All these predictions have now been validated experimentally [3]. Since the summer of 2001 we can say: (i) The CKM paradigm has become a *tested* theory. (ii) CP violation has been *demystified*: if the dynamics is sufficiently complex to support CP violation, there is no *a priori* reason, why the latter should be small; *i.e.* weak complex phases can be large. (iii) The demystification will be completed, a good thing in my view, if CP violation is found anywhere in leptodynamics.

While there are certain regions in kaon dynamics with large CP asymmetries — the interference region in $K_{\text{neut}} \rightarrow \pi\pi$ or the T odd correlation between the $\pi^+\pi^-$ and e^+e^- planes in $K_L \rightarrow \pi^+\pi^-e^+e^-$ — the statement that “CP violation in B decays is much larger than in K decays” is an empirically verified fact: while the K_L (and K_S) act like CP eigenstates to a very good approximation, this not at all true for the mass eigenstates of the $B_d - \bar{B}_d$ system.

To summarize the 2006 status more quantitatively [3, 4]:

- From $B_d \rightarrow \psi K_S$ one obtains

$$\sin 2\phi_1|_{\text{WA}} = 0.674 \pm 0.026, \text{ versus } \sin 2\phi_1|_{\text{CKM}} = 0.725 \pm 0.065. \quad (1)$$

The “battle for supremacy” has been decided: we search no longer for *alternatives* to CKM theory, but for *corrections*. At the same time baryogenesis has to be driven by dynamics other than CKM; thus we can be confident that CKM forces do not represent a monopoly.

- Direct CP violation has been established in $B_d \rightarrow K^+\pi^-$ by both BABAR and BELLE. While the BABAR and BELLE data sets on $B_d \rightarrow \pi^+\pi^-$ do not form a perfect union (yet), the BELLE analysis shows large CP violation of the indirect as well as direct variety. A time-dependent CP asymmetry in $B_d \rightarrow 2\pi$ can be expressed as a sum of \sin and $\cos \Delta M(B_d)$ terms with coefficients S and C , respectively. Without direct CP violation one obviously has $C = 0$; yet in addition also $S = -\sin 2\phi_1 \simeq -0.7$ has to hold, where the minus sign is due to the 2π and ψK_S final states having opposite CP parity. *I.e.*, once CP violation in $B_d \rightarrow \psi K_S$ has been established, one infers the existence of *direct* CP violation from $(S, C) \neq (-\sin 2\phi_1, 0)$ rather than from $(S, C) \neq (0, 0)$.

- Recently both the D0 and CDF collaborations reported a signal for $B_s - \bar{B}_s$ oscillations [5, 6]:

$$\Delta M(B_s) = \begin{cases} (19 \pm 2) \text{ ps}^{-1} & \text{D0} \\ (17.3^{+0.42}_{-0.21} \pm 0.07) \text{ ps}^{-1} & \text{CDF} \\ (18.3^{+6.5}_{-1.5}) \text{ ps}^{-1} & \text{CKM fit} \end{cases} \quad (2)$$

While the strength of the signal has not yet achieved 5σ significance, it looks most intriguing. If true, it represents another impressive triumph of CKM theory: the CP *insensitive* observables $|V(ub)/V(cb)|$ and $\Delta M(B_d)/\Delta M(B_s)$, *i.e.* observables that do *not* require CP violation for acquiring a non-zero value, imply (a) a non-flat CKM triangle and thus CP violation, see of Fig. 1 (left), that (b) is fully consistent with the observed CP sensitive observables ε_K and $\sin 2\phi_1$, see Fig. 1 (right).

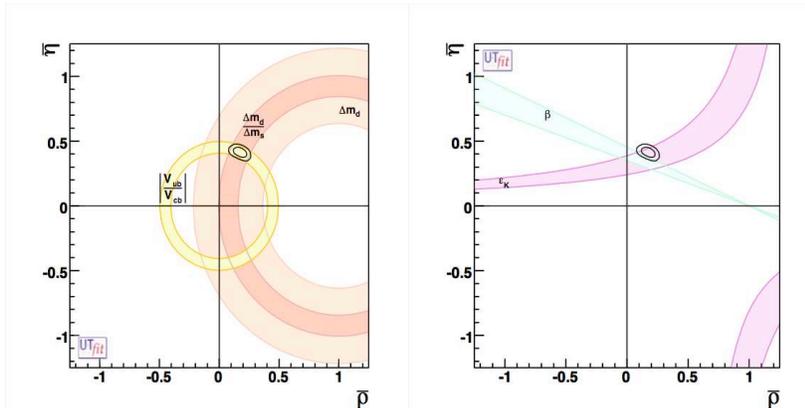


Fig. 1. Unitarity triangle from $|V(ub)/V(cb)|$ & $\Delta M(B_d)/\Delta M(B_s)$ on the left and compared to constraints from ε_K & $\sin 2\phi_1/\beta$ on the right (courtesy of M. Pierini).

These successes of the SM tell us that we *cannot count* on *numerically* massive manifestations of new dynamics in beauty transitions. Accordingly we must strive to achieve as high an accuracy level in our theoretical description as possible. The goal of high accuracy is not utopian — it can be achieved by combining a *robust* theoretical framework with *comprehensive* data as illustrated in the Appendices.

