STUDY OF COSMOLOGICAL MODEL BIANCHI I IN THE CONFORMAL POINCARÉ-GAUGE THEORY OF GRAVITATION

V.V. Karbanovski†, O.V. Nefiodova, Yu.V. Kolesnikova
Yu.N. Limonnikova, A.S. Sorokin

Murmansk State Pedagogical University
Egorova Street 15, 183720 Murmansk, Russia

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The equations for Bianchi I cosmology in the conformal Poincaré-gauge theory of gravitation are considered. It is shown that in case of anisotropy on two directions it is impossible to construct model with a matter satisfying standard physical requirements. The exact vacuum solution corresponding to "pancakes of Zel'dovich" is found.

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1. Introduction

It is known, that the GR equations for the anisotropic models of the Universe (Bianchi I)

$$dS^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + c^2(t)dz^2 \tag{1}$$

have the form (a dot denotes differentiation with respect to t)

$$k\varepsilon = -G^0{}_0 = \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc}\,, \tag{2a}$$

$$kp_1 = G^1{}_1 = -\frac{\ddot{b}}{b} - \frac{\ddot{c}}{c} - \frac{\dot{b}\dot{c}}{bc}\,, \tag{2b}$$

$$kp_2 = G^2{}_2 = -\frac{\ddot{a}}{a} - \frac{\ddot{c}}{c} - \frac{\dot{a}\dot{c}}{ac}\,, \tag{2c}$$

$$kp_3 = G^3{}_3 = -\frac{\ddot{a}}{a} - \frac{\ddot{b}}{b} - \frac{\dot{a}\dot{b}}{ab}\,, \tag{2d}$$

where $G^i{}_k = R^i{}_k - 1/2R\delta^i{}_k$, $\varepsilon = -T^0{}_0$ — the energy density of matter, $p_1 = T^1{}_1$, $p_2 = T^2{}_2$, $p_3 = T^3{}_3$ — its pressures along axes x, y, z respectively.

† e-mail: `Karbanovski_V_V@mail.ru`

As the initial point of cosmological evolution ($t = 0$) corresponds to a minimum of metric functions: $\dot{a}_0 = \dot{b}_0 = \dot{c}_0 = 0$, $\ddot{a}_0 > 0$, $\ddot{b}_0 > 0$, $\ddot{c}_0 > 0$, from (2a) we obtain $\varepsilon_0 = 0$. Then from conditions $0 \le p_i(0) \le \varepsilon (i = 1, 2, 3)$ and expressions $(2b - 2d)$ we find $\ddot{a}_0 = \ddot{b}_0 = \ddot{c}_0 = 0$. However in this case from equation

$$T^{\mu}{}_{0;\mu} = 0 \tag{3}$$

(a semicolon denotes the covariant differentiation) follows that $\dot{\varepsilon}_0 = 0$. Using requirements of the causality principle

$$0 \le \frac{\dot{p}_i}{\dot{\varepsilon}} \le 1$$

and equations (2) and (3) it is possible to prove, that all derivatives of ε and p at $t = 0$ are equal to zero. Thus, the given model can be only vacuum.

However, the (Kasner) vacuum solution for the system (2),

$$dS^2 = -dt^2 + a_0 t^{q_1} dx^2 + b_0 t^{q_2} dy^2 + c_0 t^{q_3} dz^2 \,, \tag{4}$$

is non singular only in a degenerated case $q_1 = q_2 = 0$, $q_3 = 1$ which is reduced to Minkowski Space-Time.

Here we shall consider the equations of the conformal Poincaré-gauge theory of gravitation in the Einstein gauge and in the torsionless limit [1,2]:

$$R^{\mu}{}_{\nu} - \tfrac{1}{2}\delta^{\mu}{}_{\nu} R + \kappa f C^{\mu\alpha}{}_{\nu\beta} R^{\beta}{}_{\alpha} = \kappa T^{\mu}{}_{\nu} \,, \tag{5a}$$

$$f C^{\mu\alpha}{}_{\nu\beta;\mu} = 0 \,, \tag{5b}$$

where $C^{\mu\alpha}{}_{\nu\beta}$ is the Weyl tensor and f — an arbitrary parameter.

