INCLUSION OF CONTINUUM EFFECTS IN MEAN-FIELD THEORIES*

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We present a method including continuum effects into nuclear structure mean-field calculations. We solve the Hartree–Fock–Bogoliubov problem in a basis made of the eigenstates of a Woods–Saxon potential. We show that the properties of stable nuclei remain unaffected, while the characteristic features of weakly-bound nuclei are correctly reproduced. We finally discuss the relevance of continuum effects on various nuclear observables.

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1. Introduction

With the on-going development of radioactive beam facilities, it has become possible to study properties of atomic nuclei with a large excess of one type of particle. This asymmetry results in the Fermi level being very close to the continuum threshold. It has long been expected that this would influence significantly nuclear pairing properties, in turn affecting even very basic properties of nuclei such as the binding or separation energies [1].

In the mean-field approach to the nuclear structure, continuum effects are often taken into account by enclosing the nucleus in a box with boundary conditions on the walls of the box [2]. This leads to a discretization of the continuum of positive energy states (quasi-bound states). The Hartree–Fock (HF) or Hartree–Fock–Bogoliubov (HFB) equations are then solved numerically on a lattice. Recently, an approach based on a local scale transformation of the harmonic oscillator wave-functions was also proposed [3].

In this paper we revisit the basis-expansion method by constructing a basis tailored to include continuum effects. We choose all negative-energy and a selected set of quasi-bound positive-energy eigenstates of a Woods–Saxon potential, thus assuring that the basis wave-functions are associated with a realistic potential but also possess good asymptotic properties.

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2. Description of the method

We work in the spherical HFB approximation with the finite-range effective interaction of Gogny [4]. This force presents the advantage to treat both the particle–hole (mean-field) and particle–particle channel (pairing) on the same footing and with the same parameters for the interaction. The HFB equations are solved in the basis of the eigenstates of the Woods–Saxon potential. The latter are determined by numerical integration of the Schrödinger equation on a spatial grid of points with a mesh size h (typically, $h = 0.10$ fm). Boundary conditions of the Cauchy type are imposed at $r = 0$ and $r = R_{\text{box}}$ (typically $R_{\text{box}} = 20$ fm). For positive-energy states, this automatically selects only those wave functions that have a node on the walls of the box.

In the case of spherical symmetry, the matrix elements of the Hamiltonian factorize into a radial and angular part. The full basis wave functions, including the spin degree of freedom $\vec{s}$, thus read $\psi(\vec{r},\vec{s}) = R_{n\ell}(r)\mathcal{Y}_{jm}(\theta,\varphi,\vec{s})$, where the $R_{n\ell}(r)$ are the radial wave functions and the $\mathcal{Y}_{jm}(\theta,\varphi,\vec{s})$ the tensor spherical harmonics [5]. In the standard case of the harmonic oscillator, where the wave functions are known analytically, the radial matrix elements are calculated by Gauss–Hermite quadrature. In the case of the Woods–Saxon basis, we calculate each radial matrix element by numerical integration using a Simpson 3/8 rule [6].

Tests were performed in the harmonic oscillator basis [7], where we can in the same time benchmark our program with the published results of [4], and where we can use, alternatively, Gauss–Hermite numerical integrations. We found that the replacement of "exact" integrations by numerical methods does not change the total HFB energy by more than a few keV. Moreover, the shell structure, the r.m.s. radii and all other quantities related *e.g.* to pairing are also unaffected.

More importantly, HFB calculations done in the two bases, HO and WS, for stable nuclei such as *e.g.* ^{16}O or ^{208}Pb yield remarkably close results as long as ground-state observables are concerned. Again, the nuclear binding energy or quantities such as the pairing gap or r.m.s. radius are practically unchanged when working in the WS basis. Only the position of the quasi-bound states, which are anyway a numerical artefact of the box-discretization method, is strongly influenced.

3. Results and discussion

By construction, our method is tailored to describe weakly-bound systems. Figure 1 shows the neutron density in the neutron-rich ^{132}Sn, ^{150}Sn and ^{172}Sn calculated using both the HO (squares) and WS (circles) basis — more technical information in the caption to the figure. In both cases, a box

of 20 fm was used to generate the basis states. The HO curves show an unphysical drop of density at large distance due to an improper asymptotic behavior. Conversely, calculations performed in the WS basis are capable to reproduce the characteristic long tails of neutron-rich nuclei.

