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NON-AXIAL HEXADECAPOLE DEFORMATIONS OF HEAVIEST NUCLEI*

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Potential energy of a heavy nucleus (^{250}Cf) is analysed in a 5-dimensional deformation space. This is the most general space including quadrupole and hexadecapole deformations, when the reflexion symmetry of a nucleus with respect to all three planes of the intrinsic coordinate system is assumed. Main attention is given to the influence of non-axial hexadecapole shapes on the height of the fission barrier of the analysed nucleus. It is found that this influence is small (about 0.1 MeV) for the considered nucleus, in contrast to the influence of non-axial quadrupole deformation, which is large (about 1.8 MeV).

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1. Introduction

This paper belongs to a series of studies of the potential energy of heaviest nuclei and, in particular, of the static fission-barrier heights, $B_{\rm f}^{\rm st}$, done recently in our group (*e.g.*, [1–7]). They mainly aim in learning the role of various kinds of deformation of a nucleus in determination of these heights. Generally, studies of fission barriers are recently quite intensive (*e.g.*, [8–13]).

In the present paper, we concentrate on the role of hexadecapole deformations, in particular on their non-axial components. It has been shown (*e.g.*, [6]) that the quadrupole non-axial deformation is important for the heights. It may decrease them by up to about 2 MeV. It is interesting then to learn how much non-axial hexadecapole deformations may influence these heights.

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2. Method of the calculations

Macroscopic–microscopic approach is used to describe the potential energy of a nucleus. The Yukawa-plus-exponential model is taken for the macroscopic part of the energy and the Strutinski shell correction, based on the Woods–Saxon single-particle potential, is used for its microscopic part. Details of the approach are specified in [14]. Especially important in the calculations is the deformation space admitted in them.

3. Deformation space

A 5-dimensional deformation space is used. Besides 2-dimensional quadrupole space, it includes a 3-dimensional hexadecapole space. The latter is the general hexadecapole space, if one assumes the reflexion symmetry of a nucleus with respect to all three planes of the intrinsic coordinate system [15]. The space is specified by the following expression for the nuclear radius $R(\vartheta,\varphi)$ (in the intrinsic frame of reference) in terms of spherical harmonics [15]

$$\begin{aligned} R(\vartheta,\varphi) \;=\; & R_0\Big\{1+\beta_2\left[\cos\gamma_2 \mathrm{Y}_{20}+\sin\gamma_2 \mathrm{Y}_{22}^{(+)}\right] \\ & +\frac{1}{\sqrt{12}}\beta_4\Big[(\sqrt{7}\cos\delta_4+\sqrt{5}\sin\delta_4\cos\gamma_4)\mathrm{Y}_{40}-\sqrt{12}\sin\delta_4\sin\gamma_4\mathrm{Y}_{42}^{(+)} \\ & +(\sqrt{5}\cos\delta_4-\sqrt{7}\sin\delta_4\cos\gamma_4)\mathrm{Y}_{44}^{(+)}\Big]\Big\}\,, \end{aligned} \tag{1}$$

where γ_2 is the Bohr quadrupole non-axiality parameter and the dependence of R_0 on the deformation parameters is determined by the volume-conservation condition. The functions $\mathrm{Y}_{\lambda\mu}^{(+)}$ are defined as:

$$\mathrm{Y}_{\lambda\mu}^{(+)}=\frac{1}{\sqrt{2}}\left[\mathrm{Y}_{\lambda\mu}+(-1)^{\mu}\mathrm{Y}_{\lambda-\mu}\right]\,,\qquad \text{for}\quad \mu\neq 0\,. \tag{2}$$

The regions of variation of the deformation parameters are

$$\beta_2\geq 0\,,\quad 0^\circ\leq\gamma_2\leq 60^\circ\,, \tag{3}$$

$$\beta_4\geq 0\,,\quad 0^\circ\leq\delta_4\leq 180^\circ\,,\quad 0^\circ\leq\gamma_4\leq 60^\circ\,. \tag{4}$$

4. Results

Figure 1 shows a contour map of the potential energy of the nucleus ^{250}Cf projected on the plane ($\beta_2,\ \gamma_2$). As usually in the macro–micro calculations, the energy is normalised in such a way that its macroscopic part is equal to zero at the spherical shape of a nucleus. Minimum of the energy, $E_{\min}=-3.61\,\mathrm{MeV}$, is obtained at the quadrupole deformation: $\beta_2^0=0.24$,

$\gamma_2^0 = 0$, and the saddle-point energy, $E_\mathrm{s} = 1.60\,\mathrm{MeV}$, at $\beta_2^\mathrm{s} = 0.49$, $\gamma_2^\mathrm{s} = 16°$. Strong dependence of the energy on deformation, seen in the figure, is mainly due to the main (quadrupole) component of the deformation.

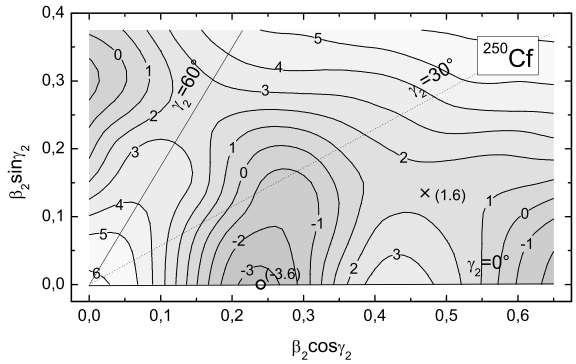

Fig. 1. Contour map of the total energy, $E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4^{\min})$, of the nucleus ^{250}Cf, projected on the plane (β_2, γ_2). Numbers at the contour lines give the values of the energy in MeV. Positions of the equilibrium (circle) and saddle (cross) points are indicated. Values of the energy at these points are given in parentheses.

