LEPTON FLAVOR VIOLATING $Z \to l_1^+ l_2^-$ DECAYS WITH THE LOCALIZED NEW HIGGS DOUBLET IN THE EXTRA DIMENSION

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We predict the branching ratios of $Z \to e^\pm \mu^\pm$, $Z \to e^\pm \tau^\pm$ and $Z \to \mu^\pm \tau^\pm$ decays in the framework of the 2HDM with the inclusion of one and two extra dimensions, by considering that the new Higgs doublet is localized in the extra dimension with a Gaussian profile. We observe that their BRs are at the order of the magnitude of 10^{-10}, 10^{-8} and 10^{-5} with the inclusion of a single extra dimension, in the given range of the free parameters. These numerical values are slightly suppressed in the case that the localization points of new Higgs scalars are different than origin.

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1. Introduction

The lepton flavor violating (LFV) interactions are sensitive to the physics beyond the standard model (SM) and they are rich theoretically since they exist at least in the one loop level. The Z decays with different lepton flavor outputs, such as $Z \to e\mu$, $Z \to e\tau$ and $Z \to \mu\tau$, are among the candidates of LFV decays and they are clean in the sense that they are free from the long distance effects. In the literature, there is an extensive work on these decays [1–14]. The theoretical studies on such Z decays have been stimulated by the Giga-Z option of the Tesla project which aims to increase the production of Z bosons at resonance.

In the framework of the SM the lepton flavor is conserved and its extension with massive neutrinos, so called νSM model, permits the LFV interactions with the lepton mixing mechanism [15]. However, in this model, the theoretical predictions of the branching ratios (BRs) of these LFV Z decays

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are extremely small when the internal neutrinos are light [1, 2]

$$\begin{aligned} \mathrm{BR}\,(Z \to e^{\pm}\mu^{\pm}) \sim \mathrm{BR}\,(Z \to e^{\pm}\tau^{\pm}) \;&\sim\; 10^{-54}\,, \\ \mathrm{BR}\,(Z \to \mu^{\pm}\tau^{\pm}) \;&<\; 4\times 10^{-60}. \end{aligned} \tag{1}$$

These numbers are far from the experimental limits obtained at LEP 1 [3]:

$$\begin{aligned} \mathrm{BR}\,(Z \to e^{\pm}\mu^{\pm}) \;&<\; 1.7\times 10^{-6} \;\; [4]\,, \\ \mathrm{BR}\,(Z \to e^{\pm}\tau^{\pm}) \;&<\; 9.8\times 10^{-6} \;\; [4,5]\,, \\ \mathrm{BR}\,(Z \to \mu^{\pm}\tau^{\pm}) \;&<\; 1.2\times 10^{-5} \;\; [4,6] \end{aligned} \tag{2}$$

and from the improved ones at Giga-Z [7]:

$$\begin{aligned} \mathrm{BR}\,(Z \to e^{\pm}\mu^{\pm}) \;&<\; 2\times 10^{-9}\,, \\ \mathrm{BR}\,(Z \to e^{\pm}\tau^{\pm}) \;&<\; f\times 6.5\times 10^{-8}\,, \\ \mathrm{BR}\,(Z \to \mu^{\pm}\tau^{\pm}) \;&<\; f\times 2.2\times 10^{-8} \end{aligned} \tag{3}$$

with $f = 0.2-1.0$. Here the BRs are obtained for the decays $Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1$, namely,

$$\mathrm{BR}\,(Z \to l_1^{\pm} l_2^{\pm}) = \frac{\Gamma(Z \to \bar{l}_1 l_2 + \bar{l}_2 l_1)}{\Gamma_Z}\,. \tag{4}$$

The extensions of νSM with one heavy ordinary Dirac neutrino [2], and with two heavy right-handed singlet Majorana neutrinos [2] ensure to enhance the BRs of the corresponding LFV Z decays. The possible enhancements in their BRs have been obtained in other models; in the Zee model [8], the two Higgs doublet model (2HDM), without (with) the inclusion of the extra dimension [9]([10]), the supersymmetric models [11, 12] and the topcolor assisted technicolor model [13].

In this work, we study the LFV processes $Z \to e^{\pm}\mu^{\pm}$, $Z \to e^{\pm}\tau^{\pm}$ and $Z \to \mu^{\pm}\tau^{\pm}$ in the framework of the 2HDM with the inclusion of a single (two) extra dimension(s). Here the LFV interactions are induced by the internal new neutral Higgs bosons h^0 and A^0 at least in the one loop level. The extension of the Higgs sector brings new contribution to the BRs of the considered decays. On the other hand, the inclusion the extra dimensions enhances the BRs since the particle spectrum is extended after the compactification of the extra dimensions.

The extra dimension idea was originated from the study of Kaluza–Klein [16] which was related to the unification of electromagnetism and the gravity and the motivation increased with the study on the string theory which was formulated in a space-time of more than four dimensions. Since the extra dimensions are hidden to the experiments at present, the most favorable description is the compactification these new dimensions to the surfaces with small radii. In the case of that the extra dimensions are at the order of submilimeter distance, for two extra dimensions, the hierarchy problem in the fundamental scales could be solved and the true scale of quantum gravity would be no more the Planck scale but in the order of electroweak (EW) scale [17, 18].

The effects of extra dimensions on various physical processes have been studied in the literature extensively [17–43]. In the extra dimension scenarios, the compactification procedure causes to appear new particles, namely Kaluza–Klein (KK) modes in the theory. If all the fields feel the extra dimensions, so called universal extra dimensions (UED), the extra dimensional momentum, therefore the KK number at each vertex, is conserved. If some fields feel the extra dimensions but not all in the theory, those extra dimensions are called non-universal extra dimensions and this is the case where the KK number at vertices is not conserved. The non-conservation of the KK number at the vertex results in the possibility of the existence of the tree level interaction of KK modes with the ordinary particles. In another scenario, some fields are considered to be localized in the extra dimension(s). In the split fermion scenario [32–39], the fermions are assumed at different points in the extra dimension with Gaussian profiles and this ensures a possible solution to the hierarchy of fermion masses by considering the overlaps of fermion wave functions in the extra dimensions. The localization of the Higgs doublet in the extra dimension has been considered in [40], by introducing an additional localizer field. In [41] the branching ratios of the radiative LFV decays have been studied in the split fermion scenario, with the assumption that the new Higgs doublet is restricted to the 4D brane or to a part of the bulk in one and two extra dimensions, in the framework of the 2HDM. [42] is devoted to analysis of the BRs of the radiative LFV decays in the case that the new Higgs scalars were localized in the extra dimension with the help of the localizer field and the SM Higgs was considered to have a constant profile. In the recent work [43], the radiative LFV decays were studied with the assumption that the new Higgs doublet was localized in the extra dimension with a Gaussian profile, by an unknown mechanism, however, the other particle zero modes have uniform profile in the extra dimension.

The present work is devoted to the BRs of the LFV Z decays in the 2HDM, with the inclusion of one and two extra dimensions by considering that the new Higgs doublet is localized in the extra dimension with a Gaussian profile, by an unknown mechanism, however, the other particle zero modes have uniform profile in the extra dimension. First, we assume that the new Higgs doublet is localized around origin and, second, we take the localization point as different than the origin but near to that. We observe that the BRs of the LFV Z decays $Z \to e^{\pm}\mu^{\pm}$, $Z \to e^{\pm}\tau^{\pm}$ and $Z \to \mu^{\pm}\tau^{\pm}$ reach to the values at the order of the magnitude of 10^{-10}, 10^{-8} and 10^{-5} with the inclusion of a single extra dimension, in the given range of the free parameters. These numerical values are slightly suppressed in the case that the localization points of new Higgs scalars are different than origin.

