PROBABILISTIC TELEPORTATION OF A TWO-PARTICLE STATE BY TWO THREE-PARTICLE GENERAL W STATES

Dong Li, Xiu Xiao-Ming, Gao Ya-Jun

Department of Physics, Bohai University, Jinzhou 121000, China

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A scheme for teleporting an unknown two-particle state is proposed when two general W states are utilized as quantum channels. In this scheme, besides the sender's Bell state measurements, the recipient need introduce an auxiliary particle, perform Von Neumann measurements and carry out appropriate unitary transformation. Finally, the recipient can realize teleportation with different probabilities of successful teleportaion by selecting different particles to recover the original state. In order to gain the biggest probability of successful teleportation, it is useful to select proper particles to reconstruct the state to be teleported.

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1. Introduction

Quantum teleportation is a process of transmission of an unknown quantum state from one place to another without propagation of the associated physical object through the intervening space. In 1993, an international team of six scientists including Charles Bennett [1] first proposed teleportation of an unknown single particle state via a previously shared Einstein–Podolsky–Rosen (EPR) pair with the help of only two classical bits transmitted through a classical channel. The success of the first experimental teleportation realized by Anton Zeilinger's group of the university of Insbruck in 1997 was the cover story of many journals, including "Scientific American", and even newspapers all around the world. Since then, quantum teleportation is always an interesting topic due to its important applications in quantum computation and quantum communication. A number of feasible schemes for teleporting an unknown atomic state or particle state are presented [2–5]. In these schemes, the quantum channel is represented by a maximally entangled state and the original state can be transferred to a recipient deterministically. Considering that an entangled state may be non-maximally entangled, schemes of teleportation by a non-maximally entangled quantum channel are proposed, such schemes can be realized with a certain probability [6–9].

W state is such a non-maximally entangled quantum state. The virtue of the three-particle W state is that it retains a maximal amount of bipartite entanglement when any one of the three qubits is traced out [10]. And the teleportation using the general W state has been studied in a few ways [11, 12].

In this paper, we present a scheme for teleporting an unknown two-particle state when two general W states are utilized as quantum channels. In this scheme, the recipient needs to introduce an auxiliary particle and perform three times Von Neumann measurements. The maximal probability of the successful teleportation is different if the recipient selects different particles to recover the information of the unknown state. The virtue of the scheme is that we can realize teleportation with the biggest probability by selecting proper particles on which recovering the information.

2. The scheme of teleporting an unknown two-particle state

Supposed that Alice wants to teleport an unknown two-particle state to Bob. The state can be expressed as

$$|\Phi\rangle_{12} = (x_0\,|00\rangle + x_1\,|01\rangle + x_2\,|10\rangle + x_3\,|11\rangle)_{12}\,, \tag{1}$$

where the coefficients are complex number and satisfy normalized condition:

$$|x_0|^2 + |x_1|^2 + |x_2|^2 + |x_3|^2 = 1\,.$$

Two three-particle general W states are shared by Alice and Bob as quantum channels. They can be expressed as:

$$\begin{aligned} |W\rangle_{345} &= (a\,|100\rangle + b\,|010\rangle + c\,|001\rangle)_{345}\,, \\ |W\rangle_{678} &= (d\,|100\rangle + e\,|010\rangle + f\,|001\rangle)_{678}\,. \end{aligned} \tag{2}$$

Without loss of generality, the real coefficients a, b, c, d and f satisfy:

$$\begin{aligned} &|a|^2 + |b|^2 + |c|^2 = 1\,, \qquad |a| > |b| > |c|\,, \\ &|d|^2 + |e|^2 + |f|^2 = 1\,, \qquad |d| > |e| > |f|\,. \end{aligned} \tag{3}$$

Particles $(1, 2, 3, 6)$ belong to Alice, and particles $(4, 5, 7, 8)$ belong to Bob. The state of the system composed of particles $(1, 2, 3, 4, 5, 6, 7, 8)$ is:

$$|\Psi\rangle = |\Phi\rangle_{12} \otimes |W\rangle_{345} \otimes |W\rangle_{678}\,. \tag{4}$$