3. Lifetimes of beauty hadrons

Based on the Heavy Quark Expansion (HQE), which is sketched in Appendix A, the lifetime ratios of beauty hadrons have been predicted in the old-fashioned sense, *i.e.* *before* meaningful measurements had been undertaken.

TABLE I

Weak lifetime ratios of beauty hadrons.

	$1/m_b$ expected	data
$\frac{\tau(B^+)}{\tau(B_d)}$	$\sim 1 + 0.05 \left(\frac{f_B}{200 \text{ MeV}}\right)^2$ '92 [19] 1.06 \pm 0.02 [20]	1.076 \pm 0.008 [21]
$\frac{\bar{\tau}(B_s)}{\tau(B_d)}$	1 \pm $\mathcal{O}(0.01)$ '94 [22]	0.958 \pm 0.039 [21]
$\frac{\tau(\Lambda_b^-)}{\tau(B_d)}$	≥ 0.9 '93 [23] $\simeq 0.94$ & ≥ 0.88 '96 [24, 25]	0.806 \pm 0.047 WA '05 [21] 1.037 \pm 0.058 CDF '06 [26] 0.870 \pm 0.102 \pm 0.041 D0 '06 [27]
$\tau(B_c)$	$\sim (0.3 - 0.7)$ psec '94ff [29]	0.45 \pm 0.12 psec [21]
$\frac{\Delta\Gamma(B_s)}{\bar{\Gamma}(B_s)}$	0.22% \times $\left(\frac{f(B_s)}{220 \text{ MeV}}\right)^2$ '87 [31] 0.12 \pm 0.05% '04 [20]	0.65 \pm 0.3 CDF 0.26 \pm 0.14 [28]

I list in Table I the original predictions together with later updates and the data. Several comments are in order: (i) The prediction on $\tau(B^+)/\tau(B_d)$ is in pleasing agreement with rather accurate data. (ii) The largest deviation from uniform lifetimes occurs for B_c mesons: their lifetime is that of charm hadrons as expected in a naive additive quark model picture and predicted by the HQE, where the absence of a $1/m_Q$ correction is essential. (iii) The long saga on the Λ_b lifetime appears to have taken a surprising turn. The authors of the original prediction and its refinement “stuck to their guns” about $\tau(B_d)$ exceeding $\tau(\Lambda_b)$ by not significantly more than 10 %, when for several years the data seemed to clearly indicate otherwise: as late as 2005 the world average still read $\tau(\Lambda_b)/\tau(B_d) = 0.806 \pm 0.047$. During that time other theorists gave different predictions [32]. I am eagerly awaiting future data, in particular also on $\tau(\Xi_b^{0,-})$ [33]. (iv) The prediction that the average lifetime of the B_s mass eigenstates should differ from $\tau(B_d)$ by merely a percent or two is a carefully analyzed, yet not an iron-clad one. Previous data indicated a somewhat lower value, which would also have been more consistent with the first results on $\Delta\Gamma(B_s)$, see below. Yet it appears data are moving closer to the original theoretical prediction. (v) The measured values for $\Gamma(B_s \rightarrow D_s^{(*)}\bar{D}_s^{(*)})$ can give a reasonable ballpark estimate for $\Delta\Gamma(B_s)$; yet a real prediction is best obtained by evaluating the “quark-box diagram” [20, 31]. Two qualifying remarks are important though: (α) A ratio of 0.25 is almost a “unitarity” bound on $\Delta\Gamma(B_s)/\Gamma(B_s)$, unless $\bar{\Gamma}(B_s)$ is significantly larger than $\Gamma(B_d)$ contrary to expectations, see

above. (β) While the quark box diagrams used for evaluating ΔM as well as $\Delta\Gamma$ look very similar, the dynamical situation is quite different in the two cases. ΔM is controlled by off-shell transitions and thus involves a considerable amount of averaging over hadronic channels making duality a good approximation [18]. On the other hand $\Delta\Gamma$ is given by on-shell transitions with less averaging, which could enhance the limitations to duality considerably. Furthermore, $\Delta\Gamma$ as obtained from the quark box diagram is considerably reduced by GIM cancellations; however, those could be modified very significantly for the on-shell modes due to the proximity of the $D_s^{(*)}\bar{D}_s^{(*)}$ thresholds. I am *not* suggesting that employing the quark box diagram for evaluating $\Delta\Gamma(B)$ is unreasonable. I am concerned about the following: while in the ratio $\Delta M(B_s)/\Delta\Gamma(B_s)$ some uncertainties like bag factors and decay constants cancel, $\Delta\Gamma(B_s)$ might suffer from further theoretical uncertainties, which could significantly bias the prediction for it as well as for $\Delta M(B_s)/\Delta\Gamma(B_s)$.

The HQE has been successful even on a quantitative level in predicting the lifetimes of beauty hadrons, which after all are dominated by non-leptonic transitions. The basic feature that nonperturbative corrections arise first in order $1/m_b^2$ holds for both semi-leptonic and non-leptonic widths [17], yet in the latter there are more perturbative QCD corrections, and limitations to quark-hadron duality are likely to be somewhat larger on average (and possibly significantly larger in some cases).