The paper is organized as follows: In Section 2, we present the effective vertex and the BRs of LFV Z decays in the 2HDM with the inclusion of extra dimensions. Section 3 is devoted to discussion and our conclusions. In appendix section, we give the explicit expressions of the factors appearing in the effective vertex.

2. The effect of the localization of the new Higgs doublet on the lepton flavor violating $Z \to l_1^+ l_2^-$ decays in the framework of the two Higgs doublet model.

The LFV Z boson decays $Z \to l_1^- l_2^+$ exist at least in the one loop level and, therefore, the theoretical values of the BRs are extremely small in the SM. With the extension of the Higgs sector in which the flavor changing neutral current (FCNC) at tree level is permitted, there appear additional contributions to the BRs of the LFV processes. The multi Higgs doublet models are among the candidates for such models and, in the present work, we consider the 2HDM with FCNC at tree level. The inclusion of the the spatial extra dimension further enhances the BRs, since the particle spectrum is extended after its compactification and the KK modes of the fields which are accessible to the extra dimension bring additional contributions. Here, the idea is to consider that the new Higgs scalars are localized in the extra dimension, with Gaussian profiles, by an unknown mechanism, and, the other particles have constant zero mode profiles in the extra dimension.

The Yukawa Lagrangian responsible for the LFV interactions in a single extra dimension reads,

$$\mathcal{L}_{\rm Y} = \xi^{E}_{5\,ij} \bar{l}_{iL} \phi_2 E_{jR} + {\rm h.c.}\,, \tag{5}$$

where L and R denote chiral projections, $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_2 is the new scalar doublet and $\xi^{E}_{5\,ij}$ are the FV Yukawa couplings in five dimensions, where i, j are family indices of leptons, l_i and E_j are lepton doublets and singlets respectively. These fields are the functions of x^μ and y, where y is the coordinate represents the fifth dimension. Here we choose the Higgs doublets ϕ_1 and ϕ_2 as

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array} \right) + \left(\begin{array}{c} \sqrt{2} \chi^+ \\ i \chi^0 \end{array} \right) \right] ; \quad \phi_2 = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \sqrt{2} H^+ \\ H_1 + i H_2 \end{array} \right) , \tag{6}$$

and their vacuum expectation values as

$$\langle \phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} ; \quad \langle \phi_2 \rangle = 0 \,. \tag{7}$$

This choice makes it possible to collect the SM (new) particles in the first (second) doublet and H_1 and H_2 becomes the mass eigenstates h^0 and A^0, respectively since no mixing occurs between two CP-even neutral bosons H^0 and h^0 at tree level.

The five dimensional lepton doublets and singlets and the SM Higgs field are expanded into their KK modes with the compactification of the extra dimension on an orbifold S^1/Z_2 with radius R and they read

$$\begin{aligned}
\phi_1(x,y) &= \frac{1}{\sqrt{2\pi R}} \left\{ \phi_1^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \phi_1^{(n)}(x) \cos\left(\frac{ny}{R}\right) \right\} , \\
l_i(x,y) &= \frac{1}{\sqrt{2\pi R}} \\
&\times \left\{ l_{iL}^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[l_{iL}^{(n)}(x) \cos\left(\frac{ny}{R}\right) + l_{iR}^{(n)}(x) \sin\left(\frac{ny}{R}\right) \right] \right\} , \\
E_i(x,y) &= \frac{1}{\sqrt{2\pi R}} \\
&\times \left\{ E_{iR}^{(0)}(x) + \sqrt{2} \sum_{n=1}^{\infty} \left[E_{iR}^{(n)}(x) \cos\left(\frac{ny}{R}\right) + E_{iL}^{(n)}(x) \sin\left(\frac{ny}{R}\right) \right] \right\} ,
\end{aligned} \tag{8}$$

where $\phi_1^{(0)}(x)$, $l_{iL}^{(0)}(x)$ and $E_{iR}^{(0)}(x)$ are the four dimensional Higgs doublet, lepton doublets and lepton singlets respectively. On the other hand the new Higgs scalar profiles read,

$$S(x,y) = A e^{-\beta y^2} S(x) \,, \tag{9}$$

and the mechanism behind the localization is unknown[1]. Here the normalization constant A is

$$A = \frac{(2\,\beta)^{1/4}}{\pi^{1/4} \sqrt{\mathrm{Erf}[\sqrt{2\,\beta}\,\pi\, R]}} \,. \tag{10}$$

[1] We consider the zero mode Higgs scalars, h^0, A^0, and we do not take into account the possible KK modes of Higgs scalars since the mechanism for the localization is unknown and we expect that the those contributions are small due to their heavy masses.

The strength of the localization of the new Higgs doublet in the extra dimension is regulated by the parameter $\beta = 1/\sigma^2$, where σ, $\sigma = \rho\, R$, is the Gaussian width of $S(x,y)$ in the extra dimension. Here the function $\mathrm{Erf}[z]$ is the error function, which is defined as

$$\mathrm{Erf}[z] = \frac{2}{\sqrt{\pi}} \int\limits_0^z e^{-t^2}\, dt\,. \tag{11}$$

The modified Yukawa interactions in four dimensions can be obtained by integrating the combination $\bar{f}^{(0(n))}_{iL\,(R)}(x,y)\, S(x,y)\, f^{(n(0))}_{jR\,(L)}(x,y)$, appearing in the Lagrangian (Eq. (5)), over the fifth dimension as

$$I = \int\limits_{-\pi R}^{\pi R} dy\;\, \bar{f}^{(0(n))}_{iL\,(R)}(x,y)\, S(x,y)\, f^{(n(0))}_{jR\,(L)}(x,y)\,, \tag{12}$$

where $f^{(n(0))}_{jR\,(L)}$ are the KK basis (zero-mode) for lepton fields (Eq. (8)), and finally, we get

$$I = V_n\, \bar{f}^{(0(n))}_{iL\,(R)}(x)\, S(x)\;\, f^{(n(0))}_{jR\,(L)}(x)\,, \tag{13}$$

with the factor V_n

$$V_n = A\, c_n\,, \tag{14}$$

and the function A which is defined in Eq. (10). The function c_n in Eq. (14) is obtained as

$$c_n = e^{-n^2/4\,\beta\, R^2}\, \frac{\left(\mathrm{Erf}\left[\frac{i\,n+2\,\beta\,\pi\,R^2}{2\,\sqrt{\beta}\,R}\right] + \mathrm{Erf}\left[\frac{-i\,n+2\,\beta\,\pi\,R^2}{2\,\sqrt{\beta}\,R}\right]\right)}{4\,\sqrt{\beta\,\pi}\,R}\,. \tag{15}$$

Notice that the factor A is embedded into the definition of the the Yukawa couplings ξ^E_{ij} in four dimensions as

$$\xi^E_{ij} = A\, \xi^E_{5\,ij}\,, \tag{16}$$

where $\xi^E_{5\,ij}$ are the Yukawa couplings in five dimensions (see Eq. (5)).