In order to teleport the state $|\Phi\rangle_{12}$, Alice must carry out the Bell state measurements on particles (1,3) and particles (2,6), respectively. After the two Bell state measurements, the state $|\Phi\rangle$ will be projected into the following sixteen possible results:

$$
\begin{aligned}
{}_{26}\langle\Phi^{\pm}|_{13}\,\langle\Phi^{\pm}\,|\Psi\rangle \;=\; & \tfrac{1}{2}\Big[x_0cf\,|0101\rangle + x_0ce\,|0110\rangle + x_0bf\,|1001\rangle + x_0be\,|1010\rangle \\
& \pm^{2}(x_1cd\,|0100\rangle + x_1bd|1000\rangle) \\
& \pm^{1}(x_2af\,|0001\rangle + x_2ae\,|0010\rangle) \pm^{2}\pm^{1}x_3ad|0000\rangle\Big]_{4578}, \quad (5)\\
{}_{26}\langle\Psi^{\pm}|_{13}\langle\Phi^{\pm}|\Psi\rangle \;=\; & \tfrac{1}{2}\Big[x_0bd\,|1000\rangle + x_0cd\,|0100\rangle \pm^{2}(x_1be\,|1010\rangle \\
& + x_1bf|1001\rangle + x_1ce|0110\rangle + x_1cf|0101\rangle) \\
& \pm^{1}x_2ad\,|0000\rangle \pm^{2}\pm^{1}(x_3ae|0010\rangle + x_3af|0001\rangle)\Big]_{4578}, \quad (6)\\
{}_{26}\langle\Phi^{\pm}|_{13}\langle\Psi^{\pm}|\Psi\rangle \;=\; & \tfrac{1}{2}\Big[x_0ae|0010\rangle + x_0af|0001\rangle \pm^{2} x_1ad|0000\rangle \\
& \pm^{1}(x_2bf\,|1001\rangle + x_2be\,|1010\rangle + x_2cf\,|0101\rangle + x_2ce\,|0110\rangle) \\
& \pm^{2}\pm^{1}(x_3cd\,|0100\rangle + x_3bd\,|1000\rangle)\Big]_{4578}, \quad (7)\\
{}_{26}\langle\Psi^{\pm}|_{13}\langle\Psi^{\pm}|\Psi\rangle \;=\; & \tfrac{1}{2}\Big[x_0ad\,|0000\rangle \pm^{2}(x_1af\,|0001\rangle + x_1ae\,|0010\rangle) \\
& \pm^{1}(x_2cd\,|0100\rangle + x_2bd\,|1000\rangle) \pm^{2}\pm^{1}(x_3bf\,|1001\rangle \\
& + x_3be\,|1010\rangle + x_3cf\,|0101\rangle + x_3ce\,|0110\rangle)\Big]_{4578}, \quad (8)
\end{aligned}
$$

where $\pm^1$ and $\pm^2$ correspond to the superscripts for the Bell state composed of particles $(1, 3)$ and particles $(2, 6)$, respectively.

For instance, if Alice's measurement results are $|\Psi^+\rangle_{13}$ and $|\Phi^-\rangle_{26}$, the corresponding result is:

$$
\begin{aligned}
& \tfrac{1}{2}\Big[x_0ae\,|0010\rangle + x_0af\,|0001\rangle - x_1ad\,|0000\rangle + (x_2bf\,|1001\rangle + x_2be\,|1010\rangle \\
& + x_2cf\,|0101\rangle + x_2ce\,|0110\rangle) - (x_3cd\,|0100\rangle + x_3bd\,|1000\rangle)\Big]_{4578}. \quad (9)
\end{aligned}
$$

After receiving the measurement results of Alice via classical channel, Bob will operate two von Neumann measurements on his particle 5 and particle 8. If the result is $|1\rangle_5$ or $|1\rangle_8$, the teleportation failed. But if the results are $|0\rangle_5$ and $|0\rangle_8$, the state of particles (4,7) will be projected into:

$$
\begin{aligned}
{}_8\langle 0|\,{}_5\langle 0|_{26}\langle\Phi^{\pm}|_{13}\langle\Phi^{\pm}|\Psi\rangle \;=\; & \tfrac{1}{2}(x_0be\,|11\rangle \pm^{2} x_1bd\,|10\rangle \\
& \pm^{1}x_2ae\,|01\rangle \pm^{2}\pm^{1}x_3ad\,|00\rangle)_{47}\,, \quad (10)
\end{aligned}
$$