4. On future lessons in B decays

Since we *cannot count on quantitatively massive* manifestations of New Physics in B decays, we must combine high accuracy with high sensitivity in heavy flavour studies. Some tools for attaining such a goal are briefly addressed in Appendix A.

4.1. Rare B decays

$\Gamma(B \rightarrow l\nu X_c)$: the *inclusive* semi-leptonic width has been calculated with about 3% theoretical uncertainty for $l = e, \mu$ [34]. The only evaluation for $l = \tau$ has been given twelve years ago [35], and the only measurement is equally old. Now we have the tools to compute $\Gamma(B \rightarrow \tau\nu X_c)/\Gamma(B \rightarrow e\nu X_c)$ much more precisely. Measuring it with commensurate precision allows a search for New Physics, in particular in the form of an extended Higgs sector, since charged Higgs exchange would affect $B \rightarrow \tau\nu X_c$ most significantly.

A variant of such a probe is to compare the *exclusive* rates for $B \rightarrow \tau\nu D$ versus $B \rightarrow e\nu D$ [36], since the former unlike the latter could be affected by a charged Higgs as heavy as several hundred GeV. There is one complication,

though: contrary to claims in the literature hadronization effects do *not* drop out from the ratio. Yet this problem can be overcome through Uraltsev's "BPS" approximation [37], as explained in Appendix A.

$B \rightarrow \bar{l}lX$: While the *inclusive* transitions can be measured only at a B factory, the *exclusive* channels $B \rightarrow l^+l^-K/K^*$ — rates, lepton spectra, CP and forward-backward asymmetries — can be studied at the LHC, in particular by LHCb with its superb particle identification.

Urging to measure $\Gamma(B \rightarrow \nu\bar{\nu}X_s)$, even exclusively, is *not* the result of the frivolous nature of theorists [38]. For the dynamical information to be gained there is in general quite independent from that in $B \rightarrow l^+l^-X_s$. Alas, it is in the domain of a Super- B factory.

4.2. Flavour-changing neutral currents in B_s decays

— an independent chapter in nature's book on fundamental dynamics

The B factories have been much more successful than anticipated in the quality of their measurements. Among many other achievements they have determined the three angles of the CKM triangle with higher accuracy than expected from $B_{d,u}$ decays. We have to focus now on finding and subsequently identifying non-CKM corrections.

Originally it was thought that B_s decays are needed in an essential way to construct the CKM triangle, namely to extract the angle ϕ_3 and the side $|V(td)/V(ts)|$. For the latter this is true, as mentioned above. Yet the angle ϕ_3 is being determined with good accuracy in $B^\pm \rightarrow D^{\text{neut}}K^\pm$. I view the statement that we will extract ϕ_3 from B_s as somewhat missing the point. Our primary goal is to search for New Physics. While within the SM similar quark box diagrams affect rare transitions and oscillations for B_d and B_s decays, we have to "think outside the box" — pun intended. For *a priori* there is no reason why New Physics should affect B_s and B_d transitions with similar weights as those within the SM. B_s channels should, therefore, be analyzed in an *autonomous* way.

$\Delta M(B_s)$ and $\Delta\Gamma(B_s)$ have been addressed already. I will give four examples where New Physics can still impact on B_s decays in a numerically massive way:

- The rate for $B_s \rightarrow \mu^+\mu^-$ with $\text{BR}(B_s \rightarrow \mu^+\mu^-)|_{\text{SM}} \sim 3 \times 10^{-9}$ can be greatly enhanced in some SUSY scenarios by $(\text{tg}\beta)^6$ [39], which could produce a rate right at the experimental upper bound of 10^{-7} .
- The time-dependent CP asymmetries in $B_s \rightarrow \psi\phi/\psi\eta^{(\prime)}$ are reliably predicted to be small in the SM [1], namely below 4 %. For on the leading Cabibbo level only quarks of the second and third family contribute, and by themselves they cannot induce CP violation. Yet New Physics could produce a CP asymmetry as large as several $\times 10$ %, even with the observed value of $\Delta M(B_s)$ close to the SM prediction.

- With oscillations leading to “wrong-sign” leptons, $\bar{B}_s \rightarrow l^+ \nu X$ and $B_s \rightarrow l^- \nu X$ one can probe for a CP asymmetry there. Within the SM it has to be tiny $\sim \mathcal{O}(10^{-5})$, since it is suppressed by $\Delta\Gamma/\Delta M$ and by the leading contributions again coming from quarks of only the second and third family. Yet the second suppression factor could be vitiated by New Physics leading to a semi-leptonic CP asymmetry two orders of magnitude larger.
- The mode $B_s \rightarrow \phi\phi$ is the analogue of $B_d \rightarrow \phi K_S$: within the SM its CP asymmetry has to basically coincide with that of $B_s \rightarrow \psi\phi$, *i.e.* be very small; yet since it is driven by a one-loop process, *i.e.* with a suppressed SM amplitude, it is quite susceptible to New Physics. Ultimately it offers one intrinsic advantage over $B_d \rightarrow \phi K_S$: once one has differentiated the contributions from $l = 0, 1, 2$ partial waves, one can analyze in which partial wave a possible direct CP asymmetry arises. LHCb will be particularly well suited for this task.