In the following, we consider that the new Higgs scalars are localized in the extra dimension at the point y_{H}, $y_{\mathrm{H}} = \alpha\,\sigma$ near to the origin, namely,

$$S(x,y) = A_{\mathrm{H}}\, e^{-\beta(y-y_{\mathrm{H}})^2}\, S(x) \tag{17}$$

with the normalization constant

$$A_{\rm H} = \frac{2\,(\beta)^{1/4}}{(2\pi)^{1/4}\sqrt{{\rm Erf}[\sqrt{2\,\beta}\,(\pi\,R+y_{\rm H})]+{\rm Erf}[\sqrt{2\,\beta}\,(\pi\,R-y_{\rm H})]}}\,. \tag{18}$$

The integration of the combination $\bar{f}^{(0(n))}_{iL\,(R)}(x,y)\,S(x,y)\,f^{(n(0))}_{jR\,(L)}(x,y)$ over extra dimension brings the factor V_n appearing in Eq. (13) as

$$V_n = A_{\rm H}\,c_n\,, \tag{19}$$

where $A_{\rm H}$ is the normalization constant defined in Eq. (18) and the function c_n reads

$$c_n = e^{-n^2/4\,\beta\,R^2}\,\cos\left[\frac{n\,y_{\rm H}}{R}\right]\,\frac{\left({\rm Erf}\left[\frac{i\,n+2\,\beta\,\pi\,R^2}{2\,\sqrt{\beta}\,R}\right]+{\rm Erf}\left[\frac{-i\,n+2\,\beta\,\pi\,R^2}{2\,\sqrt{\beta}\,R}\right]\right)}{4\,\sqrt{\beta\,\pi}\,R}\,. \tag{20}$$

Similar to the previous case, we define the Yukawa couplings in four dimensions as

$$\xi^E_{ij} = A_{\rm H}\,\xi^E_{5\,ij}\,. \tag{21}$$

Now, we would like to make the same analysis in the case of two spatial extra dimensions. The six dimensional lepton doublets and singlets and the SM Higgs fields are expanded into their KK modes with the compactification of the extra dimension on an orbifold $(S^1\times S^1)/Z_2$, with radius R and they read

$$\begin{aligned}
\phi_1(x,y,z) &= \frac{1}{2\pi R}\left\{\phi_1^{(0,0)}(x)+2\sum_{n,s}\phi_1^{(n,s)}(x)\cos\left(\frac{ny}{R}+\frac{sz}{R}\right)\right\}\,,\\
l_i(x,y,z) &= \frac{1}{2\pi R}\left\{l_{iL}^{(0,0)}(x)+2\sum_{n,s}\left[l_{iL}^{(n,s)}(x)\cos\left(\frac{ny}{R}+\frac{sz}{R}\right)\right.\right.\\
&\qquad\left.\left.+l_{iR}^{(n,s)}(x)\sin\left(\frac{ny}{R}+\frac{sz}{R}\right)\right]\right\},\\
E_i(x,y,z) &= \frac{1}{2\pi R}\left\{E_{iR}^{(0,0)}(x)+2\sum_{n,s}\left[E_{iR}^{(n,s)}(x)\cos\left(\frac{ny}{R}+\frac{sz}{R}\right)\right.\right.\\
&\qquad\left.\left.+E_{iL}^{(n,s)}(x)\sin\left(\frac{ny}{R}+\frac{sz}{R}\right)\right]\right\},
\end{aligned} \tag{22}$$

where $\phi_1^{(0,0)}(x)$, $l_{iL}^{(0,0)}(x)$ and $E_{iR}^{(0,0)}(x)$ are the four dimensional Higgs doublet, lepton doublets and lepton singlets respectively. Here the summation is done over the indices n, s but both are not zero at the same time. Similar to a single extra dimension case, the new Higgs scalar profiles read,

$$S(x,y,z) = A' e^{-\beta(y^2+z^2)} S(x)\,, \tag{23}$$

and the mechanism behind its localization is unknown. Here, the normalization constant A' is

$$A' = \frac{(2\,\beta)^{1/2}}{\pi^{1/2}\,\mathrm{Erf}[\sqrt{2\,\beta}\,\pi\,R]}\,. \tag{24}$$

The modified Yukawa interactions in four dimensions can be obtained by integrating the combination $\bar{f}_{iL\,(R)}^{(0,0(n,s))}(x,y)\,S(x,y,z)\,f_{jR\,(L)}^{(n,s(0,0))}(x,y)$ over the fifth and sixth dimensions:

$$I = \int_{-\pi R}^{\pi R} dy \int_{-\pi R}^{\pi R} dz\ \bar{f}_{iL\,(R)}^{(0,0(n,s))}(x,y,z)\,S(x,y,z)\,f_{jR\,(L)}^{(n,s(0,0))}(x,y,z)\,, \tag{25}$$

where

$$I = V_{n,s}\,\bar{f}_{iL\,(R)}^{(0,0(n,s))}(x)\,S(x)\ f_{jR\,(L)}^{(n,s(0,0))}(x)\,, \tag{26}$$

with the factor $V_{n,s}$

$$V_{n,s} = A'\,c_{n,s}\,, \tag{27}$$

and the function A' which is defined in Eq. (24). The function $c_{n,s}$ in Eq. (27) is obtained as:

$$c_{n,s} = e^{-n^2+m^2/4\,\beta\,R^2} \times \frac{\left(\mathrm{Erf}\left[\frac{in+2\beta\pi R^2}{2\sqrt{\beta}R}\right]+\mathrm{Erf}\left[\frac{-in+2\beta\pi R^2}{2\sqrt{\beta}R}\right]\right)\left(\mathrm{Erf}\left[\frac{is+2\beta\pi R^2}{2\sqrt{\beta}R}\right]+\mathrm{Erf}\left[\frac{-is+2\beta\pi R^2}{2\sqrt{\beta}R}\right]\right)}{16\beta\pi R^2}\,. \tag{28}$$

Here the Yukawa couplings ξ_{ij}^E in four dimensions read

$$\xi_{ij}^E = A'\,\xi_{6\,ij}^E\,, \tag{29}$$

where $\xi_{6\,ij}^E$ are the Yukawa couplings in six dimensions.

The internal neutral Higgs particles h^0 and A^0 play the main role in the existence of the $Z \to l_1^- l_2^+$ decay, theoretically. In Fig. 1 the necessary 1-loop diagrams, the self energy and vertex diagrams, are given. The inclusion of extra dimensions brings additional lepton KK mode contributions. The general effective vertex for the interaction of on-shell Z-boson with a fermionic current reads

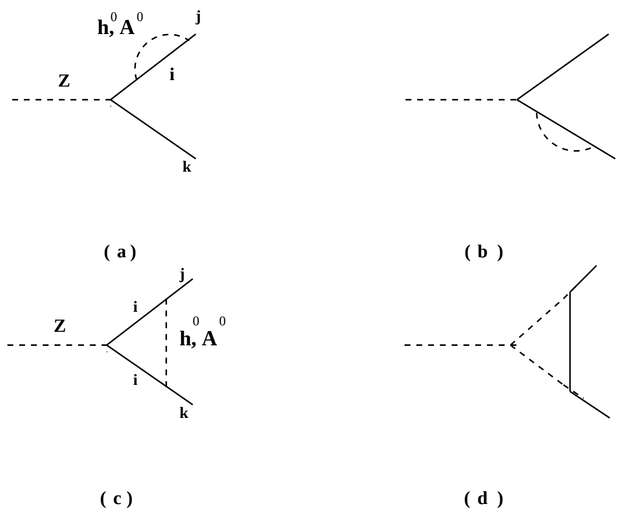

Fig. 1. One loop diagrams contribute to $Z \to k^+ j^-$ decay due to the neutral Higgs bosons h_0 and A_0 in the 2HDM. i represents the internal, j (k) outgoing (incoming) lepton, dashed lines the vector field Z, h_0 and A_0 fields. In 5 (6) dimensions there exist also the KK modes of lepton and Higgs fields.

