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E(5) AND X(5) DYNAMICAL SYMMETRIES FROM A MICROSCOPIC PERSPECTIVE*

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Microscopic mean field approach based on ATDHFB theory has been applied to describe low energy collective properties of ^{104}Ru, ^{102}Pd and ^{154}Gd nuclei, which recently have been regarded as good examples of phenomenological E(5) or X(5) symmetries.

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1. Introduction

In the frame of Interacting Boson Model there are known several dynamical symmetries connected with subgroups of the U(6) group (SU(3), U(5), SO(6)). Recently proposed new theoretical schemes [1, 2], referred to as E(5) and X(5), have slightly different character, in particular they do not correspond directly to any subgroup of U(6). Their origin lies in analytically solvable classes of the Bohr Hamiltonian, which can be obtained from the IBM Hamiltonian through a coherent state formalism. In fact the term E(5) can be treated as a compact (however sometimes misleading) name for an infinitely deep potential well in 5 dimensions. Several attempts have been made to find nuclei whose energies of collective quadrupole states and E2 transition probabilities agree with predictions of these schemes. On the other hand, modern mean field theories based on effective nucleon–nucleon interactions (of Skyrme or Gogny type) or the RMF theory (Relativistic Mean Field) also offer an explanation of quadrupole collective excitations by means of the Adiabatic Time Dependent HFB theory or the Generating Coordinate Method. It would be interesting to check if and how symmetries proposed on a phenomenological ground can manifest in microscopic theories.

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In this work we discuss three test cases of a tentative experimental evidence for the new dynamical symmetries. Examples of the E(5) symmetry are: a chain of ruthenium isotopes $^{96-110}$Ru, in particular the ^{104}Ru nucleus, and the ^{102}Pd nucleus. The X(5) symmetry is represented by the ^{154}Gd nucleus. In case of Ru and Pd nuclei we have employed the Relativistic Mean Field theory while in case of the ^{154}Gd nucleus we have done calculations with the Skyrme interaction.

Let us also mention several other nuclei that were studied in searching for the experimental evidence of the dynamical symmetries, *e.g.* ^{134}Ba [3], ^{108}Pd [4], ^{128}Xe [5] in the case of E(5) and ^{150}Nd [6], ^{152}Sm [7], ^{126}Ba [8], ^{162}Yb [9] in the case of X(5).

2. Theory

The general scheme of our approach can be summarized as follows. We make HFB calculations with linear constraints on components of a quadrupole mass distribution tensor. Deformation variables β, γ are determined through the mean values of these components by $\beta\cos\gamma = D\langle Q_{20}\rangle$, $\beta\cos\gamma = D\sqrt{3}\langle Q_{22}\rangle$ with $D = \sqrt{\pi/5}/A\langle r^2\rangle$. Then the collective generalized Bohr Hamiltonian containing mass parameters and moments of inertia depending on β and γ is constructed within the frame of Adiabatic Time Dependent HFB theory. Important aspect of our method is that we do not fit any free parameters to obtain energies and $B(\mathrm{E}2)$ transition probabilities. We use (one of) standard RMF or Skyrme force parametrization and we determine a strength of the pairing interaction from experimental mass differences. Moreover, such method gives a clear interpretation of the β, γ collective variables in contrast to phenomenological models like the IBM. Details can be found in [10, 11].

3. Results

3.1. The $^{96-110}$Ru isotopes

In the paper [12] authors discuss a sequence of IBM Hamiltonians describing nuclei in the chain of $^{96-110}$Ru isotopes and then a sequence of potential energy surfaces (PES) obtained through coherent states approach. This sequence represents a transition from spherical (U(5)) to γ unstable shapes (SO(6)) with ^{104}Ru as a tentative critical point with properties close to the E(5) symmetry.

We have studied the same chain of isotopes using the Relativistic Mean Field theory with the NL3 parameters and with a strength of the pairing interaction slightly less (3%) than in [11]. Microscopically calculated potential energy surfaces for $^{96-110}$Ru nuclei exhibit some common features. They are rather flat especially in the γ direction, with shallow minima occurring for nonzero deformation. In Fig. 1 we show for considered nuclei positions of these minima in the β, γ plane and sections of potential surfaces obtained by putting $\gamma = \gamma_{\min}$. Similar qualitative results of PES for some of considered nuclei were obtained also in other microscopic models (*e.g.* [10,13,14]). The full plot of the potential energy for ^{104}Ru is given in Fig. 1 (right panel). Our results support a general qualitative picture of a transition from (almost) spherical ^{96}Ru to deformed heavier Ru isotopes but the ^{104}Ru nucleus does not seem to be an exceptional point.