4.3. On the capabilities of hadronic collider experiments

While CMS, ATLAS and LHCb should be able to search for $B_s \rightarrow \mu^+ \mu^-$, it is not clear, if even LHCb can probe for $B_s \rightarrow \tau^+ \tau^-$, despite the latter’s branching ratio being larger by two orders of magnitude.

The relatively low value reported by CDF/D0 for $\Delta M(B_s)$ should allow also ATLAS and CMS to track B_s oscillations. Whether this will yield enough sensitivity to hunt (time-dependent) CP asymmetries in $B_s \rightarrow \psi\phi/\eta$ or in $B_s \rightarrow D_s K$ with the latter [former] probing for New Physics in $\Delta B = 1\&2$ [$\Delta B = 2$] dynamics, will depend on the quality of the flavour tagging and particle identification; likewise for $\bar{B}_s \rightarrow l^+ X$ versus $B_s \rightarrow l^- X$. To be able to study rates, lepton spectra and asymmetries in $B \rightarrow l^+ l^- K/K^*/\pi/\rho$ and $B_s \rightarrow l^+ l^- \eta/\phi, K^*$ will require efficient triggers, good flavour tagging and particle identification. I expect LHCb to be well up to the task.

5. The dark horses — charm quarks and τ leptons

B decays (and similarly for kaons) with their large CKM suppression are a most natural place to search for New Physics. Yet we have to search in unconventional places as well.

5.1. On the future promise of charm

Accurate measurements of leptonic as well as semi-leptonic charm decays will teach us novel lessons about nonperturbative QCD, calibrate and hopefully validate our theoretical tools that then can be employed with more confidence in B studies. This is the foundation of the CLEO-c program. Yet

there is much more beyond this “guaranteed profit”: New Physics could induce flavour changing neutral currents that are considerably less suppressed for up- than for down-type quarks. Only charm allows the full range of probes for New Physics in general and flavour-changing neutral currents in particular: (i) Since top quarks do not hadronize [10], there can be no $T^0 - \bar{T}^0$ oscillations. More generally, hadronization, while hard to bring under theoretical control, enhances the observability of CP violation [40]. (ii) As far as u quarks are concerned, π^0 , η and η' decays electromagnetically, not weakly. They are their own antiparticles and thus cannot oscillate. CP asymmetries are mostly ruled out by CPT invariance.

My basic contention: *Charm transitions provide a unique portal for finding the intervention of New Physics in flavour dynamics with the experimental situation being a priori quite favorable apart from the absence of Cabibbo suppression. Yet even that handicap can be overcome by statistics.*

I am quite skeptical that the observation of $D^0 - \bar{D}^0$ oscillations by themselves can establish the intervention of New Physics, since the SM predictions for $x_D = \Delta M_D/\Gamma_D$ and $y_D = \Delta\Gamma_D/2\Gamma_D$ yield values $\sim \mathcal{O}(10^{-3})$ [41], and might allow even 10^{-2} [42], when the data read $x_D \leq 0.03$ and $y_D = 0.01 \pm 0.005$. Nevertheless one should make every effort to observe it, mainly because it can provide independent validation for a signal of CP violation involving $D^0 - \bar{D}^0$ oscillations. Such an effect would represent conclusive proof for the intervention of New Physics.

Since baryogenesis implies the existence of New Physics in CP violating dynamics, we better undertake dedicated searches for CP asymmetries in charm decays, where the “background” from known physics is between absent and small [8, 45]. Most experimental facts help a search for CP violation due to New Physics, be it of the direct or indirect variety, be it in partial widths or final state distributions. I will list just two examples:

- One can search for a time-dependent difference in the rates for the doubly Cabibbo suppressed modes $D^0 \rightarrow K^+\pi^-$ versus $\bar{D}^0 \rightarrow K^-\pi^+$ [43]. With LHCb expecting to record about 5×10^7 tagged $D^* \rightarrow D + \pi \rightarrow K^+K^- + \pi$ events in a nominal year of 10^7 s [44], one will achieve very high sensitivity for New Physics.
- In $\bar{D} \rightarrow K\bar{K}\pi^+\pi^-$ one can measure the angle ϕ between the $K\bar{K}$ and $\pi^+\pi^-$ planes and probe for a difference in the ϕ distribution for D and \bar{D} decays. Since one can measure T odd and even moments separately for D and \bar{D} , one should be able to control systematics in the detection efficiencies of particles and antiparticles [45].

5.2. τ decays — an almost unique opportunity

Lepton Flavour Violating (LFV) modes like $\tau \rightarrow l\gamma/3l$ with $l = e, \mu$ require New Physics to occur. While searching for $\tau \rightarrow l\gamma$ appears beyond the capabilities of the LHC, $\tau \rightarrow 3l$ with its present upper bound $\text{BR}(\tau \rightarrow 3l) \leq \text{few} \times 10^{-7}$ does not. For semi-leptonic B transitions produce about $\mathcal{O}(10^{12})$ τ leptons per year. While the width for $\tau \rightarrow 3l$ tends to be smaller than for $\tau \rightarrow l\gamma$ in most New Physics models, there are exceptions; more importantly the former is typically within an order of magnitude of the latter. If for illustrative purposes one makes two *ad-hoc* assumptions, namely that (a) New Physics makes up half of the observed $B_d \rightarrow \phi K_S$ amplitude and (b) the corresponding lepton coupling is equal in size, one arrives at $\text{BR}(\tau \rightarrow 3\mu) \sim \mathcal{O}(10^{-8})$ after this crude exercise.

