PHASE TRANSITIONS IN THE CONFIGURATION MIXED INTERACTING BOSON MODEL: U(5)–O(6) MIXING*

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The phase diagram for the configuration mixed Interacting Boson Model is investigated for the special case of U(5)–O(6) mixing using the methods provided by Catastrophe Theory. It will be shown that this phase diagram exhibits properties not present when only a single configuration is considered.

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1. Introduction

In several regions of the nuclear chart, a systematic lowering of collective intruder bands is observed when approaching proton or neutron midshell. Macroscopically, this can be understood as the coexistence of different potential energy minima within a small energy interval. From microscopical point of view, this phenomenon can be explained invoking particle-hole excitations across the closed shell. The combined effect of monopole, pairing, and quadrupole interaction may cause the intrusion of the bands built on these p–h excitations to very low energies [1], hence leading to mixing between the different bands, *i.e.* configuration mixing. One of the most spectacular examples is the Pb-region where both microscopic [2, 3] and macroscopic [4, 5] approaches succesfully describe the dramatic lowering of collective intruder bands towards neutron midshell.

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Configuration mixing can easily be incorporated in the Interacting Boson Model (IBM) [6], an algebraic model that approximates particle pairs coupled to 0^+ and 2^+ as s and d bosons. A nice feature of this model is the fact that information on nuclear collective motion can be extracted by means of the coherent state formalism. The main goal of this paper is a general study of the concept of configuration mixing and its relation to shape coexistence as configuration mixing will not always give rise to macroscopical shape coexistence. Depending on the IBM-parameters, the strength of the mixing, and the unperturbed excitation energy of the intruder configurations, the potential energy surface calculated with the coherent state formalism [7–9] can exhibit a single minimum or coexisting minima. Using Catastrophe Theory [10], we will construct phase-diagrams for the special case of U(5)–O(6) mixing and mark out the regions with different behaviour of the potential energy surface.

2. Theoretical framework

2.1. The potential energy surface for U(5)–O(6) mixing

The potential energy surface (PES) for the configuration mixed system E_- is obtained as the lowest eigenvalue of the potential energy matrix [11]

$$\begin{bmatrix} \epsilon N \frac{\beta^2}{1+\beta^2} & \Omega \\ \Omega & -|\kappa| \left[(N+2)\frac{(5+\beta^2)}{1+\beta^2} + (N+2)(N+1)\frac{4\beta^2}{(1+\beta^2)^2} \right] + \Delta \end{bmatrix}, \quad (1)$$

with $\Omega = \sqrt{(N+1)(N+2)}\alpha$ the mixing strength [11], N the total number of bosons, ϵ and κ IBM-parameters [6], β the axial quadrupole deformation, and Δ the initial excitation energy of the intruder band corrected for pairing and change in monopole interaction. In the absence of mixing, the U(5)-limit corresponds to a PES with a spherical minimum while the O(6)-limit gives rise to a γ-independent deformed minimum in the PES, where γ denotes the nonaxial quadrupole deformation.

2.2. Criticality conditions

If one examines the properties of a system, phase transitions might occur when the order parameters of the system under study are varied and pass through a critical value. In thermodynamics, temperature is the order parameter while for the PES describing U(5)–O(6) mixing, the parameters $\Delta, \epsilon, \Omega, N, \kappa$ are the order parameters. Such kind of phase transitions are referred to as "quantum phase transitions" [10, 12].

To determine the critical values of the order parameters of the system, one needs to determine the locus of points for which the conditions

$$\frac{\partial E_-}{\partial \beta} = 0\,, \qquad \frac{\partial^2 E_-}{\partial \beta^2} = 0\,, \tag{2}$$

are fulfilled.

In regions where the PES has several minima, it is from physics point of view important to know which minimum is the global one. Maxwell points [10] mark out where the global minimum jumps from one local minimum to another.