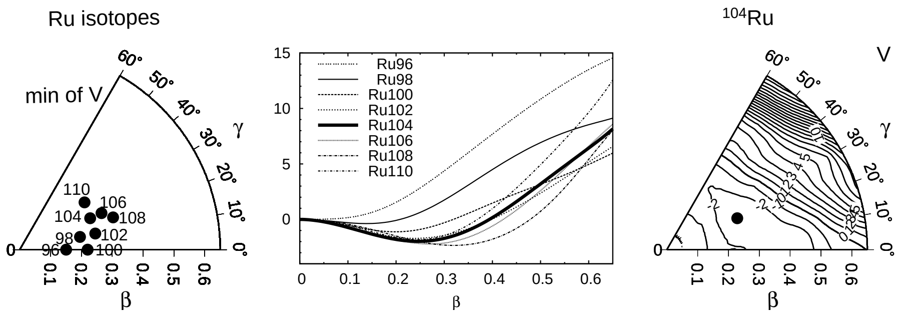

Fig. 1. Position of minima of the potential energy V (left panel), plots of V for $\gamma = \gamma_{\min}$ (middle panel) for $^{96-110}$Ru isotopes and the plot of V for the ^{104}Ru (right panel).

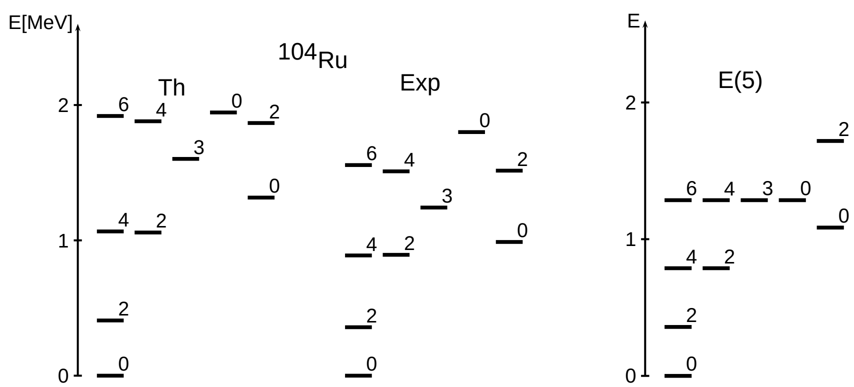

Fig. 2. Theoretical (this work) and experimental energy levels of the ^{104}Ru nucleus. The right panel contains E(5) symmetry results fitted to the energy of the 2_1 level. The levels are grouped into bands in the same manner as in [1].

Calculated energy levels of ^{104}Ru together with experimental ones [14] are shown in Fig. 2. The theoretical spectrum is stretched a bit but an overall agreement is quite good, especially when one keeps in mind that we do not use any free parameters. Comparison with the E(5) predictions shows larger discrepancies than in the case of ^{102}Pd discussed in the next subsection, however one should keep in mind that E(5) symmetry in ^{104}Ru was in some sense predicted in Ref. [12] by studying the chain of Ru isotopes rather than by direct inspection of the energy levels.

3.2. The ^{102}Pd nucleus

The ^{102}Pd nucleus was proposed in [15] as a very good example of the E(5) symmetry. Below we present a plot of calculated and experimental [15] energy levels of ^{102}Pd (Fig. 3). Theoretical method applied now is identical as in the previous subsection. The agreement with the experiment is a bit worse than in the case of ^{104}Ru but still acceptable.

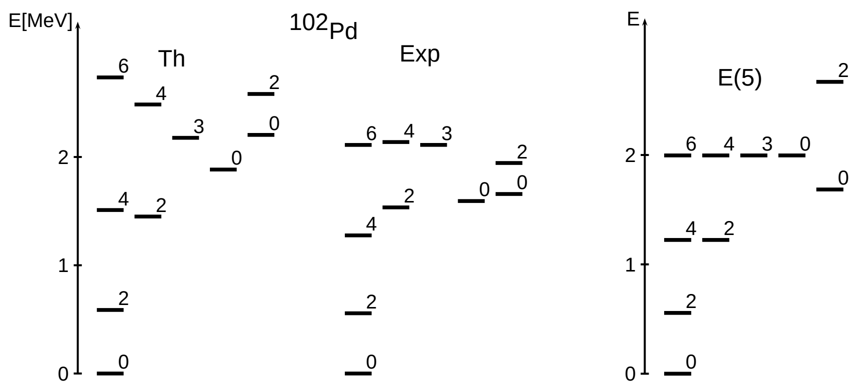

Fig. 3. Energy levels of the ^{102}Pd nucleus, see also caption to Fig. 2.