For a line element (1) they are reduced to the following system [3]

$$-\kappa\varepsilon = G^0_0 + \kappa f \left(AG^1_1 + BG^2_2 - (A+B)G^3_3\right) \,, \tag{6a}$$

$$\kappa P_1 = G^1_1 + \kappa f \left(AG^0_0 - (A+B)G^2_2 + BG^3_3\right) \,, \tag{6b}$$

$$\kappa P_2 = G^2_2 + \kappa f \left(BG^0_0 - (A+B)G^1_1 + AG^3_3\right) \,, \tag{6c}$$

$$\kappa P_3 = G^3_3 + \kappa f \left(-(A+B)G^0_0 + BG^1_1 + AG^2_2\right) \,, \tag{6d}$$

$$\dot{A} + (2A + B)h_2 + (A - B)h_3 = 0 \,, \tag{7a}$$

$$\dot{B} + (A + 2B)h_1 + (B - A)h_3 = 0 \,, \tag{7b}$$

here $A = C^{01}{}_{01}$, $B = C^{02}{}_{02}$, $h_1 = \frac{\dot{a}}{a}$, $h_2 = \frac{\dot{b}}{b}$, $h_3 = \frac{\dot{c}}{c}$.

In the next section we investigate this problem for material and vacuum equations in case of anisotropy only on two directions, *i.e.* $a = b$.

2. Anisotropic cosmological models in the conformal Poincaré-gauge gravitation

In case $a = b$ equations (6), (7) become

$$\begin{aligned} 3h_1^2 + 2h_1\eta + \kappa f \frac{\gamma}{a^3}\left(2h_1\eta + 3\frac{\gamma}{a^3}\right) &= \kappa\varepsilon\,, &(8a)\\ -2\dot{h}_1 - 2h_1\eta - 3h_1^2 - 3\frac{\gamma}{a^3} + \kappa f\frac{\gamma}{a^3}\left(\dot{h}_1 + h_1\eta + 3\frac{\gamma}{a^3}\right) &= \kappa p_1\,, &(8b)\\ -2\dot{h}_1 - 3h_1^2 + \kappa f\frac{\gamma}{a^3}\left(-2\dot{h}_1 - 3\frac{\gamma}{a^3}\right) &= \kappa p_3\,, &(8c)\\ p_1 &= p_2 &(8d) \end{aligned}$$

$$\dot{\eta} = -\eta^2 - h_1\eta + 3\frac{\gamma}{a^3}\,, \tag{9}$$

where γ is an integration constant of equations (6) and

$$\eta = h_3 - h_1\,. \tag{10}$$

We investigate the possibility of constructing, for this case the model containing the "usual" matter. In particular, for it the condition

$$p_3 \geq 0 \tag{11}$$

should be satisfied. However, during the evolution of the Universe with the finite rate of expansion there should be a moment of time τ, when $\dot{h}_1(\tau) = 0$. In this case, equation (8c) becomes

$$-3h_1^2(\tau) - 3\frac{\kappa f\gamma^2}{a^6(\tau)} = \kappa p_3(\tau)\,. \tag{12}$$

But in [3] it is shown also that for present problem the constant f is positive. Then it is obvious that conditions (11) and (12) are incompatible. Hence, in a considered problem the cosmological model for the "usual" matter cannot be constructed. However, it is necessary to note, that the question about generalization of such conclusion to the case of "full anisotropy" ($a \neq b$) remains open.

We shall consider now the vacuum problem, when $\varepsilon = p_1 = p_3 = 0$. It can play a role of some "background" for material constructions.