$$\Gamma_\mu = \gamma_\mu (f_{\rm V} - f_{\rm A}\, \gamma_5) + \frac{i}{m_W}\, (f_{\rm M} + f_{\rm E}\, \gamma_5)\, \sigma_{\mu\,\nu}\, q^\nu \,, \tag{30}$$

where q is the momentum transfer, $q^2 = (p - p')^2$, $f_{\rm V}$ ($f_{\rm A}$) is vector (axial-vector) coupling, $f_{\rm M}$ ($f_{\rm E}$) magnetic (electric) transitions of unlike fermions. Here p ($-p'$) is the four momentum vector of lepton (anti-lepton). The vector (axial-vector) $f_{\rm V}$ ($f_{\rm A}$) couplings and the magnetic (electric) transitions $f_{\rm M}$ ($f_{\rm E}$) including the contributions coming from a single extra dimension can be obtained as

$$\begin{aligned} f_{\rm V} &= \sum_{i=1}^{3} \left(f_{i{\rm V}}^{(0)} + 2 \sum_{n=1}^{\infty} f_{i{\rm V}}^{(n)} \right) , \\ f_{\rm A} &= \sum_{i=1}^{3} \left(f_{i{\rm A}}^{(0)} + 2 \sum_{n=1}^{\infty} f_{i{\rm A}}^{(n)} \right) , \end{aligned}$$

$$
\begin{aligned}
f_{\rm M} &= \sum_{i=1}^{3}\left(f_{i\rm M}^{(0)}+2\sum_{n=1}^{\infty}f_{i\rm M}^{(n)}\right),\\
f_{\rm E} &= \sum_{i=1}^{3}\left(f_{i\rm E}^{(0)}+2\sum_{n=1}^{\infty}f_{i\rm E}^{(n)}\right),
\end{aligned}
\tag{31}
$$

where $f^{(0)}_{i({\rm V,A,M,E})}$ are the couplings without lepton KK mode contributions and they can be calculated by taking $n=0$ in Eq. (A.4). On the other hand the couplings $f^{(n)}_{i({\rm V,A,M,E})}$ are the ones due to the KK modes of the leptons (see Eq. (A.4)). Here the summation over the index i represents the sum due to the internal lepton flavors, namely, e,μ,τ. We present $f^{(n)}_{i({\rm V,A,M,E})}$ in the Appendix, by taking into account all the masses of internal leptons and external lepton (anti-lepton). If we consider two extra dimensions where all the particles are accessible, the couplings $f^{(n)}_{i({\rm V,A,M,E})}$ appearing in Eq. (31) should be replaced by $f^{(n,s)}_{i({\rm V,A,M,E})}$ and they read

$$
\begin{aligned}
f_{\rm V} &= \sum_{i=1}^{3}\left(f_{i\rm V}^{(0,0)}+4\sum_{n,s}^{\infty}f_{i\rm V}^{(n,s)}\right),\\
f_{\rm A} &= \sum_{i=1}^{3}\left(f_{i\rm A}^{(0,0)}+4\sum_{n,s}^{\infty}f_{i\rm A}^{(n,s)}\right),\\
f_{\rm M} &= \sum_{i=1}^{3}\left(f_{i\rm M}^{(0,0)}+4\sum_{n,s}^{\infty}f_{i\rm M}^{(n,s)}\right),\\
f_{\rm E} &= \sum_{i=1}^{3}\left(f_{i\rm E}^{(0,0)}+4\sum_{n,s}^{\infty}f_{i\rm E}^{(n,s)}\right),
\end{aligned}
\tag{32}
$$

where the summation would be done over $n,s=0,1,2\ldots$ except $n=s=0$ (see appendix for their explicit forms).

Finally, the BR for $Z\to l_1^-\,l_2^+$ can be written in terms of the couplings $f_{\rm V}$, $f_{\rm A}$, $f_{\rm M}$ and $f_{\rm E}$ as

$$
{\rm BR}(Z\to l_1^- l_2^+)=\frac{1}{48\pi}\frac{m_Z}{\Gamma_Z}\left\{|f_{\rm V}|^2+|f_{\rm A}|^2+\frac{1}{2\cos^2\theta_W}(|f_{\rm M}|^2+|f_{\rm E}|^2)\right\},
\tag{33}
$$

where Γ_Z is the total decay width of Z boson. In our numerical analysis we consider the BR due to the production of sum of charged states, namely

$$
{\rm BR}\,(Z\to l_1^\pm\,l_2^\pm)=\frac{\Gamma(Z\to(\bar{l}_1\,l_2+\bar{l}_2\,l_1)}{\Gamma_Z}\,.
\tag{34}
$$

3. Discussion

The LFV Z decays $Z \to l_1^{\pm} l_2^{\pm}$, $l_1 \neq l_2$, are rare decays in the sense that they exist at least in the one loop level and they are rich theoretically since the physical parameters of these decays contain number of free parameters of the model used. In the framework of the 2HDM the internal leptons and new scalar bosons drive the interaction and the corresponding physical quantities are sensitive to the Yukawa couplings[2] $\bar{\xi}^{E}_{N,ij}$, $i,j = e,\mu,\tau$, which are among the free parameters of the model. These couplings should be restricted by using present and forthcoming experiments. Here, we assume that the couplings which contain τ index are dominant respecting the Cheng-Sher scenario [44] and, therefore, we consider only the internal τ lepton in the loop diagrams. In addition to this, we take the Yukawa couplings $\bar{\xi}^{E}_{N,ij}$ as symmetric with respect to the indices i and j. As a result, among the Yukawa couplings, we need the numerical values for $\bar{\xi}^{E}_{N,\tau e}$, $\bar{\xi}^{E}_{N,\tau\mu}$ and $\bar{\xi}^{E}_{N,\tau\tau}$. Furthermore, the new Higgs masses are also free parameters of the model and we take their numerical values as $m_{h^0} = 100$ GeV, $m_{A^0} = 200$ GeV.

In the present work, we study the LFV decays $Z \to l_1^{\pm} l_2^{\pm}$, $l_1 \neq l_2$ in the framework of the 2HDM with the addition extra dimensions. Our assumption is that the new Higgs scalars are localized in the extra dimension with Gaussian profiles by an unknown mechanism, however, the other particles have uniform zero mode profiles in the extra dimension. Here we choose one (two) extra dimension(s) which are compactified on to orbifold S^1/Z_2 ($(S^1 \times S^1)/Z_2$) with the compactification scale $1/R$, which is another free parameter. The direct limits from searching for KK gauge bosons imply $1/R > 800$ GeV, the precision electro weak bounds on higher dimensional operators generated by KK exchange place a far more stringent limit $1/R > 3.0$ TeV [22] and, from $B \to \phi\, K_S$, the lower bounds for the scale $1/R$ have been obtained as $1/R > 1.0$ TeV, from $B \to \psi\, K_S$ one got $1/R > 500$ GeV, and from the upper limit of the BR , BR $(B_s \to \mu^+\mu^-) < 2.6 \times 10^{-6}$, the estimated limit was $1/R > 800$ GeV [36]. On the other hand, the localization of new Higgs doublet is regulated by the parameter σ, which is the Gaussian width in the extra dimension, and it is chosen so that it does not contradict with the experimental results. Here, we take the compactification scale $1/R$ in the range 200 GeV $\leq 1/R \leq$ 1000 GeV and choose the Gaussian width $\sigma = \rho\, R$ at most $0.05\, R$. Notice that throughout our calculations we use the input values given in Table I.