For ^{102}Pd we present in addition results of calculations of B(E2) probabilities (Table I). In the frame of the microscopic theory they can be obtained without any new parameter (effective charge *etc.*). Table I also contains predictions of the E(5) symmetry (up to a constant factor, fixed by the B(E2) of the $2_1 \to 0_1$ transition). Both E(5) symmetry and microscopic results are in a very good agreement with the experiment.

TABLE I

B(E2) transition probabilities (in W.u.) for ^{102}Pd.

$J_\mathrm{i} \quad J_\mathrm{f}$	Exp	Th	E(5)	$J_\mathrm{i} \quad J_\mathrm{f}$	Exp	Th	E(5)
$2_1 \to 0_1$	33(2)	33.6	33	$4_2 \to 2_1$	3(1)	0.04	0
$4_1 \to 2_1$	51(3)	54.7	55	$4_2 \to 2_2$	45(9)	37.2	38
$2_2 \to 2_1$	15(2)	30.3	55	$0_2 \to 2_1$	$< 4\times10^{-4}$	0.8	0
$2_2 \to 0_1$	2(1)	1.7	0	$0_2 \to 2_2$	96(40)	168.8	73
$4_2 \to 4_1$	< 8	21.7	35	$0_3 \to 2_1$	13(3)	17.6	28

3.3. The ^{154}Gd nucleus

The X(5) symmetry has been proposed to describe transitional nuclei between the spherical vibrator and the axially deformed rotor. One of possible examples of nuclei with X(5) properties is ^{154}Gd [8, 16]. We performed calculations for this nucleus using the SIII Skyrme interaction plus the seniority force in the *p*–*p* channel. As in the previous subsection we present a comparison of theoretical and experimental [16] results for energy levels (Fig. 4) and B(E2) transition probabilities (Table II).

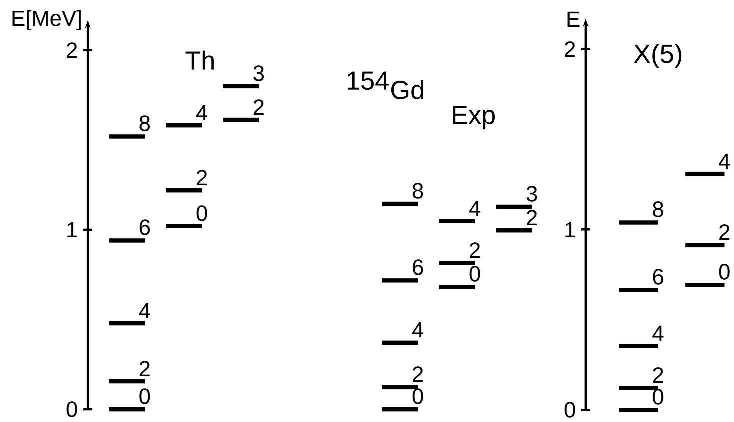

Fig. 4. Theoretical (this work) and experimental energy levels of the ^{154}Gd nucleus. The X(5) symmetry results are fitted to the energy of the 2_1 level. The bands are analogous to presented in [2].

TABLE II

B(E2) transition probabilities (in W.u.) for ^{154}Gd.

J_i J_f	Exp	Th	E(5)	J_i J_f	Exp	Th	E(5)
$2_1 \to 0_1$	157.8(6)	144.7	158	$2_2 \to 0_2$	54.0(36)	159.1	125
$4_1 \to 2_1$	245.2(58)	220.0	250	$\to 4_1$	20.0(11)	16.8	57
$6_1 \to 4_1$	266(7)	262.2	313	$\to 2_1$	6.3(4)	9.0	14
$8_1 \to 6_1$	312(17)	297.1	359	$\to 0_1$	0.9(1)	1.9	3
$0_2 \to 2_1$	45.1(33)	46.1	100	$4_2 \to 2_2$	187(13)	240.1	190
				$\to 6_1$	14.2(9)	18.0	44
				$\to 4_1$	5.0(5)	9.6	9
				$\to 2_1$	0.54(4)	1.1	1.5

One can observe in the present case a larger effect of 'stretching' of the theoretical spectrum. This effect, resulted from too small values of mass parameters, and possible explanations for it were discussed in [10]. Again, theoretical B(E2)'s agree very well with the experimental ones.

4. Conclusions

Results presented in the previous section show that mean field theories describe quite accurately quadrupole collective excitations in nuclei regarded as good examples of the E(5) or X(5) symmetry. However, microscopically calculated potential energies do not comply with assumptions necessary for these symmetries (*e.g.* a square well shape in the β direction). Moreover, microscopic mass parameters (not discussed in the present paper) depend on nuclear deformation, unlike the phenomenological ones. Definite answer to the question if the E(5) and X(5) symmetries are only clever schemes useful to order and classify experimental data or there is a deeper background behind their patterns needs further studies.

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