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INDUCED PAIRING INTERACTION IN NUCLEI AND IN NEUTRON STARS*

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For an electron going from X to Y, the pole of the propagator for a free particle is $p^2 = m^2$. However, making measurements at X and Y one could not tell if the electron had emitted or absorbed any number of photons. Such process, the simplest of which corresponds to emission and subsequent absorption of a photon (self-energy process) causes a shift in the position of the pole. Physically, this means that nothing is really free and that what one measures, the experimental mass, is not the "bare mass" but something else which includes the effect of virtual processes mentioned above. If this is true for electrons moving in vacuum it is even more true in the case of nucleons moving inside a nucleus. Bouncing inelastically off the nuclear surface, a nucleon can set it into vibrations exciting a normal mode of the system which it can reabsorb at a later time. The density of levels at the Fermi energy reflects this interweaving between fermionic and bosonic degrees of freedom and the resulting effective nucleon mass. If the vibration is reabsorbed by a second nucleon it leads to an exchange of bosons and thus to an effective interaction, *e.g.* in the pairing channel. Within the scenario discussed above, both quantum electrodynamics (QED) and nuclear field theory (NFT), are effective field theories. Now, while the photon is an elementary particle and the exchange of photons can be accurately parametrized in terms of the Coulomb interaction, collective surface vibrations do not exist outside the nucleus and there is no simple neither accurate parametrization of the pairing induced interaction, let alone of the $^1S_0$ NN-potential in terms of quarks and gluons. On the other hand, NFT based on effective interactions and HFB plus QRPA, provide a remarkably detailed picture of the induced pairing interaction, with which, together with *e.g.* the v_{14} NN-potential, one is able to calculate rather accurately the pairing gap of superfluid systems, including compact objects like (the inner crust of) neutron stars. The corresponding results are likely to provide the basis of a DFT of pairing in nuclei.

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1. Towards an *ab initio* theory of nuclear structure and reactions

The starting point of nuclear theory is the two-nucleon interaction. Several interactions already in use satisfy the criterion of fitting the two-nucleon data. In particular, the Argonne v_{14} and v_{18} bare NN-potentials [1]. And as far as nuclear structure properties and reactions at few MeV/nucleon are concerned, the "low-k" version of these potentials or their regularization through *e.g.* the unitary correlation operator method (UCOM) should be adequate [2, 4, 5]. In particular, making use of v_{18} within the framework of UCOM one can solve the HF equations and, in the resulting single-particle basis, calculate the linear response [3–5] of the system by solving the random phase approximation equations (RPA). It is likely that a similar approach can be implemented making use of $v_{\mathrm{low}-k}$. Furthermore, it is expected that the extension of the work mentioned above to a quasiparticle basis, *i.e.* HFB and QRPA, could be carried out in the near future. In this way one would obtain the (zeroth order) approximation to a full *ab initio* field theoretical solution [6] of the nuclear many-body problem to be carried out iteratively. The interweaving of the single-particle and collective motion gives rise to self-energy [7], induced interaction and vertex correction processes leading to dressed quasiparticles and effective interactions [8, 10, 11] as well as to renormalization of the vibrational modes [9, 11, 12] and, through zero-point fluctuations (ZPF), renormalization of the nuclear masses [13,14], radii [15, 16], *etc.*

In these calculations (see *e.g.* [14, 17]) not only densities, spin, isospin and charge exchange modes are to be considered, but also the modes (pairing vibrations) arising by considering the pairing field also as a dynamic variable (see [18] and references therein).

Direct reactions between heavy ions are controlled by form-factors and optical potentials. These quantities can be calculated *ab initio* making use of the dressed single-particle states, densities and the matrix elements of $v_{\mathrm{low}-k}$. The accuracy of the corresponding results can be assessed through unitary relations [19, 20]. While it is unlikely that all the dynamical effects contained in such *ab initio* solution of the nuclear many-body problem can be parametrized in terms of simple functions which only depend on the normal and abnormal (pairing) densities of the system, it is likely that many aspects of such a solution can be encoded into an optimal functional providing the basis of a density functional theory (DFT [21, 22], see *e.g.* also [23, 24] and references therein) of the atomic nucleus.

Such a project has been partially carried out for heavy ion reactions at few MeV/nucleon in Ref. [20], while many of the elements needed for a LDA and a TDLDA of the nuclear structure based on DFT have been worked out in Ref. [6, 7, 9], paying special emphasis to pairing correlations.