[2] In the following we use the dimensionful coupling $\bar{\xi}^{E}_{N,ij}$ in four dimensions, with the definition $\xi^{E}_{N,ij} = \sqrt{\frac{4\, G_F}{\sqrt{2}}}\, \bar{\xi}^{E}_{N,ij}$ where N denotes the word "neutral".

TABLE I

The values of the input parameters used in the numerical calculations.

Parameter	Value
m_μ	0.106 (GeV)
m_τ	1.78 (GeV)
m_W	80.26 (GeV)
m_Z	91.19 (GeV)
m_{h^0}	100 (GeV)
m_{A^0}	200 (GeV)
$G_{\rm F}$	$1.16637 10^{-5}(\mathrm{GeV}^{-2})$
Γ_Z	2.490 (GeV)
$\sin\theta_W$	$\sqrt{0.2325}$

In our analysis, we first consider that the new Higgs doublet is localized around the origin in a single extra dimension. Furthermore, we choose the localization point is near to the origin, at the point $y_{\rm H} = \alpha\,\sigma$, and study its effect on the BRs. We continue to analyze the same physical quantity with the inclusion of two extra dimensions.

Fig. 2 is devoted to the parameter $\rho = \sigma/R$ dependence of the BR $(Z \to l_1^\pm\, l_2^\pm)$ for $1/R = 500$ GeV. Here the lower–intermediate–upper solid (dashed, small dashed) lines represent the BR $(Z \to \mu^\pm\, e^\pm)$ – BR $(Z \to \tau^\pm\, e^\pm)$ – BR $(Z \to \tau^\pm\, \mu^\pm)$ for $\bar{\xi}^{E}_{{\rm N},\tau e} = 0.1$ GeV, $\bar{\xi}^{E}_{{\rm N},\tau\mu} = 10$ GeV – $\bar{\xi}^{E}_{{\rm N},\tau e} = 0.1$ GeV, $\bar{\xi}^{E}_{{\rm N},\tau\tau} = 100\,$GeV – $\bar{\xi}^{E}_{{\rm N},\tau\mu} = 10\,$GeV, $\bar{\xi}^{E}_{{\rm N},\tau\tau} = 100$ GeV without lepton KK modes in a single extra dimension (with lepton KK modes in a single extra dimension, with lepton KK modes in two extra dimensions). It is observed that BR is at the order of the magnitude of 10^{-14}–10^{-12}–10^{-8} without lepton KK modes in a single extra dimension, for the parameter $\rho \sim 0.05$ and its sensitivity to the parameter ρ is strong. With the inclusion of lepton KK modes, the BR enhances to the values of the order of 10^{-10}–10^{-8}–10^{-5} and this is almost four order enhancement in the BRs. For two extra dimensions, the numerical value of the BR is slightly smaller compared to the single extra dimension case, since there is an additional suppression factor (see the exponential factor in Eq. (28)) appears in the expressions. Now, we study the dependence of the BR of the LFV Z decays to the Yukawa couplings, regulating the lepton–lepton-new Higgs interactions.

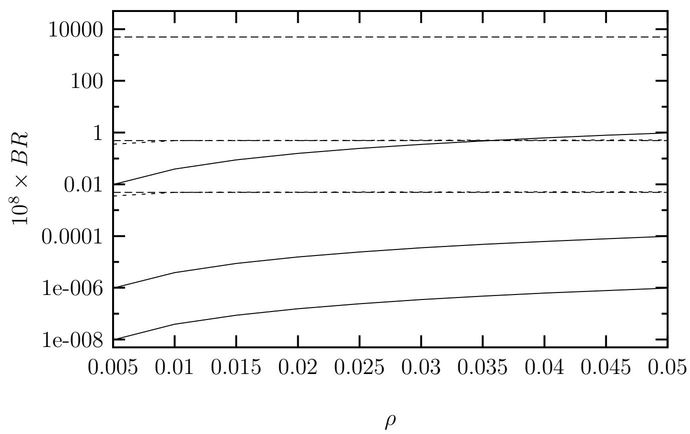

Fig. 2. BR $(Z \to l_1^{\pm} l_2^{\pm})$ with respect to the parameter ρ for $1/R = 500$ GeV. Here the lower-intermediate-upper solid (dashed, small dashed) lines represent the BR$(Z \to \mu^{\pm} e^{\pm})$ – BR$(Z \to \tau^{\pm} e^{\pm})$ – BR$(Z \to \tau^{\pm} \mu^{\pm})$ for $\bar{\xi}^E_{N,\tau e} = 0.1$ GeV, $\bar{\xi}^E_{N,\tau\mu} = 10$ GeV – $\bar{\xi}^E_{N,\tau e} = 0.1$ GeV, $\bar{\xi}^E_{N,\tau\tau} = 100$ GeV – $\bar{\xi}^E_{N,\tau\mu} = 10$ GeV, $\bar{\xi}^E_{N,\tau\tau} = 100$ GeV without lepton KK modes in a single extra dimension (with lepton KK modes in a single extra dimension, with lepton KK modes in two extra dimensions).

Figs. 3, 4 represent the Yukawa coupling $\bar{\xi}^E_{N,\tau e}$;$\bar{\xi}^E_{N,\tau\tau}$ dependence of the BR $(Z \to \mu^{\pm} e^{\pm})$, BR $(Z \to \tau^{\pm} e^{\pm})$;BR $(Z \to \tau^{\pm} \mu^{\pm})$ for $\bar{\xi}^E_{N,\tau\mu} = 10\,\mathrm{GeV}$, $\bar{\xi}^E_{N,\tau\tau} = 100\,\mathrm{GeV}$;$\bar{\xi}^E_{N,\tau\mu} = 10\,\mathrm{GeV}$, $\rho = 0.01$ and $1/R = 500\,\mathrm{GeV}$. In Fig. 3 the lower–upper solid (dashed, small dashed) line represent the BRs $(Z \to \mu^{\pm} e^{\pm})$ – BR$(Z \to \tau^{\pm} e^{\pm})$ without lepton KK modes in a single extra dimension (with lepton KK modes in a single extra dimension, with lepton KK modes in two extra dimensions). Fig. 4 represents the same curves for BR $(Z \to \tau^{\pm} \mu^{\pm})$, for $\bar{\xi}^E_{N,\tau\mu} = 10\,\mathrm{GeV}$. The BR is strongly sensitive to the Yukawa coupling $\bar{\xi}^E_{N,\tau e}$;$\bar{\xi}^E_{N,\tau\tau}$ and in the interval $0.005 \leq \bar{\xi}^E_{N,\tau e} \leq 0.05$;$50 \leq \bar{\xi}^E_{N,\tau\tau} \leq 100$ it enhances almost two; one order of magnitude. These figures also show that the inclusion of lepton KK modes causes the BR to increase considerably.

Finally, we study the effects of the position of the localization point of the new Higgs doublet on the BR of the considered decays.