An even more ambitious task is to probe for CP violation in τ decays. As already mentioned a new source of CP violation is needed to implement baryogenesis. Furthermore, leptogenesis might be the primary process; in that case it is essential to identify CP violation in leptodynamics. I see a realistic chance for success in three areas only: neutrino oscillations, the electric dipole moment of electrons and τ decays, in particular in the channels $\tau \rightarrow \nu K\pi$. For while those are Cabibbo suppressed in the SM, they should be particularly sensitive to exchanges of charged Higgs bosons. A CP asymmetry can arise not merely in the partial widths, and known dynamics has to induce a 0.0032 asymmetry in $\tau \rightarrow \nu K_S\pi$ [46], but also in the final state distributions [47]; the τ spin can be used as a powerful observable in $e^+e^- \rightarrow \tau^+\tau^-$ by employing the spin alignment of the τ pair or, better still, having the electron beam polarized. None of this can be achieved at the LHC. Since an optimistic, yet not unrealistic range is given by the 10^{-3} level [48], this is a noble task for a Super-Flavour Factory.

For proper perspective one should note that the rates for LFV modes are quadratic in New Physics amplitudes; CP asymmetries in τ decays, on the other hand, have to be only linear, since the SM provides the other amplitude. Searching for a LFV rate on the 10^{-8} level is thus of comparable sensitivity to New Physics as a CP asymmetry of order 10^{-3} in a Cabibbo suppressed mode.

6. Summary and outlook — on the need for a Super-Flavour Factory

There have been many good news for the SM over the last five years. In particular its paradigm of large CP asymmetries, both indirect and direct ones, in B decays has been validated. Through CKM dynamics the SM provides at least the lion's share of the CP asymmetries observed in K_L and B decays. The SM appears to have scored another impressive success of a new quality with an observable given purely by quantum correc-

tions: the value seen for $\Delta M(B_s)$ is quite consistent with the prediction, and together with another CP insensitive quantity, $|V(ub)/V(cb)|$, it constrains the CP observables $|\varepsilon_K|$ and $\sin 2\phi_1$ very close to their measured values. While giving theorists the unwelcome feeling of “deja vu all over again”, it represents good news for dedicated experimentalists. For even with $\Delta M(B_s)|_{\text{exp}} \simeq \Delta M(B_s)|_{\text{SM}}$ the time-dependent CP asymmetry in $B_s \rightarrow \psi\phi/\eta$ can still exceed the small value predicted by the SM by an order of magnitude. In addition the “moderate” value of $\Delta M(B_s)$ should allow ATLAS and CMS to participate in the hunt for New Physics through resolving the oscillations in $B_s \rightarrow \psi\phi$ and $B_s \rightarrow l^+l^-\phi$. While in these processes and a few others like $B_s \rightarrow \mu^+\mu^-$ and $B_s \rightarrow l^-X$ versus $\bar{B}_s \rightarrow l^+X$ the deviations from SM expectations can be large, I expect New Physics to induce typically smallish effects only. Thus high premium has to be placed on accuracy on the experimental as well as theoretical side, the latter concerning making predictions and interpreting the data. Heavy quark theory and its $1/m_Q$ expansions implemented through the OPE, augmented by (hopefully) validated lattice QCD and calibrated by a large body of “flanking” measurements should allow us to attain this ambitious goal. These elements are sketched in the Appendix below.

I can hardly over-emphasize that B_s transitions represent an independent chapter in Nature’s Book on Fundamental Dynamics and, therefore, fully deserve a comprehensive and detailed program of research.

A large discovery potential for New Physics exists also in τ decays, LFV and CP violation, and in weak charm decays mainly through CP studies. We know the SM *cannot* implement baryogenesis. Charm is the only up-type quark that allows a full probe of New Physics through flavour changing neutral currents, and only recently have we entered a domain with a realistic chance to see something novel.

There arise two questions to the LHC community: (i) Can LHCb take up the charm challenge, *i.e.* trigger with sufficient efficiency on charm to exploit the statistical muscle of the LHC for high quality charm studies? (ii) Can ATLAS and CMS go after $\tau \rightarrow 3l$?

Let me conclude with some glimpses of the “Big Picture”. I am confident that there resides indeed New Physics around the TeV scale (*cpNP*), and LHC will find its footprints. Identifying its features has to be our central goal. This *cpNP* can affect flavour dynamics *significantly*, though not necessarily *massively*. Analyses of heavy flavour decays, in the quark as well as lepton sector, are likely to reveal some of these salient features and thus provide probes of the *cpNP* complementary to high p_\perp observations. I view a continuing dedicated program of heavy flavour studies essential, *not* a luxury, where the high sensitivity of CP studies in particular is mainly *instrumentalized* to probe the *cpNP*.