In what follows we shall discuss some of the central results concerning the role long-range pairing correlations effects, not easily incorporated in Kohn–Sham-like equations, play in the (state dependent) pairing gap of nuclei.

2. Long-range pairing correlations in nuclei

The bare nucleon–nucleon interaction is essential for the production of pair correlations in nuclei, but the induced interaction due to phonon exchange also contributes. This is easy to see from the BCS [25] relations (see also Fig. 1).

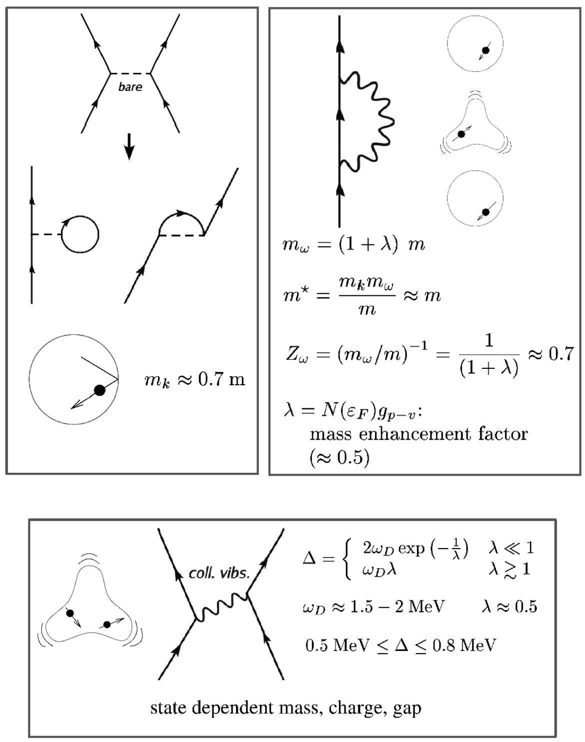

Fig. 1. Upper left: schematic representation of the process which, starting from a two-body bare NN-potential (*e.g.* $v_{\text{low}-k}$), gives rise to a Hartree–Fock mean field and associated effective k-mass smaller than the nucleon bare mass. Upper right: schematic representation of a diagram describing the coupling of a nucleon to collective vibrations of the nucleus. Such a process gives rise to an ω-mass controlled by the mass enhancement factor λ, product of the density of levels at the Fermi energy $N(\varepsilon_{\text{F}})$ and of the particle-vibration coupling strength g_{p-v} [18]. The quantity $Z_\omega = (m/m_\omega)$ is the spectroscopic factor associated with single-particle levels close to the Fermi energy. Typical values of this quantity determined through one-particle stripping and pick up reactions is ~ 0.7, implying a mass enhancement factor $\lambda \sim 0.5$ and an effective mass m^* of the order of the bare mass, as required by the experimental findings (*e.g.* density of levels at the Fermi energy). (Lower part): induced pairing interaction arising from the exchange of collective vibrations between pairs of nucleons moving in time reversal states around the Fermi energy.

$$m_\omega = m(1+\lambda)\,, \tag{1}$$

$$Z_\omega = (1+\lambda)^{-1}\,, \tag{2}$$

and

$$\Delta = \begin{cases} 2\omega_{\rm D}\ \exp(-1/\lambda) & \lambda \ll 1\,, \\ \omega_{\rm D}\lambda & \lambda \gtrsim 1\,, \end{cases} \tag{3}$$

where m_ω is the effective mass associated with the coupling of nucleons to collective vibrations (ω-mass), Z_ω the associated spectroscopic factor, while Δ is the BCS pairing gap in the weak and strong coupling approximations, respectively. Because $Z_\omega \sim 0.7$, $\lambda \sim 0.5$ for nuclei along the stability valley, the contribution to Δ associated with polarization effects is $\sim$ 0.5–0.8 MeV, that is, about half of the experimental value of the pairing gap. In carrying out the above estimates the typical value $\omega_{\rm D}$ = 2–2.5 MeV was used. For exotic halo (highly polarizable) nuclei $\lambda \sim 1$.In this case the induced interaction accounts for essentially all of the pairing correlations in these systems.

