SEARCH FOR LESS IMPORTANT DEFORMATIONS IN THE SHAPES OF HEAVIEST NUCLEI*

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Potential energy of the superheavy nucleus ^{262}Sg is analysed in a 5-dimensional deformation space. The space includes two components of the quadrupole deformation and three components of the hexadecapole deformation. The scope of the study is to find the component which has only a small influence on the energy. The analysis indicates that the best candidate for it is one (γ_4) of two non-axial hexadecapole deformations.

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1. Introduction

An accurate theoretical analysis of the properties of heavy nuclei within a macro–micro approach requires the use of a multidimensional deformation space (*e.g.*, [1, 2]). This makes the analysis quite complex and time-consuming. It is important then to learn which kind of the deformation has a relatively small influence on the analysed properties and, as a consequence, may be disregarded in the analysis without appreciable decrease of its accuracy.

Our recent studies concentrate on the heights of the fission barriers (*e.g.*, [3–5], *cf.* also works of other groups, *e.g.*, [6–8]), for which the choice of the deformation space to be used in the calculations is a basic question. The investigations of the role of the hexadecapole non-axial deformations in the potential energy of heaviest nuclei, performed by us recently [9, 10], suggest that one of these deformations (γ_4) may be less important.

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The objective of the present paper is to check, if this suggestion is correct. To this aim, we choose the nucleus ^{262}Sg ($Z = 106$), which is in some distance (in proton Z and neutron N numbers) from both the nuclei: ^{250}Cf [9] and ^{284}114 [10] studied previously, and in which the role of hexadecapole deformations is important. The hexadecapole (multipolarity: $\lambda = 4$) deformation of a quite general type, not discussed earlier, is considered. The potential energy of the nucleus ^{262}Sg, which is the basic quantity for the study of its properties (*e.g.*, the fission barrier), is analysed.

2. Method of the calculations

A macroscopic–microscopic approach is used to describe the potential energy of a nucleus. The Yukawa-plus-exponential model [11] is taken for the macroscopic part of the energy and the Strutinski shell correction, based on the Woods–Saxon single-particle potential [12], is used for its microscopic part. Details of the approach are specified in [13].

Especially important in the calculations is the deformation space admitted in them. In this study, a 5-dimensional space is used. Besides 2-dimensional quadrupole space, it includes a 3-dimensional hexadecapole space. The hexadecapole space is of a general type, if one assumes the reflexion symmetry of a nucleus with respect to all three planes of the intrinsic coordinate system [14]. Our total space is specified by the following expression for the nuclear radius $R(\vartheta,\varphi)$ (in the intrinsic frame of reference) in terms of spherical harmonics

$$
\begin{aligned}
R(\vartheta,\varphi) \;=\; R_0\Big\{ & 1+\beta_2\left[\cos\gamma_2 \mathrm{Y}_{20}+\sin\gamma_2 \mathrm{Y}_{22}^{(+)}\right] \\
& +\frac{1}{\sqrt{12}}\beta_4\Big[(\sqrt{7}\cos\delta_4+\sqrt{5}\sin\delta_4\cos\gamma_4)\mathrm{Y}_{40}-\sqrt{12}\sin\delta_4\sin\gamma_4 \mathrm{Y}_{42}^{(+)} \\
& +(\sqrt{5}\cos\delta_4-\sqrt{7}\sin\delta_4\cos\gamma_4)\mathrm{Y}_{44}^{(+)}\Big]\Big\}\,, \qquad (1)
\end{aligned}
$$

where γ_2 is the Bohr quadrupole non-axiality parameter and the dependence of R_0 on the deformation parameters is determined by the volume-conservation condition. The functions $\mathrm{Y}_{\lambda\mu}^{(+)}$ are defined as:

$$
\mathrm{Y}_{\lambda\mu}^{(+)}=\frac{1}{\sqrt{2}}\left[\mathrm{Y}_{\lambda\mu}+(-1)^{\mu}\mathrm{Y}_{\lambda-\mu}\right], \qquad \text{for} \quad \mu\neq 0\,. \qquad (2)
$$

The regions of variation of the deformation parameters are

$$
\beta_2 \geq 0\,, \quad 0^\circ \leq \gamma_2 \leq 60^\circ\,, \qquad (3)
$$

$$
\beta_4 \geq 0\,, \quad 0^\circ \leq \delta_4 \leq 180^\circ\,, \quad 0^\circ \leq \gamma_4 \leq 60^\circ\,. \qquad (4)
$$