To saturate the discovery potential in B decays we need numerically reliable and precise tools. In the last fifteen years we have made great strides in that respect by developing various aspects of heavy quark theory and will continue to do so. One lesson we can take from there is that the availability of precise data and the challenges provided by them drive (at least some) theorists to strive for higher accuracy.

In charm and τ decays on the other hand one can hope for numerically massive deviations from SM predictions, since the latter are a bit on the dull side; yet one has to push the experimental sensitivity as high as possible.

LHC's high p_{\perp} program represents largely hypothesis-probing research; B studies have significant aspects also of hypothesis-probing research, in particular once LHC finds New Physics directly, while charm and τ studies are of the hypothesis-generating variety.

Let me add one look at the "Grand Picture". Heavy flavour studies continue to be of fundamental importance, its lessons cannot be obtained any other way, thus they cannot become obsolete and they can sweep out dynamical scales up to the 100 TeV domain, *i.e.* well beyond the direct reach of the LHC. The LHC is and has to be the centerpiece of our efforts for quite a while to come. Yet it has three natural daughters: the "straight daughter" or ILC; the "Cinderella" or tau-charm factory; the "beautiful daughter" or Super-Flavour Factory $e^+e^- \rightarrow \Upsilon(4S, 5S) \rightarrow b\bar{b}, c\bar{c}, \tau^+\tau^-$ with a luminosity of around $10^{36} \text{ cm}^{-2} \text{ s}^{-1}$. The latter will provide a data base of the required size and quality not only to make precise measurements, but more importantly to interpret them accurately. We are at the beginning of a most exciting adventure, where we can be certain to find exciting new phenomena, and we are most privileged to participate.

Kraków is one of the truly great cities of the world not merely because of the beauty of its architecture, the civic sense of its citizens or its long history *per se*, but also for a reason touching us more directly as citizens of academia: its Jagiellonian University founded in 1364 is practically a founding and certainly an elite member of the academy in central Europe. It has done our ideals and aspirations proud, despite having had neighbors that all too often were less than benign. I consider it always a privilege to participate in a scientific meeting in Kraków, and I am grateful to the organizers, my colleagues and friends, to have granted me this privilege again. This work was supported by the NSF under grant PHY03-55098.

Appendix A

Calibrating and validating our theoretical tools

Theory actually faces two types of challenges:

- *Generic* TeV scale New Physics scenarios would already have manifested themselves, in particular through flavor changing neutral currents, since those are so highly suppressed within the SM. Apparently we are missing an important message about flavor dynamics, this is the “New Flavour Problem”. The fact that studies of heavy flavor decays represent largely hypothesis-generating rather than hypothesis-probing research is illustrated by the common use of classifications like “minimal-flavor violation”, “next-to-minimal-flavor violation” *etc.*

- To obtain precise SM predictions and likewise to interpret the data in a reliable way we have to bring nonperturbative QCD under theoretical control. This is the challenge I will address below.

Heavy quark theory

Heavy quark theory based on heavy quark symmetry and heavy quark expansions (HQE) in *inverse* powers of the heavy quark mass m_Q , thus combining a global symmetry with a dynamical treatment, is one of the most active and quickly progressing fields of QCD, although it is not often appreciated by the rest of the QCD community. Its central tool is the operator product expansion (OPE), which expresses (mostly inclusive) observables through a series of expectation values of local heavy flavor operators O_i with coefficients that can be calculated in short distance dynamics:

$$\text{observable} (H_Q \rightarrow f) = \sum_i c_i(f) \langle H_Q | O_i | H_Q \rangle. \quad (\text{A.1})$$

The $c_i(f)$ contain in particular the CKM parameters and m_Q ; more specifically the coefficients of the higher dimensional operators are suppressed by increasing powers of $1/m_Q$ (or the energy release $1/(m_b - m_c)$ for $b \rightarrow c$ transitions). The sometimes heard statement that the underlying concept of quark-hadron duality represents an *additional ad-hoc* assumption is not even wrong, it just misses the point, as explained in considerable detail in Ref. [18]. There it had been predicted that limitations to duality in evaluating $\Gamma_{\text{SL}}(B)$ cannot exceed 0.5 %.

As far as *fully integrated* widths are concerned, the leading nonperturbative corrections are $\sim \mathcal{O}(1/m_Q^2)$ rather than $\mathcal{O}(1/m_Q)$ as is the case for hadronic masses and differential distributions [17]. This result, which is intimately connected with color being a locally gauged quantum number, is

essential for the goal of high accuracy: for $Q = b$ one then has nonperturbative corrections of order $(A_{\text{NP}}/m_b)^2 \sim (1/5)^2 = 0.04$, and one needs to control them merely on the 20% level to achieve an overall accuracy of 1%. Furthermore, it is not necessarily foolish to apply a HQE to charm widths to obtain at least semiquantitative predictions.