3. Results and discussion

An analytical solution for the criticality conditions can be found by expanding the PES for configuration mixing around $\beta = 0$ [12]. The Taylor expansion around this point is given by

$$\begin{aligned} E_- \;=\; & -\zeta - \sqrt{\zeta^2 + \Omega'^2} + \frac{1}{\sqrt{\zeta^2 + \Omega'^2}} \Big[\epsilon' N \left(\sqrt{\zeta^2 + \Omega'^2} - \zeta \right) - 4N(N+2) \\ & \times \left(\sqrt{\zeta^2 + \Omega'^2} + \zeta \right) \Big] \beta^2 + \dots \,, \end{aligned} \tag{3}$$

where $\zeta = -\Delta/|\kappa| + 5(N+2)$, $\epsilon' = \epsilon/|\kappa|$, and $\Omega' = 2\Omega/|\kappa|$ were subsituted. If the coefficient in β^2 vanishes identically, the criticality conditions are fulfilled. This leads to the following relation between ζ, Ω', and ϵ'

$$\epsilon' = -4(N+2) \frac{\zeta + \sqrt{\zeta^2 + \Omega'^2}}{\zeta - \sqrt{\zeta^2 + \Omega'^2}}\,. \tag{4}$$

For $\beta \neq 0$, the criticality conditions have to be solved numerically. The resulting phase diagrams are shown in Fig. 1 for different excitation energies of the intruder states. For small $\Delta/|\kappa|$, we can clearly distinguish three different regions, one with a spherical minimum, one with a deformed minimum and a large region with a coexisting deformed and spherical minimum. When $\Delta/|\kappa|$ increases, the physical region with a deformed minimum becomes smaller and when the slope of the line of Maxwell points changes sign, a phase with spherical minima starts growing at the origin of the phase diagram. Eventually, this small region of sphericity merges with the larger one and the area of shape coexistence is splitted. For high $\Delta/|\kappa|$, coexistence for small values of $\epsilon/|\kappa|$ disappears and the remaining region of shape coexistence shifts to high values of $\epsilon/|\kappa|$.

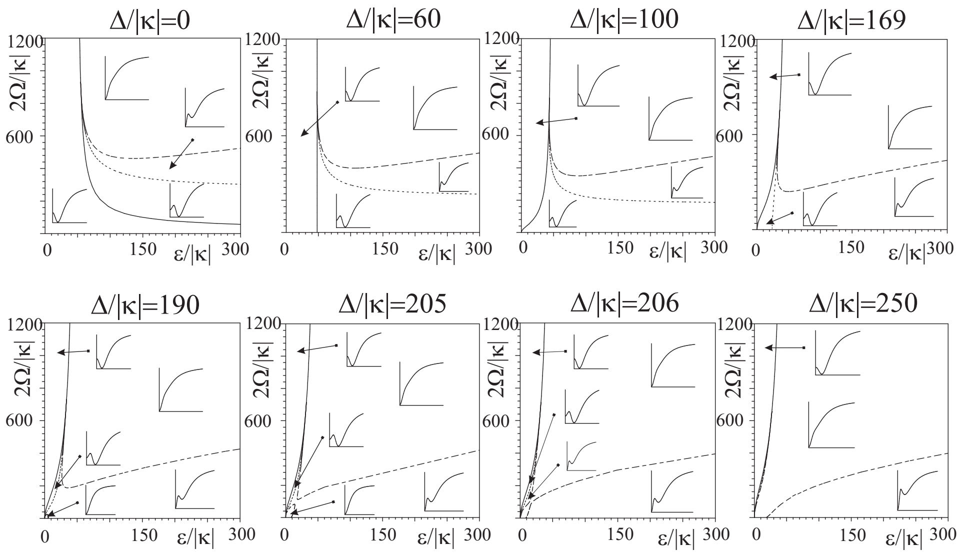

Fig. 1. Phase diagram in case of U(5)–O(6) mixing for several values of $\Delta/|\kappa|$ and for N=10. The locus of points which are determined analytically are shown with a full line. Numerical solutions of the criticality conditions are displayed with a dashed line, while the Maxwell points are indicated with a dotted line. The inset figures illustrate the arbitrary shape of the potential energy surface as a function of β. For small regions in the phase diagram, the arrows indicate which arbitrary shape is connected with those phases.

4. Conclusions

We have determined the phase-diagram for U(5)–O(6) mixing and discussed the evolution with varying excitation energies of the intruder states. Whereas the pure U(5) and O(6) limit can be associated with a spherical and a deformed minimum, an additional phase with shape coexistence can be found in the case of mixing. Analysis of mixing between the other IBM symmetry limits and applications in the Pt-Pb-Hg region are in progress.

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