The potential energy is calculated at the following grid points:

$$\begin{aligned}
\beta_2 \cos\gamma_2 &= 0(0.05)0.65\,,\\
\beta_2 \sin\gamma_2 &= 0(0.075)0.375\,,\\
\beta_4 \cos\delta_4 &= -0.20(0.05)0.20\,,\\
\beta_4 \sin\delta_4 &= 0(0.075)0.225\,,\\
\gamma_4 &= 0^\circ(20^\circ)60^\circ\,,
\end{aligned} \tag{5}$$

i.e. at $14\times 6\times 9\times 4\times 4 = 12096$ points. Numbers in the parentheses specify the step with which the calculation is done for a given variable.

Then, the energy is interpolated (by the standard SPLIN3 procedure of the IMSL library) to the five times denser grid in each variable. Thus, we finally have the values of the potential energy at 12096×5^5 grid points.

3. Results

Fig. 1 shows a contour map of the potential energy of the nucleus ^{262}Sg, when only quadrupole deformations β_2 and γ_2 are taken into account. As usually in the macro–micro calculations, the energy is normalised in such a way that its macroscopic part is equal to zero at the spherical shape of a nucleus. One can see that the equilibrium point is obtained at the axially symmetric deformation ($\gamma_2 = 0$), while the saddle point (denoted by "x") appears at a non-axial shape of the nucleus. The energy at this point is by 2.3 MeV lower than in the case of the axial symmetry (point "+"). Thus, the quadrupole non-axiality decreases the barrier height $B_{\rm f}^{\rm st}$ of ^{262}Sg by 2.3 MeV.

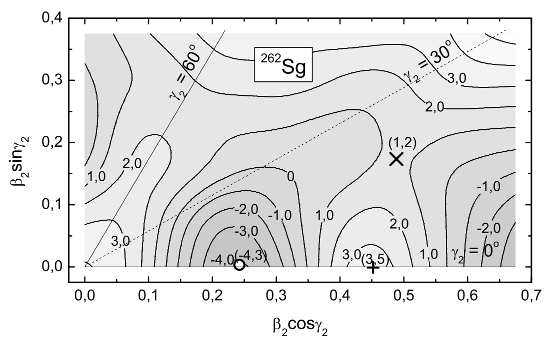

Fig. 1. Contour map of the potential energy $E(\beta_2, \gamma_2)$ of the nucleus ^{262}Sg, when only the quadrupole deformations β_2 and γ_2 are taken into account. Numbers at the contour lines give the values of the energy in MeV. Positions of the equilibrium (circle), axial (denoted by "+") and non-axial (denoted by "x") saddle points are indicated. Values of the energy at these points are given in parentheses.

After the inclusion of the hexadecapole deformations, the potential energy is shown in Fig. 2. It is seen that the structure of the energy is changed with respect to that of Fig. 1, especially in the region of the saddle point. The position of this point in the β_2 and γ_2 degrees of freedom, and also its energy, are significantly changed.

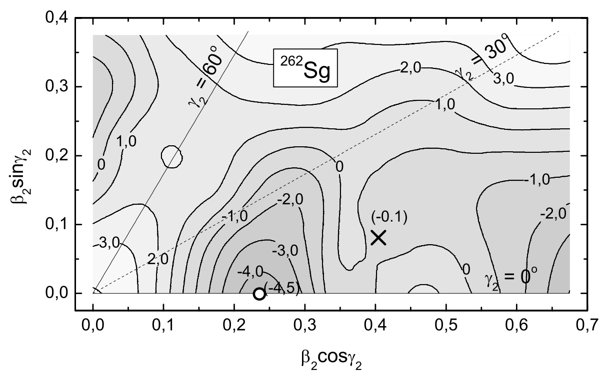

Fig. 2. Contour map of the potential energy $E(\beta_2,\gamma_2;\beta_4^{\rm min},\delta_4^{\rm min},\gamma_4^{\rm min})$ of the nucleus ^{262}Sg, projected on the plane (β_2,γ_2), when hexadecapole deformations are also considered.