Fig. 5 represents the parameter α dependence of BR $(Z \to \mu^{\pm} e^{\pm})$ – BR$(Z \to \tau^{\pm} e^{\pm})$ – BR$(Z \to \tau^{\pm} \mu^{\pm})$ for the Yukawa couplings $\bar{\xi}^E_{N,\tau\mu} = 10$ GeV, $\bar{\xi}^E_{N,\tau e} = 0.1$ GeV – $\bar{\xi}^E_{N,\tau\tau} = 100$ GeV, $\bar{\xi}^E_{N,\tau e} = 0.1\,\mathrm{GeV}$ – $\bar{\xi}^E_{N,\tau\tau} = 100\,\mathrm{GeV}$, $\bar{\xi}^E_{N,\tau\mu} = 10\,\mathrm{GeV}$ and $\rho = 0.01$, $1/R = 500\,\mathrm{GeV}$. Here the solid (dashed) line represents the BR without lepton KK modes in a single extra dimension (with lepton KK modes in a single extra dimension). Without

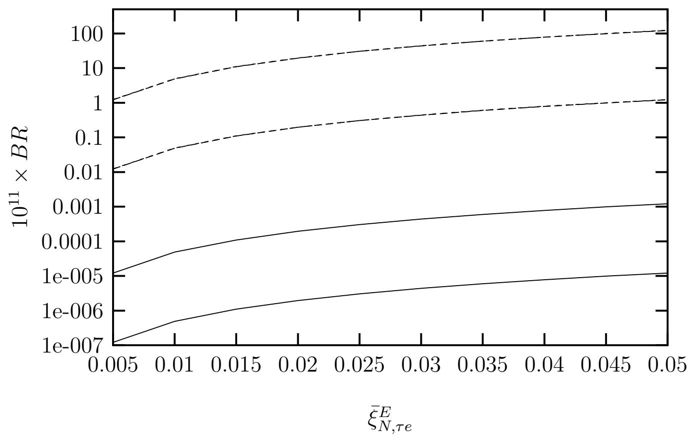

Fig. 3. BR $(Z \to \mu^{\pm}\, e^{\pm})$–BR $(Z \to \tau^{\pm}\, e^{\pm})$ with respect to $\bar{\xi}^{E}_{\mathrm{N},\tau e}$ for $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10\,\mathrm{GeV} - \bar{\xi}^{E}_{\mathrm{N},\tau\tau} = 100\,\mathrm{GeV}$, and $\rho = 0.01$, $1/R = 500\,\mathrm{GeV}$. Here lower-upper solid (dashed, small dashed) line represent the BRs $(Z \to \mu^{\pm}\, e^{\pm}) - \mathrm{BR}\,(Z \to \tau^{\pm}\, e^{\pm})$ without lepton KK modes in a single extra dimension (with lepton KK modes in a single extra dimension, with lepton KK modes in two extra dimensions).

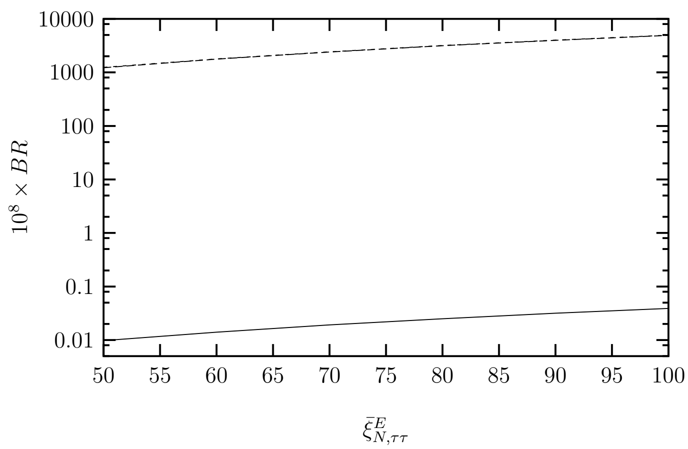

Fig. 4. BR $(Z \to \tau^{\pm}\, \mu^{\pm})$ with respect to $\bar{\xi}^{E}_{\mathrm{N},\tau\tau}$ for $\bar{\xi}^{E}_{\mathrm{N},\tau\mu} = 10\,\mathrm{GeV}$, and $\rho = 0.01$, $1/R = 500\,\mathrm{GeV}$. Here lower–upper solid (dashed, small dashed) line represent the BR without lepton KK modes in a single extra dimension (with lepton KK modes in a single extra dimension, with lepton KK modes in two extra dimensions).

lepton KK modes, the BR is not sensitive to the parameter α for the interval $0.1 \leq \alpha \leq 1$. The inclusion of lepton KK modes makes the BR sensitive to the parameter α and the increasing values of this parameter cause to decrease the BR almost one order for the considered interval taken.

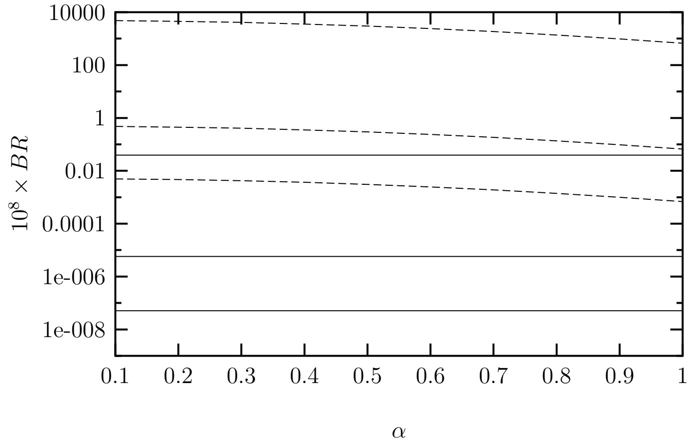

Fig. 5. BR $(Z \to \mu^\pm e^\pm)$ – BR $(Z \to \tau^\pm e^\pm)$ – BR $(Z \to \tau^\pm \mu^\pm)$ with respect to α for the Yukawa couplings $\bar{\xi}^E_{N,\tau\mu} = 10\,\mathrm{GeV}$, $\bar{\xi}^E_{N,\tau e} = 0.1\,\mathrm{GeV} - \bar{\xi}^E_{N,\tau\tau} = 100\,\mathrm{GeV}$, $\bar{\xi}^E_{N,\tau e} = 0.1\,\mathrm{GeV} - \bar{\xi}^E_{N,\tau\tau} = 100\,\mathrm{GeV}$, $\bar{\xi}^E_{N,\tau\mu} = 10\,\mathrm{GeV}$ and $\rho = 0.01$, $1/R = 500\,\mathrm{GeV}$. Here the solid (dashed) line represents the BR without lepton KK modes in a single extra dimension (with lepton KK modes in a single extra dimension).

As a summary, the BR $(Z \to \mu^\pm e^\pm)$ $((Z \to \tau^\pm e^\pm)$, $(Z \to \tau^\pm \mu^\pm))$ enhances up to the values of the order of 10^{-10} $(10^{-8}, 10^{-5})$ with the inclusion of lepton KK modes in a single extra dimension. For two extra dimensions, the numerical value of the BRs are slightly smaller compared to the single extra dimension case. On the other hand the inclusion of lepton KK modes makes the BRs sensitive to the parameter α and the increasing values of this parameter cause to decrease the BR almost one order for the considered interval of this parameter. With the forthcoming more accurate experimental measurements of the these decays, the valuable information can be obtained to detect the effects due to the extra dimensions and the possible localization of the Higgs doublet.