In this case solution (8c) has the form

$$\dot{a}^2 = \frac{\kappa f\gamma^2 + C_0 a^2}{a(a^3 + \kappa f\gamma)}\,, \tag{13}$$

here C_0 — a constant of integration.

From (8b) and (8c) we obtain

$$2\left(1+\frac{\kappa f\gamma}{a^3}\right)\left(\frac{\kappa f\gamma}{a^3}-2\right)h_1\eta-9h_1^2\frac{\kappa f\gamma}{a^3}-\frac{6\gamma}{a^3}\left(1+\frac{\kappa f\gamma}{a^3}\right)+\frac{3\kappa f\gamma^2}{a^6}\left(4+\frac{\kappa f\gamma}{a^3}\right)=0\,. \tag{14}$$

Using (8a) and (14), we find

$$\left(h_1^2-\frac{\gamma}{a^3}\right)\left(1-2\frac{\kappa f\gamma}{a^3}\right)=0\,. \tag{15}$$

For $a=(2\kappa f\gamma)^{1/3}$ we have $h_1=0$. Equation (8c) can satisfy this condition only for $\gamma=0$. However, such variant reduces to corresponding problem of GR. Therefore, we shall consider the case

$$h_1^2=\frac{\gamma}{a^3}\,. \tag{16}$$

Then from expressions (13) and (16) we obtain

$$C_0=\kappa f\gamma$$

and

$$\dot{a}^2=\frac{\gamma}{a}\,. \tag{17}$$

For the case $\dot{a}>0$ the solution of equation (17) has the form

$$a^{3/2}=\frac{3}{2}\sqrt{\gamma}t+C_1\,. \tag{18}$$

Substituting (17) into equation (8a), we find

$$\left(\frac{3\gamma}{a^3}+2\frac{\sqrt{\gamma}}{a^{3/2}}\eta\right)\left(1+\frac{\kappa f\gamma}{a^3}\right)=0\,. \tag{19}$$

From (19) it follows

$$\eta=-\frac{3\sqrt{\gamma}}{2a^{3/2}}\,. \tag{20}$$

It is easy to see, that function (20) satisfies equation (9). From formula (10), we obtain

$$c=C_2ae^{\int\eta dt}\,, \tag{21}$$

here C_2 — a constant of integration. Using (17) in (21), we find

$$c=\frac{C_2}{\sqrt{a}}\,. \tag{22}$$

As the result it is possible to write down the expression for the vacuum solution of equations (8) and (9)

$$dS^2 = -dt^2 + \left(C_1 + \frac{3}{2}\sqrt{\gamma}t\right)(dx^2 + dt^2) + C_2\left(C_1 + \frac{3}{2}\sqrt{\gamma}t\right)^{-1/3} dz^2\,. \quad (23)$$

Thus the metric (23), as well as the Kasner solution in a similar problem within GR,

$$dS^2 = -dt^2 + a_0 t^{q_1} dx^2 + b_0 t^{q_2} dy^2 + C_0 t^{q_3} dz^2\,, \quad (24)$$

possess the asymptotical singularity which corresponds to "pancakes of Zel'dovich". It is obvious that solution (23), as a special case ($\gamma = 0$, $C_1 = C_2 = 1$), contains the Minkowski metric. The possibility for generalization of the obtained result to the general case ($a \neq b \neq c$) is now investigated.

3. Conclusion

Thus in the paper:

1. it is shown that in case of anisotropy on two directions it is impossible to construct model with matter satisfying standard physical requirements;
2. the exact vacuum solution corresponding to "pancakes of Zel'dovich" is found.

REFERENCES

[1] B.N. Frolov, *Acta Phys. Pol. B* **9**, 823 (1978).
[2] B.N. Frolov, V.V. Karbanovski, *Phys. Lett.* **A169**, 1 (1992).
[3] V.V. Karbanovski *et al.*, *Acta Phys. Pol. B* **36**, 2143 (2005).