Influence of the hexadecapole component of the deformation on the energy is illustrated in Fig. 2. One can see that this influence is relatively small for the considered nucleus. This component decreases the energy by about 0.5 MeV at the equilibrium point and by about 0.4 MeV at the saddle point. As a result, it changes (increases) the barrier height B_f^st only by about 0.1 MeV, in contrast to the quadrupole non-axial deformation, which lowers B_f^st by about 1.8 MeV [16].

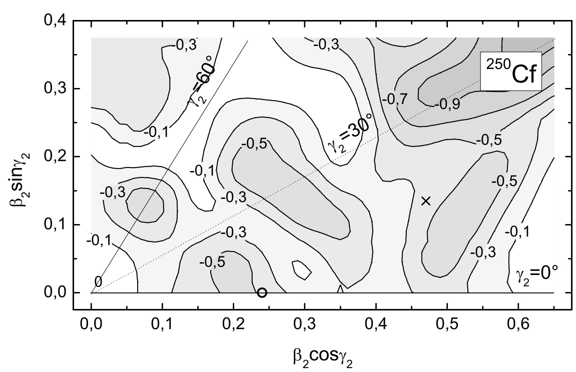

Fig. 2. Same as in Fig. 1, but for the difference: $E(\beta_2, \gamma_2; \beta_4^{\min}, \delta_4^{\min}, \gamma_4^{\min}) - E(\beta_2, \gamma_2; \beta_4 = 0)$, *i.e.* for the total effect of the hexadecapole deformation on the energy of ^{250}Cf.

Effect of the non-axial hexadecapole deformations on the energy is shown in Fig. 3. One can see that this effect is rather small, smaller than 0.7 MeV (in its absolute value) in the whole considered region of deformations. In particular, these deformations decrease the height $B_{\rm f}^{\rm st}$ by only about 0.15 MeV. Here, $\delta_4 = \delta_4^0$ ($\mathrm{tg}\delta_4^0 = \sqrt{5/7}$) and γ_4=0 correspond to hexadecapole shapes which are axially symmetric [15].

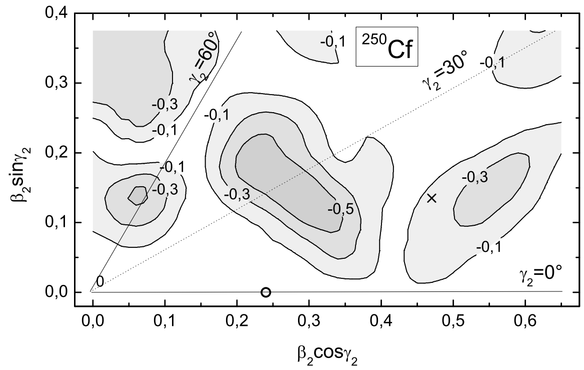

Fig. 3. Same as in Fig. 1, but for the difference: $E(\beta_2,\gamma_2;\beta_4^{\rm min},\delta_4^{\rm min},\gamma_4^{\rm min}) - E(\beta_2,\gamma_2;\beta_4^{\rm min},\delta_4^0,\gamma_4^0)$, *i.e.* for the effect on energy of the hexadecapole non-axial deformations of ^{250}Cf described by the parameters δ_4 and γ_4.

Finally, Fig. 4 shows the effect of the (non-axial) hexadecapole deformation γ_4 on the energy. One can see that this effect is very small. In particular, it does not change the energy in the ground state and in the saddle point, and, as a consequence, leaves the barrier height $B_{\rm f}^{\rm st}$ unchanged.

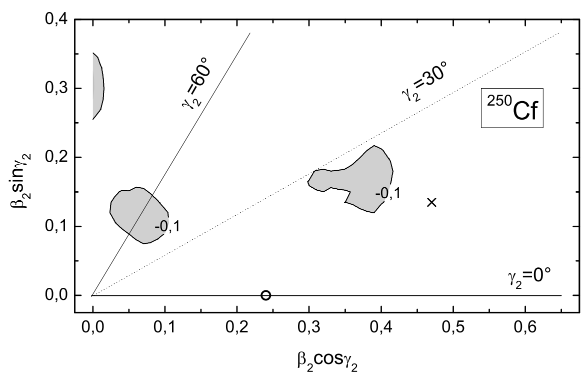

Fig. 4. Same as in Fig. 1, but for the difference: $E(\beta_2,\gamma_2;\beta_4^{\rm min},\delta_4^{\rm min},\gamma_4^{\rm min}) - E(\beta_2,\gamma_2;\beta_4^{\rm min},\delta_4^{\rm min},\gamma_4 = 0)$, *i.e.* for the effect on energy of the hexadecapole non-axial deformation of ^{250}Cf described by the parameter γ_4.

In conclusion, one can say that the hexadecapole non-axial deformations have only a small influence on the energy of the considered nucleus ^{250}Cf. This especially concerns the deformation γ_4. If the latter conclusion, concerning γ_4, is general (which should be checked), the calculations of the energy may be simplified by omission of γ_4, without an appreciable decrease in the accuracy of the results.

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