Appendix

The explicit expressions appearing in the text

Here we present the explicit expressions for $f_{i\mathrm{V}}^{(n)}$, $f_{i\mathrm{A}}^{(n)}$, $f_{i\mathrm{M}}^{(n)}$ and $f_{i\mathrm{E}}^{(n)}$ [9] (see Eq. (31)):

$$
\begin{aligned}
f_{i\mathrm{V}}^{(n)} &= \frac{g}{64\pi^2\cos\theta_W}\int_0^1 dx \frac{1}{m^2_{l_2^+}-m^2_{l_1^-}}\Bigg\{c_{\mathrm{V}}(m_{l_2^+}+m_{l_1^-}) \\
&\times\Bigg(\Big(-m_i\eta_i^+ + m_{l_1^-}(-1+x)\eta_i^V\Big)\ln\frac{L^{\mathrm{self}}_{1,h^0}}{\mu^2} + \Big(m_i\eta_i^+ - m_{l_2^+}(-1+x)\eta_i^V\Big)\ln\frac{L^{\mathrm{self}}_{2,h^0}}{\mu^2} \\
&+\Big(m_i\eta_i^+ + m_{l_1^-}(-1+x)\eta_i^V\Big)\ln\frac{L^{\mathrm{self}}_{1,A^0}}{\mu^2} - \Big(m_i\eta_i^+ + m_{l_2^+}(-1+x)\eta_i^V\Big)\ln\frac{L^{\mathrm{self}}_{2,A^0}}{\mu^2}\Bigg) \\
&+c_{\mathrm{A}}(m_{l_2^+}-m_{l_1^-})\Bigg(\Big(-m_i\eta_i^- + m_{l_1^-}(-1+x)\eta_i^A\Big)\ln\frac{L^{\mathrm{self}}_{1,h^0}}{\mu^2} \\
&+\Big(m_i\eta_i^- + m_{l_2^+}(-1+x)\eta_i^A\Big)\ln\frac{L^{\mathrm{self}}_{2,h^0}}{\mu^2} + \Big(m_i\eta_i^- + m_{l_1^-}(-1+x)\eta_i^A\Big)\ln\frac{L^{\mathrm{self}}_{1,A^0}}{\mu^2} \\
&+\Big(-m_i\eta_i^- + m_{l_2^+}(-1+x)\eta_i^A\Big)\ln\frac{L^{\mathrm{self}}_{2,A^0}}{\mu^2}\Bigg)\Bigg\} \\
&-\frac{g}{64\pi^2\cos\theta_W}\int_0^1 dx\int_0^{1-x} dy\Bigg\{m_i^2(c_{\mathrm{A}}\eta_i^A - c_{\mathrm{V}}\eta_i^V)\left(\frac{1}{L^{\mathrm{ver}}_{A^0}}+\frac{1}{L^{\mathrm{ver}}_{h^0}}\right) \\
&-(1-x-y)m_i\Bigg(c_{\mathrm{A}}(m_{l_2^+}-m_{l_1^-})\eta_i^-\left(\frac{1}{L^{\mathrm{ver}}_{h^0}}-\frac{1}{L^{\mathrm{ver}}_{A^0}}\right) \\
&+c_{\mathrm{V}}(m_{l_2^+}+m_{l_1^-})\eta_i^+\left(\frac{1}{L^{\mathrm{ver}}_{h^0}}+\frac{1}{L^{\mathrm{ver}}_{A^0}}\right)\Bigg) \\
&-(c_{\mathrm{A}}\eta_i^A + c_{\mathrm{V}}\eta_i^V)\Bigg(-2+(q^2xy+m_{l_1^-}m_{l_2^+}(-1+x+y)^2)\left(\frac{1}{L^{\mathrm{ver}}_{h^0}}+\frac{1}{L^{\mathrm{ver}}_{A^0}}\right) \\
&-\ln\frac{L^{\mathrm{ver}}_{h^0}}{\mu^2}\frac{L^{\mathrm{ver}}_{A^0}}{\mu^2}\Bigg) - (m_{l_2^+}+m_{l_1^-})(1-x-y)\Bigg(\frac{\eta_i^A(xm_{l_1^-}+ym_{l_2^+})+m_i\eta_i^-}{2L^{\mathrm{ver}}_{A^0h^0}} \\
&+\frac{\eta_i^A(xm_{l_1^-}+ym_{l_2^+})-m_i\eta_i^-}{2L^{\mathrm{ver}}_{h^0A^0}}\Bigg) + \frac{1}{2}\eta_i^A\ln\frac{L^{\mathrm{ver}}_{A^0h^0}}{\mu^2}\frac{L^{\mathrm{ver}}_{h^0A^0}}{\mu^2}\Bigg\}, \qquad (\mathrm{A.1})
\end{aligned}
$$

$$
\begin{aligned}
f_{i\mathrm{A}}^{(n)} &= \frac{-g}{64\pi^2 \cos\theta_W} \int_0^1 dx \frac{1}{m_{l_2^+}^2 - m_{l_1^-}^2} \Bigg\{ c_{\mathrm{V}} (m_{l_2^+} - m_{l_1^-}) \\
&\times \Bigg((m_i \eta_i^- + m_{l_1^-}(-1+x)\eta_i^A) \ln \frac{L_{1,A^0}^{\mathrm{self}}}{\mu^2} + (-m_i \eta_i^- + m_{l_2^+}(-1+x)\eta_i^A) \ln \frac{L_{2,A^0}^{\mathrm{self}}}{\mu^2} \\
&+ (-m_i \eta_i^- + m_{l_1^-}(-1+x)\eta_i^A) \ln \frac{L_{1,h^0}^{\mathrm{self}}}{\mu^2} + (m_i \eta_i^- + m_{l_2^+}(-1+x)\eta_i^A) \ln \frac{L_{2,h^0}^{\mathrm{self}}}{\mu^2} \Bigg) \\
&+ c_{\mathrm{A}} (m_{l_2^+} + m_{l_1^-}) \\
&\times \Bigg((m_i \eta_i^+ + m_{l_1^-}(-1+x)\eta_i^V) \ln \frac{L_{1,A^0}^{\mathrm{self}}}{\mu^2} - (m_i \eta_i^+ + m_{l_2^+}(-1+x)\eta_i^V) \ln \frac{L_{2,A^0}^{\mathrm{self}}}{\mu^2} \\
&+ (-m_i \eta_i^+ + m_{l_1^-}(-1+x)\eta_i^V) \ln \frac{L_{1,h^0}^{\mathrm{self}}}{\mu^2} \\
&+ (m_i \eta_i^+ - m_{l_2^+}(-1+x)\eta_i^V) \frac{\ln L_{2,h^0}^{\mathrm{self}}}{\mu^2} \Bigg) \Bigg\} \\
&+ \frac{g}{64\pi^2 \cos\theta_W} \int_0^1 dx \int_0^{1-x} dy \Bigg\{ m_i^2 (c_{\mathrm{V}} \eta_i^A - c_{\mathrm{A}} \eta_i^V) \left(\frac{1}{L_{A^0}^{\mathrm{ver}}} + \frac{1}{L_{h^0}^{\mathrm{ver}}} \right) \\
&- m_i (1-x-y) \Bigg(c_{\mathrm{V}} (m_{l_2^+} - m_{l_1^-}) \eta_i^- + c_{\mathrm{A}} (m_{l_2^+} + m_{l_1^-}) \eta_i^+ \Bigg) \left(\frac{1}{L_{h^0}^{\mathrm{ver}}} - \frac{1}{L_{A^0}^{\mathrm{ver}}} \right) \\
&+ (c_{\mathrm{V}} \eta_i^A + c_{\mathrm{A}} \eta_i^V) \Bigg(-2 + (q^2 xy - m_{l_1^-} m_{l_2^+} (-1+x+y)^2) \\
&\times \left(\frac{1}{L_{h^0}^{\mathrm{ver}}} + \frac{1}{L_{A^0}^{\mathrm{ver}}} \right) - \ln \frac{L_{h^0}^{\mathrm{ver}}}{\mu^2} \frac{L_{A^0}^{\mathrm{ver}}}{\mu^2} \Bigg) \\
&- (m_{l_2^+} - m_{l_1^-})(1-x-y) \Bigg(\frac{\eta_i^V (x m_{l_1^-} - y m_{l_2^+}) + m_i \eta_i^+}{2 L_{A^0 h^0}^{\mathrm{ver}}} \\
&+ \frac{\eta_i^V (x m_{l_1^-} - y m_{l_2^+}) - m_i \eta_i^+}{2 L_{h^0 A^0}^{\mathrm{ver}}} \Bigg) - \frac{1}{2} \eta_i^V \ln \frac{L_{A^0 h^0}^{\mathrm{ver}}}{\mu^2} \frac{L_{h^0 A^0}^{\mathrm{ver}}}{\mu^2} \Bigg\},
\end{aligned} \tag{A.2}
$$