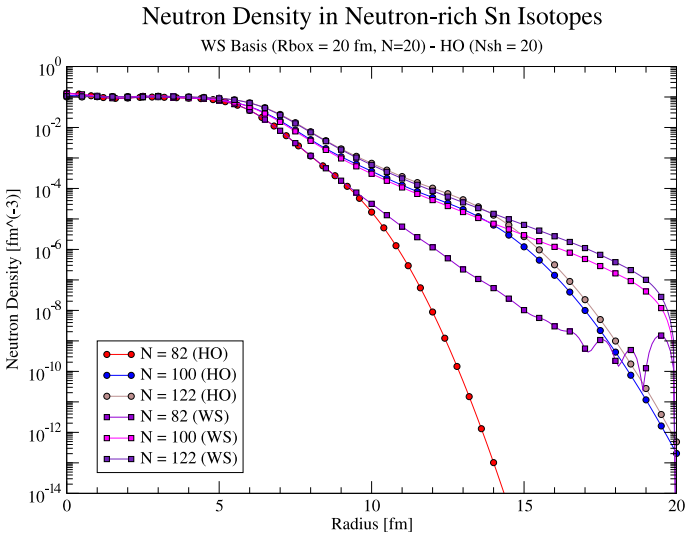

Fig. 1. Neutron density in several neutron-rich Sn isotopes as function of the radius. Curves marked with squares were obtained in the WS basis with $\ell_{\max} = 10$ and $n_{\max} = 20$ while curves marked with a circle were obtained in the harmonic oscillator basis with $N_{\rm sh} = 20$.

Figure 1 shows that our method is well-suited to describe neutron-rich nuclei indeed. Moreover, by construction, it is also particularly appropriate to compare the two different techniques, HO- and WS-basis, within the same framework of the HFB theory. Figure 2 reports the pairing energy in the chain of sodium isotopes from the proton-rich to neutron-rich nuclei.

It is worth noticing that the type of basis used barely changes the final result, even in those very neutron-rich nuclei where pairing properties are expected to be strongly influenced by the continuum. This may imply that, irrespective of whether the microscopic interaction may or may not describe ground-state properties correctly, standard HO basis-expansion techniques can be used at no risk *at least in light nuclei*. Heavier nuclei with even larger N/Z ratios are currently under investigation and preliminary results seem to indicate a more significant influence of the basis.

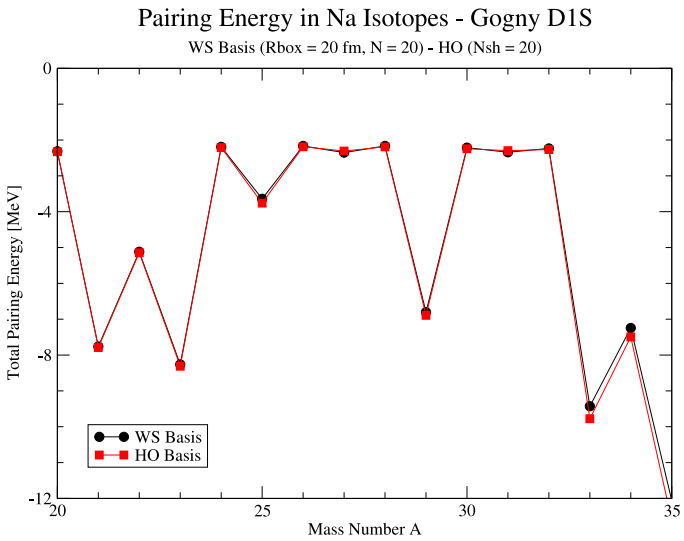

Fig. 2. Nuclear pairing energy in the isotopic chain of Na isotopes in the HFB approach with the D1S Gogny interaction. For the WS-basis calculations, $\ell_{\max} = 6, n_{\max} = 20$ while the number of shells is $N_{\rm sh} = 20$ for HO-basis calculations.

4. Conclusion

We solve the spherical HFB equations in the basis of the eigenstates of a Woods–Saxon potential. By construction, this basis imbeds continuum effects. In light weakly-bound nuclei, only the densities are found to be altered significantly, while all other quantities are unchanged.

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