Applications to semi-leptonic and radiative B decays

It has been demonstrated that high numerical accuracy can be achieved in our theoretical description [34]. From inclusive semi-leptonic and radiative B decays one has inferred [12]

$$m_b^{\text{kin}}(1 \text{ GeV}) = (4.59 \pm 0.04) \text{ GeV} \hat{=} \pm 1.0 \%, \quad (\text{A.2})$$

$$m_c^{\text{kin}}(1 \text{ GeV}) = (1.14 \pm 0.06) \text{ GeV} \hat{=} \pm 5.0 \%, \quad (\text{A.3})$$

$$|V(cb)|_{\text{incl}} = (41.96 \pm 0.67) \times 10^{-3} \hat{=} \pm 1.6 \%, \quad (\text{A.4})$$

where the last line should be compared with what we know about the Cabibbo angle studied for a much longer period:

$$|V(us)| = 0.2252 \pm 0.0022 \hat{=} \pm 1.0 \%. \quad (\text{A.5})$$

The robust theoretical framework required for such achievements has been provided by Heavy Quark Theory [11] implemented through the Wilsonian OPE and augmented by SV and other sum rules. Yet equally important was the impact of high quality data on total rates and distributions allowing to measure energy and hadronic mass moments in $B \rightarrow l\nu X_c, \gamma X_s$.

To achieve such accuracy levels one had to determine and even define the heavy quark mass very carefully, since the weak decay widths depend on the fifth power of it. A few concise comments on this complex issue have to suffice here. (i) The *pole* mass cannot be used, since renormalon effects due to its infrared instability in full QCD induce irreducible uncertainties parametrically larger than the leading nonperturbative corrections. (ii) The \overline{MS} mass is most appropriate, when the relevant scales are larger than m_Q like in $Z \rightarrow b\bar{b}$ or $H \rightarrow b\bar{b}$; yet it is ill-suited for H_Q decays, where the relevant scales are necessarily lower than m_Q . On the other hand it can be employed as a reference point for m_Q extracted from different processes. (iii) The *kinetic* mass is defined by the scale dependence

$$\frac{dm_Q(\mu)}{d\mu} = \frac{-16\alpha_S(\mu)}{3\pi} + \dots, \quad (\text{A.6})$$

i.e., a linear scale dependence in the IR region. It is the most appropriate quantity for H_Q decays, and its framework has been well developed now.

(iv) The $1S$ mass has principal shortcomings as explained in Ref. [49]. I view it as inferior to the *kinetic* mass. Therefore, I was quite surprised, to put it very mildly, that PDG has declared by “ordre du mufti” to list only the $1S$, but not the *kinetic* mass. I would be most grateful if somebody could explain to me, what the *scientific* reason behind this decision is.

Experimental cuts have often to be imposed on kinematical variables, like on the lepton and photon energies in $B \rightarrow l\nu X$ and $B \rightarrow \gamma X$, respectively. Those can create a serious theoretical problem though: for they reduce the amount of averaging over hadronic channels and thus might reduce the quantitative validity of quark–hadron duality; they manifestly introduce another energy scale potentially making the application of the OPE more ambiguous. This issue has been addressed theoretically concerning the lower cut E_{cut} on the photon energy in $B \rightarrow \gamma X$. Ignoring the sensitivity to E_{cut} will distort the spectrum, yet such “biases” can be corrected, and the validity of the OPE thus extended [50]. These predicted effects have been found in the data [12]. Analogous complications are expected to arise, when one measures lepton energy and hadronic mass moments in $B \rightarrow l\nu X_c$ with the lepton energy cut exceeding 1.6 GeV. *It would be most instructive to study, how the measured and predicted moments deviate from each other for $E_{\text{cut}}^{\text{lept}} \geq 1.6$ GeV.* The philosophy here is similar to that of engineers, who strain an engine to the breaking point to test its reliability.

The lessons we are learning from $B \rightarrow l\nu X_c, \gamma X$ help us with extracting $|V(ub)|$ from $B \rightarrow l\nu X_u$ in general as well as specific ways. Most importantly one does not need to “re-invent the wheel”: it is one of the strengths of the OPE that the *same* HQP are to be used in both $B \rightarrow l\nu X_{c,u}, \gamma X$ and the non-leptonic rates, albeit with coefficients specific to the final state. $\Gamma(B \rightarrow l\nu X_u)$ can actually be calculated in terms of $|V(ub)|$ with higher accuracy than $\Gamma(B \rightarrow l\nu X_c)$. The problem arises on the experimental side, since the total width cannot be measured directly; severe cuts have to be imposed on kinematical variables to extract a signal from the huge $B \rightarrow l\nu X_c$ background. While the lepton energy endpoint region provides a clean signal for $|V(ub)| \neq 0$, interpreting it with high numerical reliability is quite another matter. At least one has to perform such an analysis separately for B_d and B_u decays, since they are affected differently by weak annihilation [51]. Theoretically more promising ways are to analyze the hadronic recoil spectrum in $B \rightarrow l\nu X$ with and without cuts on q^2 [52, 53]. While I am skeptical about the accuracy presently claimed, I am optimistic that we can achieve *defensible* 5% (or even better) precision over the next few years.

On the powers of the Dalitz plot

Bringing hadronization under theoretical control obviously represents a stiff challenge. Yet since hadronization also enhances many signals for CP violation [40], we should view it as an essential even if quirky ally we can deal with by treating rich and complex data *judiciously*. Rather than viewing the Dalitz plot method as a prehistoric remnant used by people too old to learn C++, it should be recognized as a mature and powerful high-sensitivity tool. It will be crucial in saturating the discovery potential in B (and D) decays, as sketched by a few examples.