3. Solution of the Dyson equation for ^{120}Sn

In what follows we discuss results obtained by carrying out the nuclear field theoretical (NFT) program discussed in Sec. 1, in connection with the pairing channel. The limitations of these results are to be found in the fact that the mean field properties as well as the linear response of the system under study (^{120}Sn) were treated phenomenologically. Today, this situation has changed radically in keeping with the fact that results of mean field and QRPA calculations of finite nuclei carried out *ab initio* making use of $v_{{\rm low}-k}$ have been published in the literature [4, 5].

In Ref. [9] some of the realistic features of the bare nucleon motion were introduced in terms of the k-mass ($\sim 0.7m$). As expected, the RPA solution in this basis does not reproduce important features of the experimental data. In particular the energy of the low-lying quadrupole surface vibration is too high, and the associated B(E2)-transition probability too small (see Table I). Overall agreement with the experimental data is obtained by renormalizing the particles and holes associated with the RPA phonons through the (particle conserving, generalized Ward identities) processes (self-energy and vertex renormalization graphs) arising from the coupling to collective modes (see Fig. 2). The exchange of the resulting phonons between pairs of nucleons moving in time reversal states close to the Fermi energy gives rise to a pairing induced interaction $v_{\rm ind}$. Iterating the scattering process for this component of the force as well as for the bare interaction v_{14} making use of the Dyson equation one obtains a (state dependent) pairing gap and

TABLE I

The energy and reduced E2 transition strength of the low-lying 2+ state of ^{120}Sn, calculated according to different theoretical models, are compared to the experimental values, after [9].

	$\hbar\omega_{2^+}$ (MeV)	B(E2↑) (e^2fm^4)
RPA (Gogny)	2.9	660
RPA (SLy4)	1.5	890
RPA + renorm.	0.9	2150
Exp.	1.2	2030

quasiparticle energies (see Fig. 3) in overall agreement with the experimental data. Note that about one half of the pairing gap arises from $v_{\rm ind}$, the other half from v_{14}. This result is typical of nuclei lying along the valley of stability. For weakly bound, highly polarizable neutron drip nuclei like ^{11}Li and ^{12}Be, one finds that $v_{\rm ind}$ is essentially responsible for the whole of the two-neutron pairing correlation energies [26, 27]. Returning to well bound, stable nuclei like ^{120}Sn it is possible to parametrize $v_{\rm ind}$ in terms of a surface peaked semiclassical density functional [28].

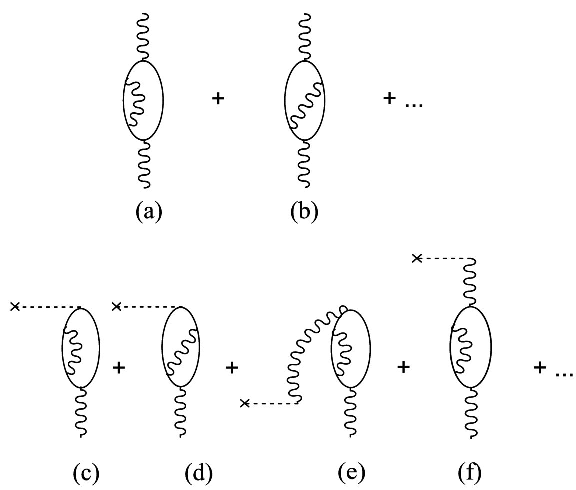

Fig. 2. Most relevant processes taken into account in the renormalization of the energy of the phonon (a)–(b) and of the associated transition strength (c)–(f).

Note that the density dependent pairing interaction used in the literature (see *e.g.* [29–32]) tries to parametrize in simple terms the summed contributions of the bare and induced interactions. A proper treatment of

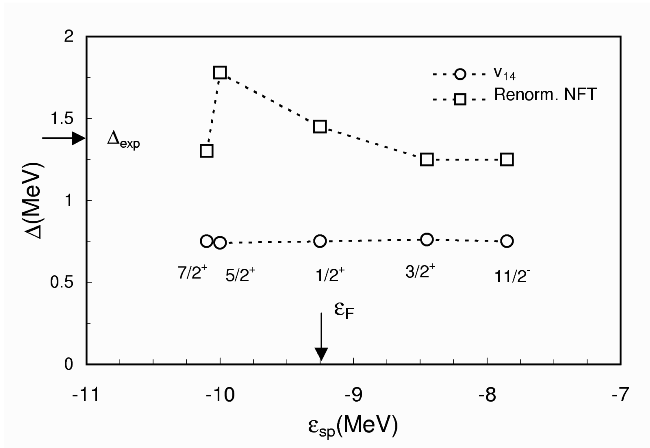

Fig. 3. The state-dependent pairing gap for the levels close to the Fermi energy obtained using BCS theory with the v_{14} Argonne potential (circles) is compared with the result obtained including renormalization effects (squares).

the separate contributions is called for (see *e.g.* [10, 30]). The mean field diagonalization of this interaction can, in principle, be thought as providing Kohn–Sham quasiparticle states which can be again parametrized in terms of a density functional.