To see explicitly the effect of the total hexadecapole deformation on the energy of ^{262}Sg, the difference between the energies of Fig. 2 and of Fig. 1 is shown in Fig. 3. One can see that the effect is rather large, up to about 3 MeV in the whole considered region of the deformations. It is relatively large in the region of the saddle point, while it is small around the equilibrium point.

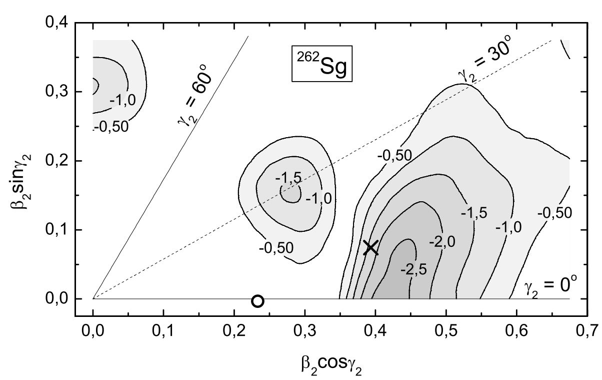

Fig. 3. Same as in Fig. 2, but for the difference: $E(\beta_2,\gamma_2;\beta_4^{\rm min},\delta_4^{\rm min},\gamma_4^{\rm min}) - E(\beta_2,\gamma_2;\beta_4=0)$, *i.e.* for the total effect of the hexadecapole deformation on the energy of ^{262}Sg.

Effect of the non-axial hexadecapole deformation δ_4 on the energy is illustrated in Fig. 4. It is seen that the effect is also rather large, not much smaller than that of the total hexadecapole deformation.

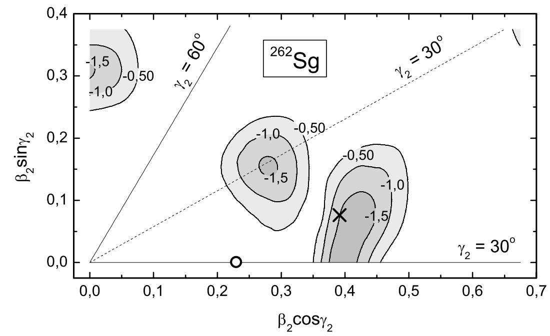

Fig. 4. Same as in Fig. 2, but for the difference: $E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4^{\min}) - E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4 = 0, \gamma_4^{\min})$, *i.e.* for the effect on energy of the hexadecapole non-axial deformation of ^{262}Sg described by the parameter δ_4.

Finally, Fig. 5 shows the effect of the non-axial hexadecapole deformation described by the parameter γ_4. One can see that this effect is small, less than about 0.3 MeV (in its absolute value) in the whole considered region of deformation, similarly as obtained for other nuclei: ^{250}Cf [9] and ^{284}114 [10].

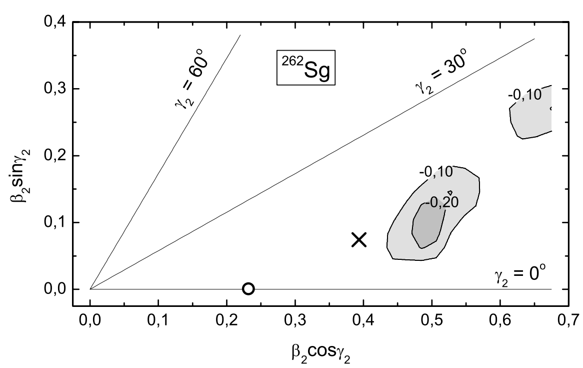

Fig. 5. Same as in Fig. 2, but for the difference: $E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4^{\min}) - E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4 = 0)$, *i.e.* for the effect on energy of the hexadecapole non-axial deformation of ^{262}Sg described by the parameter γ_4.

Concluding, one can say that the dependence of the potential energy of a heavy nucleus on deformation is a very individual property of each nucleus (because of its individual shell structure). Due to this, one should be careful in drawing general conclusions from considered examples. Still, the results obtained in this paper for ^{262}Sg and in other studies, done for ^{250}Cf and ^{284}114, strongly suggest a small influence of the non-axial deformation γ_4 on the energy. It seems to be reasonable then to omit this deformation in calculations, and check only the final result (for example, the energy at interesting us points, *e.g.*, equilibrium or saddle point), if it may be changed by this degree of freedom. This may reduce the dimension of the deformation space used in the analysis.

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