Case I: The angle ϕ_3 in the CKM unitarity triangle is being extracted from $B^\pm \rightarrow D^{\text{neut}} K^\pm$, where the neutral D mesons have been identified through (a) flavor-specific or (b) nonspecific modes, *i.e.* those common to D^0 and \bar{D}^0 . Originally one had considered only two-body channels for the latter. A new level of accuracy and reliability has been reached by relying on a full Dalitz plot analysis of $D^0/\bar{D}^0 \rightarrow K_S \pi^+ \pi^-$ as pioneered by BELLE [3]. It requires a very substantial effort, yet this investment pays handsome profits in the long run, for the very complexity of a full Dalitz plot with its many correlations provides a profound quality check thus giving us confidence in the weak parameters extracted. Increases in statistics, therefore, translate into a largely commensurate gain in information with a *defensible* estimate of the uncertainties.

Case II: In extracting the angle ϕ_2 from CP asymmetries in $B \rightarrow \pi' s$ one has to deal with the complication of two quark-level operators contributing, namely a tree as well as a (one-loop) Penguin one. The theoretically cleanest, yet experimentally very challenging method is based on analyzing $B_d \rightarrow \pi^+ \pi^- / 2\pi^0$ & $B^\pm \rightarrow \pi^\pm \pi^0$. A recent favorite has been to study $B \rightarrow 2\rho$ [3]. Experimentally it offers some advantages, yet theoretically suffers from significant drawbacks as well. For those transitions have to be inferred from $B \rightarrow 4\pi$, which will contain final states other than 2ρ , namely $\rho\sigma$, 2σ *etc.*, where it does not matter, whether the σ is a *bona fide* resonance or not. One should note that even if the σ is a genuine resonance, it *cannot* be described by a simple Breit–Wigner excitation function [15]. Imposing a cut on the dipion mass is not overly selective due to the large ρ width. For as stated repeatedly above we have to aim for an accuracy level of very few percent at most. Similar concerns affect the analysis based on $B \rightarrow \rho\pi$, since the primary reaction $B \rightarrow 3\pi$ contains coherent contributions from $B \rightarrow \sigma\pi$, $[3\pi]_{NR}$ as well [15]. Ultimately the goal has to be to perform full Dalitz plots analyses [54] of $B_d \rightarrow \pi^+ \pi^- \pi^0 / 3\pi^0$ & $B^\pm \rightarrow \pi^\pm \pi^+ \pi^- / \pi^\pm \pi^0 \pi^0$, where the multineutral final states provide an important cross check. In doing so one has to implement all the constraints from chiral dynamics applied to $\pi\pi$ scattering.

Case III: The angle ϕ_1 can be extracted also from $B_d \rightarrow \phi K_S$. Within the SM this Penguin driven channel has to exhibit basically the same CP asymmetries as in $B_d \rightarrow \psi K_S$, where it has been determined with high precision. Any deviation signals the intervention of New Physics, which actually finds fertile ground in $B_d \rightarrow \phi K_S$: its SM amplitude is considerably suppressed; the transition is driven by a single operator; we have a reliable SM prediction and finally, the ϕ constitutes a narrow resonance. The BELLE/BABAR average yields a value for the S term for $B_d \rightarrow \phi K_S$ and analogous modes like $B_d \rightarrow \eta K_S$ that is somewhat low compared to the SM prediction, yet not inconsistent with it [4]. I am most intrigued and tantalized by it, since the present central values are in a most natural range for New Physics. Again ultimately one has to analyze time-dependent Dalitz plots for $B_d \rightarrow K^+ K^- K_S$ (and cross reference them with $B_d \rightarrow 3K_S$ as well as $B^\pm \rightarrow K^\pm K^+ K^- / K^\pm K_S K_S$). One should also note that within the SM $B_d \rightarrow f(980) K_S$ has to exhibit a CP asymmetry of equal magnitude, yet opposite sign to $B_d \rightarrow \phi K_S$, since the two final states have opposite CP parity. That means that a $B_d \rightarrow f(980) K_S$ amplitude 10 % the size of that for $B_d \rightarrow \phi K_S$, thus quite insignificant in rate, would reduce the observable CP asymmetry by 20 %.

$B \rightarrow \tau \nu D$ versus $B \rightarrow \mu \nu D$

While $\Gamma(B \rightarrow e \nu D)$ is dominated by a single form factor f_+ , $\Gamma(B \rightarrow \tau \nu D)$ is affected also by the second form factor f_- , since m_τ is not negligible on the scale of $M_B - M_D$; secondly, the range of q^2 , which forms the argument in $f_{+,-}(q^2)$ is quite different for the two transitions. This complication can, however, be overcome by a novel theoretical tool, namely Uraltsev's "BPS" approximation. Applying the latter to $B \rightarrow e \nu D$ should allow to extract $|V(cb)|$ with very few percent uncertainty. Once this approximation has been validated by comparing $|V(cb)|_{\text{BPS}}$ with $|V(cb)|_{\text{incl}}$, it can be relied upon for calculating the SM value for $\Gamma(B \rightarrow \tau \nu D) / \Gamma(B \rightarrow e \nu D)$.

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