$$
\begin{aligned}
{}_8\langle 0|_5\langle 0|_{26}\langle \Psi^{\pm}|_{13}\langle \Phi^{\pm}|\Psi\rangle &= \tfrac{1}{2}(x_0 bd\,|10\rangle \pm^2 x_1 be\,|11\rangle \\
&\quad \pm^1 x_2 ad\,|00\rangle \pm^2 \pm^1 x_3 ae\,|01\rangle)_{47}\,, \qquad (11)\\
{}_8\langle 0|_5\langle 0|_{26}\langle \Phi^{\pm}|_{13}\langle \Psi^{\pm}|\Psi\rangle &= \tfrac{1}{2}(x_0 ae\,|01\rangle \pm^2 x_1 ad\,|00\rangle \\
&\quad \pm^1 x_2 be\,|11\rangle \pm^2 \pm^1 x_3 bd\,|10\rangle)_{47}\,, \qquad (12)\\
{}_8\langle 0|_5\langle 0|_{26}\langle \Phi^{\pm}|_{13}\langle \Psi^{\pm}|\Psi\rangle &= \tfrac{1}{2}(x_0 ad\,|00\rangle \pm^2 x_1 ae\,|01\rangle \\
&\quad \pm^1 x_2 bd\,|10\rangle \pm^2 \pm^1 x_3 be\,|11\rangle)_{47}\,. \qquad (13)
\end{aligned}
$$

According to the result, Bob do a unitary transformation U_1 on particles $(4, 7)$. The possible expressions of U_1 are:

$$U_1 = (|0\rangle\langle 1| \pm^1 |1\rangle\langle 0|)_4 \otimes (|0\rangle\langle 1| \pm^2 |1\rangle\langle 0|)_7\,, \qquad (14)$$

$$U_1 = (|0\rangle\langle 1| \pm^1 |1\rangle\langle 0|)_4 \otimes (|0\rangle\langle 0| \pm^2 |1\rangle\langle 1|)_7\,, \qquad (15)$$

$$U_1 = (|0\rangle\langle 0| \pm^1 |1\rangle\langle 1|)_4 \otimes (|0\rangle\langle 1| \pm^2 |1\rangle\langle 0|)_7\,, \qquad (16)$$

$$U_1 = (|0\rangle\langle 0| \pm^1 |1\rangle\langle 1|)_4 \otimes (|0\rangle\langle 0| \pm^2 |1\rangle\langle 1|)_7\,. \qquad (17)$$

They are corresponding to the result of Eqs. (10), (11), (12) and (13), respectively. After that the possible expressions of states of particles $(4, 7)$ are:

$$|\Psi\rangle_{47} = \tfrac{1}{2}(x_0 be\,|00\rangle + x_1 bd\,|01\rangle + x_2 ae\,|10\rangle + x_3 ad\,|11\rangle)_{47}\,, \qquad (18)$$

$$|\Psi\rangle_{47} = \tfrac{1}{2}(x_0 bd\,|00\rangle + x_1 be\,|01\rangle + x_2 ad\,|10\rangle + x_3 ae\,|11\rangle)_{47}\,, \qquad (19)$$

$$|\Psi\rangle_{47} = \tfrac{1}{2}(x_0 ae\,|00\rangle + x_1 ad\,|01\rangle + x_2 be\,|10\rangle + x_3 bd\,|11\rangle)_{47}\,, \qquad (20)$$

$$|\Psi\rangle_{47} = \tfrac{1}{2}(x_0 ad\,|00\rangle + x_1 ae\,|01\rangle + x_2 bd\,|10\rangle + x_3 be\,|11\rangle)_{47}\,. \qquad (21)$$

At the same time, Bob introduces an auxiliary particle a with the initial state $|0\rangle_a$, and performs another unitary transformation U_2 on particles $(4, 7, a)$ under the basis $\{|000\rangle, |010\rangle, |100\rangle, |110\rangle, |001\rangle, |011\rangle, |101\rangle, |111\rangle\}$. The unitary transformation is a 8×8 matrix, and has the form as:

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \qquad (22)$$

where A_1 and A_2 are 4×4 matrices.

$$
\begin{aligned}
A_1 &= \mathrm{diag}\,(a_1, a_2, a_3, a_4)\,,\\
A_2 &= \mathrm{diag}\left(\sqrt{1-a_1^2}, \sqrt{1-a_2^2}, \sqrt{1-a_3^2}, \sqrt{1-a_4^2}\right). \qquad (23)
\end{aligned}
$$

The coefficients a_i which determined by the result of Eqs. (18), (19), (20) and (21) (or Eqs. (5), (6), (7) and (8)) are given in Table I.