$$f_{i\mathrm{M}}^{(n)} = -\frac{g m_W}{64\pi^2 \cos\theta_W} \int\limits_0^1 dx \int\limits_0^{1-x} dy \Bigg\{ \Bigg((1-x-y)(c_{\mathrm{V}}\eta_i^V + c_{\mathrm{A}}\eta_i^A)(x m_{l_1^-} + y m_{l_2^+})$$

$$+ m_i(c_{\mathrm{A}}(x-y)\eta_i^- + c_{\mathrm{V}}\eta_i^+(x+y)) \Bigg) \frac{1}{L_{h^0}^{\mathrm{ver}}}$$

$$+ \Bigg((1-x-y)(c_{\mathrm{V}}\eta_i^V + c_{\mathrm{A}}\eta_i^A)(x m_{l_1^-} + y m_{l_2^+})$$

$$- m_i(c_{\mathrm{A}}(x-y)\eta_i^- + c_{\mathrm{V}}\eta_i^+(x+y)) \Bigg) \frac{1}{L_{A^0}^{\mathrm{ver}}}$$

$$-(1-x-y)\Bigg(\frac{\eta_i^A (x m_{l_1^-} + y m_{l_2^+})}{2} \Big(\frac{1}{L_{A^0 h^0}^{\mathrm{ver}}} + \frac{1}{L_{h^0 A^0}^{\mathrm{ver}}} \Big)$$

$$+ \frac{m_i \eta_i^-}{2} \Big(\frac{1}{L_{h^0 A^0}^{\mathrm{ver}}} - \frac{1}{L_{A^0 h^0}^{\mathrm{ver}}} \Big) \Bigg) \Bigg\}, \qquad \text{(A.3)}$$

$$f_{i\mathrm{E}}^{(n)} = -\frac{g m_W}{64\pi^2 \cos\theta_W} \int\limits_0^1 dx \int\limits_0^{1-x} dy \Bigg\{ \Bigg((1-x-y)$$

$$\times \Big(-(c_{\mathrm{V}}\eta_i^A + c_{\mathrm{A}}\eta_i^V)(x m_{l_1^-} - y m_{l_2^+}) \Big)$$

$$- m_i(c_{\mathrm{A}}(x-y)\eta_i^+ + c_{\mathrm{V}}\eta_i^-(x+y)) \Bigg) \frac{1}{L_{h^0}^{\mathrm{ver}}}$$

$$+ \Bigg((1-x-y)\Big(-(c_{\mathrm{V}}\eta_i^A + c_{\mathrm{A}}\eta_i^V)(x m_{l_1^-} - y m_{l_2^+}) \Big) + m_i(c_{\mathrm{A}}(x-y)\eta_i^+$$

$$+ c_{\mathrm{V}}\eta_i^-(x+y)) \Bigg) \frac{1}{L_{A^0}^{\mathrm{ver}}}$$

$$+(1-x-y)\Bigg(\frac{\eta_i^V}{2}(m_{l_1^-} x - m_{l_2^+} y) \Big(\frac{1}{L_{A^0 h^0}^{\mathrm{ver}}} + \frac{1}{L_{h^0 A^0}^{\mathrm{ver}}} \Big)$$

$$+ \frac{m_i \eta_i^+}{2} \Big(\frac{1}{L_{A^0 h^0}^{\mathrm{ver}}} - \frac{1}{L_{h^0 A^0}^{\mathrm{ver}}} \Big) \Bigg) \Bigg\}, \qquad \text{(A.4)}$$

where

$$L_{1,h^0}^{\mathrm{self}} = m_{h^0}^2 (1-x) + (m_i^{(n)2} - m_{l_1^-}^2 (1-x)) x,$$

$$
\begin{aligned}
L^{\rm self}_{1,A^0} &= L^{\rm self}_{1,h^0}(m_{h^0} \to m_{A^0}),\\
L^{\rm self}_{2,h^0} &= L^{\rm self}_{1,h^0}(m_{l_1^-} \to m_{l_2^+}),\\
L^{\rm self}_{2,A^0} &= L^{\rm self}_{1,A^0}(m_{l_1^-} \to m_{l_2^+}),\\
L^{\rm ver}_{h^0} &= m^2_{h^0}(1-x-y) + m_i^{(n)2}(x+y) - q^2 xy,\\
L^{\rm ver}_{h^0 A^0} &= m^2_{A^0} x + m_i^{(n)2}(1-x-y) + (m^2_{h^0} - q^2 x) y,\\
L^{\rm ver}_{A^0} &= L^{\rm ver}_{h^0}(m_{h^0} \to m_{A^0}),\\
L^{\rm ver}_{A^0 h^0} &= L^{\rm ver}_{h^0 A^0}(m_{h^0} \to m_{A^0}),
\end{aligned}
\tag{A.5}
$$

and

$$
\begin{aligned}
\eta_i^V &= c_n^2\{\xi^E_{il_1}\xi^{E*}_{il_2} + \xi^{E*}_{l_1 i}\xi^E_{l_2 i}\},\\
\eta_i^A &= c_n^2\{\xi^E_{il_1}\xi^{E*}_{il_2} - \xi^{E*}_{l_1 i}\xi^E_{l_2 i}\},\\
\eta_i^+ &= c_n^2\{\xi^{E*}_{l_1 i}\xi^{E*}_{il_2} + \xi^E_{il_1}\xi^E_{l_2 i}\},\\
\eta_i^- &= c_n^2\{\xi^{E*}_{l_1 i}\xi^{E*}_{il_2} - \xi^E_{il_1}\xi^E_{l_2 i}\}.
\end{aligned}
\tag{A.6}
$$

The parameters $c_{\rm V}$ and $c_{\rm A}$ are $c_{\rm A} = -\frac{1}{4}$ and $c_{\rm V} = \frac{1}{4} - \sin^2\theta_W$ and the masses $m_i^{(n)}$ read $m_i^{(n)} = \sqrt{m_i^2 + n^2/R^2}$, where R is the compactification radius. In Eq. (A.6) the flavor changing couplings $\xi^E_{il_j}$ represent the effective interaction between the internal lepton i, ($i = e, \mu, \tau$) and outgoing (incoming) $j = 1 (j = 2)$ one. The parameter c_n is defined in Eq. (15) for the localization of the new Higgs doublet around the origin and in Eq. (20) for the localization of the new Higgs doublet around the point $y_{\rm H}$ near to the origin. In the case of two extra dimensions c_n is replaced by $c_{n,s}$ (see Eq. (28)) and the masses $m_i^{(n)}$ are replaced by $m_i^{(n,s)}$, $m_i^{(n,s)} = \sqrt{m_i^2 + m_n^2 + m_s^2}$, with $m_n = n/R, m_s = s/R$.

Finally, the couplings $\xi^E_{l_j i}$ may be complex in general and they can be parametrized as

$$
\xi^E_{l_i j} = |\xi^E_{l_i j}| e^{i\theta_{ij}}\,, \tag{A.7}
$$

where i, l_j denote the lepton flavors and θ_{ij} are CP violating parameters which are the possible sources of the lepton EDM. However, in the present work we take these couplings real.

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