4. The variety of renormilization processes

The functional $v_{\rm pair}(\vec{r}, \vec{r}')$ embodies the exchange of low-lying collective surface vibrations of finite nuclei. The actual values of the parameters, are a consequence of the competition between the attractive character of density modes and the repulsive effects associated with the exchange of spin modes, as testified by the results shown in Fig. 4 (*cf.* Ref. [33]).

In the set up of the inner crust of neutron stars, where nuclei are immersed in a sea of free neutrons, the balance between the attractive (density modes) and repulsive (spin modes) contribution to $v_{\rm ind}$ is altered in favor of the spin modes. In fact, the main source of attractive processes associated with the exchange of low-lying surface vibrations becomes weaker and weaker with increasing Fermi energy, because the neutrons relevant for pairing correlations are less and less sensitive to the effect of the nuclear potential. This is the reason why one expects, in neutron stars, a quenching of the pairing gap associated with the bare NN-potential instead of a boost, as observed in the case of finite nuclei.

Although much work has been carried out on this subject, no consensus has been reached concerning the actual value of the associated pairing gap.

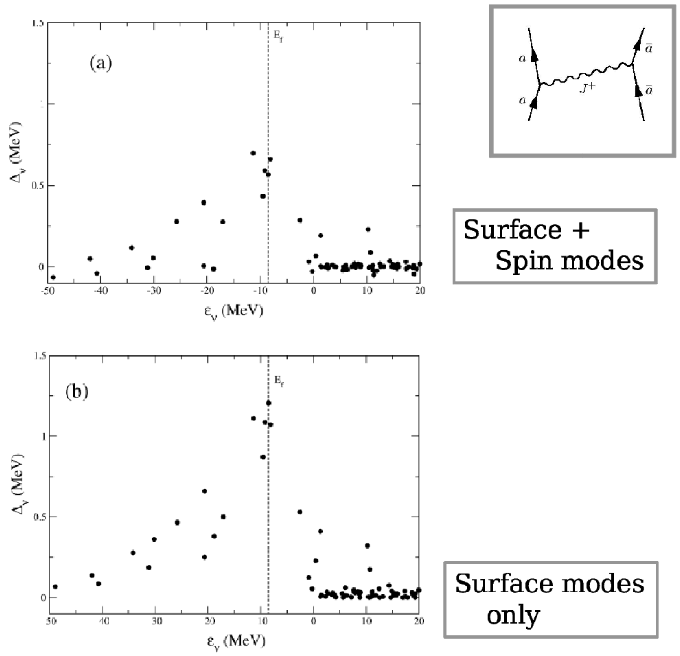

Fig. 4. The induced interaction v_{ind} arising from the coupling of surface and spin modes in ^{120}Sn is attractive and leads to a pairing gap of about 0.7 MeV (50% of the experimental value). Excluding the coupling to spin modes, the gaps increases to about 1.1 MeV. The total pairing gap should be calculated by adding to v_{ind} the bare NN-interaction $v_{\mathrm{low}-k}$. The resulting gap is, in this case, $\Delta \sim 1.5$ MeV, to be compared with the experimental value of 1.4 MeV.

5. Conclusions

Medium renormalization effects are essential to provide a quantitative description of superfluidity both in nuclei and in (the inner crust of) neutron stars. The treatment of these phenomena within the framework of HFB and QRPA to describe the fermionic and bosonic degrees of freedom, and of NFT to take into account their interweaving together with the bare NN-potential v_{14}, is able to account for the experimental findings in both stable and exotic nuclei. The availability of a *low-k* parametrization of the NN–Argonne potential is opening the possibility of carrying an essentially exact calculation based on a Dyson equation, in particular regarding the pairing channel. The results of such calculations can be used to provide an economic and accurate parametrization of a Kohn–Sham-like equation to describe both quasiparticle degree of freedom and the linear response of the condensate, as well as the state dependent pairing gap.

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