TABLE I

The unitary transformation U_2 corresponding to Alice's result as Bob wants to recover the state $|\Phi\rangle_{12}$ on particles $(4, 7)$.

	a_1	a_2	a_3	a_4
Equation (5)	1	$\frac{e}{d}$	$\frac{b}{a}$	$\frac{be}{ad}$
Equation (6)	$\frac{e}{d}$	1	$\frac{be}{ad}$	$\frac{b}{a}$
Equation (7)	$\frac{b}{a}$	$\frac{be}{ad}$	1	$\frac{e}{d}$
Equation (8)	$\frac{be}{ad}$	$\frac{b}{a}$	$\frac{e}{d}$	1

The state of particles $(4, 7, a)$ after the transformation can be expressed as one of the four results:

$$\begin{aligned} |\Psi\rangle_{47a} &= \frac{be}{2}\Big(x_0|00\rangle + x_1|01\rangle + x_2|10\rangle + x_3|11\rangle\Big)_{47}|0\rangle_a + \frac{1}{2}\Big(b\sqrt{d^2-e^2}x_1|01\rangle \\ &+ e\sqrt{a^2-b^2}x_2|10\rangle + \sqrt{a^2d^2-b^2e^2}x_3|11\rangle\Big)_{47}|1\rangle_a\,, \end{aligned} \tag{24}$$

$$\begin{aligned} |\Psi\rangle_{47a} &= \frac{be}{2}\Big(x_0|00\rangle + x_1|01\rangle + x_2|10\rangle + x_3|11\rangle\Big)_{47}|0\rangle_a + \frac{1}{2}\Big(b\sqrt{d^2-e^2}x_0|00\rangle \\ &+ \sqrt{a^2d^2-b^2e^2}x_2|10\rangle + e\sqrt{a^2-b^2}x_3|11\rangle\Big)_{47}|1\rangle_a\,, \end{aligned} \tag{25}$$

$$\begin{aligned} |\Psi\rangle_{47a} &= \frac{be}{2}(x_0|00\rangle + x_1|01\rangle + x_2|10\rangle + x_3|11\rangle)_{47}|0\rangle_a + \frac{1}{2}\Big(e\sqrt{a^2-b^2}x_0|00\rangle\,, \\ &+ \sqrt{a^2d^2-b^2e^2}x_1|01\rangle + b\sqrt{d^2-e^2}x_3|11\rangle\Big)_{47}|1\rangle_a\,, \end{aligned} \tag{26}$$

$$\begin{aligned} |\Psi\rangle_{47a} &= \frac{be}{2}(x_0|00\rangle + x_1|01\rangle + x_2|10\rangle + x_3|11\rangle)_{47}|0\rangle_a \\ &+ \frac{1}{2}\Big(\sqrt{a^2d^2-b^2e^2}x_0|00\rangle + e\sqrt{a^2-b^2}x_1|01\rangle + b\sqrt{d^2-e^2}x_2|10\rangle\Big)_{47}|1\rangle_a\,. \end{aligned} \tag{27}$$

Finally, Bob measures the auxiliary particle. If $|0\rangle_a$ is obtained, the teleportation succeed, and the probability of successful teleportation is $16 \times |(be)/2|^2 = 4|be|^2$. Now Bob recover the state $|\Phi\rangle_{12}$ on particles $(4, 7)$ with probability $4\,|be|^2$.

TABLE II

The unitary transformation U_2 corresponding to Alice's result as Bob wants to recover the state $|\Phi\rangle_{12}$ on particles $(5, 8)$.

	a_1	a_2	a_3	a_4
Equation (5)	1	$\frac{f}{d}$	$\frac{c}{a}$	$\frac{cf}{ad}$
Equation (6)	$\frac{f}{d}$	1	$\frac{cf}{ad}$	$\frac{c}{a}$
Equation (7)	$\frac{c}{a}$	$\frac{cf}{ad}$	1	$\frac{f}{d}$
Equation (8)	$\frac{cf}{ad}$	$\frac{c}{a}$	$\frac{f}{d}$	1

TABLE III

The unitary transformation U_2 corresponding to Alice's result as Bob wants to recover the state $|\Phi\rangle_{12}$ on particles $(4, 8)$.

	a_1	a_2	a_3	a_4
Equation (5)	1	$\frac{f}{d}$	$\frac{b}{a}$	$\frac{bf}{ad}$
Equation (6)	$\frac{f}{d}$	1	$\frac{bf}{ad}$	$\frac{b}{a}$
Equation (7)	$\frac{b}{a}$	$\frac{bf}{ad}$	1	$\frac{f}{d}$
Equation (8)	$\frac{bf}{ad}$	$\frac{b}{a}$	$\frac{f}{d}$	1

TABLE IV

The unitary transformation U_2 corresponding to Alice's result as Bob wants to recover the state $|\Phi\rangle_{12}$ on particles $(5, 7)$.

	a_1	a_2	a_3	a_4
Equation (5)	1	$\frac{e}{d}$	$\frac{c}{a}$	$\frac{ce}{ad}$
Equation (6)	$\frac{e}{d}$	1	$\frac{ce}{ad}$	$\frac{c}{a}$
Equation (7)	$\frac{c}{a}$	$\frac{ce}{ad}$	1	$\frac{e}{d}$
Equation (8)	$\frac{ce}{ad}$	$\frac{c}{a}$	$\frac{e}{d}$	1

It can be proved that Bob also can recover the state $|\Phi\rangle_{12}$ on particles $(5,8)$ with probability $4\,|cf|^2$, on particles $(4,8)$ with probability $4|bf|^2$, on particles $(5,7)$ with probability $4|ce|^2$. Similarly, Bob must do two Von Neumann measurements on particle 4 and particle 7, particle 5 and particle 7 or particle 4 and particle 8, and do a unitary transformation U_1 given in equations (14)–(17) on particles $(5,8)$, $(4,8)$ or $(5,7)$, respectively. But in these cases, the unitary transformations U_2 have to change a little. The corresponding expressions of U_2 are given in Table II, Table III and Table IV.

3. Discussions and conclusions

From the above analysis, it can be seen that Bob can recover the two-particle state to be teleported with different probabilities by selecting different particles to receive the state. If Bob selects particles $(5,8)$ to recover the state, the probability of successful teleportation is $4\,|be|^2$. If Bob selects particles $(4,7)$, $(4,8)$ or $(5,7)$, the probability is $4|cf|^2$, $4|bf|^2$ or $4|ce|^2$, respectively. Considering the hypothesis $|a| > |b| > |c|$, $|d| > |e| > |f|$, if he wants to realize teleportation with the biggest probability, Bob must select particles $(5,8)$ to recover the state to be teleported.

It is obvious that the maximal probability of successful teleportation will approach 1 when the parameter $|c|$ and $|f|$ of the quantum channel $|W\rangle_{345} \otimes |W\rangle_{678}$ are small enough and $|b|$ approximates to $|a|$, and $|e|$ approximates to $|d|$.

REFERENCES

[1] C.H. Bennett, G. Brassard, C. Crepeau *et al.*, *Phys. Rev. Lett.* **70**, 1895 (1993).
[2] C.P. Yang, G.C. Guo, *Chin. Phys. Lett.* **16**, 628 (1999).
[3] J.X. Fang, S.Q. Zhu, X.F. Chen *et al.*, *Commun. Theor. Phys.* **41**, 369 (2004).
[4] M. Cao, S.Q. Zhu, J X. Fang, *Commun. Theor. Phys.* **41**, 689 (2004).
[5] Y.B. Zhan, *Chin. Phys.* **13**, 1801 (2004).
[6] W.L. Li, C.F. Li, G.C. Guo, *Phys. Rev.* **A61**, 034301 (2000).
[7] H. Lu, G.C.Guo, *Phys. Lett.* **A276**, 209 (2000).
[8] T.J. Gu, Y.Z. Zheng, G.C. Guo, *Chin. Phys. Lett.* **18**, 1543 (2001).
[9] F.L. Yan, H.W. Ding, *Chin. Phys. Lett.* **23**, 17 (2006).
[10] W. Dür, G. Vidal, J.I. Cirac, *Phys. Rev.* **A62**, 062314 (2000).
[11] B.S. Shi, A. Tomita, *Phys. Lett.* **A296**, 161 (2002).
[12] H.Y. Dai, P.X. Chen, C.Z. Li, *J. Opt. B: Quantum Semiclass* **6**, 106 (2004).