A REMARK ON VECTOR MESON DOMINANCE, UNIVERSALITY AND THE PION FORM FACTOR

Alfonso R. Zerwekh

Instituto de Física, Facultad de Ciencias, Universidad Austral de Chile
Casilla 567, Valdivia, Chile
alfonsozerwekh@uach.cl

(Received December 1, 2006)

In this paper, we address the universality problem in the mass-mixing representation of vector meson dominance. First, we stress the importance of using physical (mass eigenstate) fields in order to get the correct q^2 dependence of the pion form factor. Then we show that, when a direct coupling of the (proto-)photon to the pions is included, it is not necessary to invoke universality. Our method is similar to the delocalization idea in some deconstruction theories.

PACS numbers: 12.40.Vv, 13.40.Gp, 14.40.Aq

In 1960 Sakurai [1] proposed a theory of strong interactions based on the idea of local gauge invariance, where the interaction was supposed to be mediated by vector mesons. In this scenario, the electromagnetic interaction of hadrons was introduced through a mixing of the photon and the vector mesons. This idea is known as Vector Meson Dominance (VMD) [2].

Historically, two Lagrangian realizations of VMD have been in use. The first one, due to Kroll, Lee and Zumino [3], describes the mixing of the photon and the rho meson through a term of the form

$$\mathcal{L}_{\gamma\rho} = \frac{e}{2g_\rho}\rho_{\mu\nu}F^{\mu\nu}\,. \tag{1}$$

This representation is usually called VMD-1. In general, it is viewed as the more elegant VMD realization because it is explicitly consistent with electromagnetic gauge invariance and the pion form factor calculated from it can be written as

$$F_\pi(q^2) = \left[1 - \frac{q^2}{q^2 - m_\rho^2}\frac{g_{\rho\pi\pi}}{g_\rho}\right]\,, \tag{2}$$

which satisfy the condition $F_\pi(0) = 1$ without any assumption about the coupling constants $g_{\rho\pi\pi}$ and g_ρ. The price to pay is to work with non-diagonal propagators.

The second and, in some sense, more popular realization (usually called VMD-2) is based on a mass-mixing term of the form

$$\mathcal{L}_{\gamma\rho} = -\frac{em_\rho^2}{g_\rho}\rho_\mu A^\mu \,. \tag{3}$$

This version of VMD is seen as unsatisfactory because the mixing term (3) introduces corrections to the photon propagator acquiring a non-zero mass. In order to correct this important flaw, it is necessary to add to the Lagrangian a mass term for the photon. On the other hand, when the pion form factor is calculated in this representation the results can be written as

$$F_\pi(q^2) = -\frac{m_\rho^2}{q^2 - m_\rho^2}\frac{g_{\rho\pi\pi}}{g_\rho} \,. \tag{4}$$

In this case the condition $F_\pi(0) = 1$ is satisfied only if $g_{\rho\pi\pi} = g_\rho$. This is the universality condition. It is fair to say that both version of VMD are seen as equivalent in the limit of universality.

In the rest of this paper we develop some ideas in order to evade the need of universality. We consider a simplified model in which the (neutral) rho meson is treated as an abelian field and we do not take into account the charged rho mesons in order to concentrate our attention on the main features of the mechanism. Of course, the conclusions do not depend on such a simplification.

We start by writing down the VMD-2 Lagrangian for the vector sector

$$\mathcal{L} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{1}{4}\tilde{\rho}_{\mu\nu}\tilde{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\tilde{\rho}_\mu\tilde{\rho}^\mu - \frac{\tilde{e}m_\rho^2}{g_\rho}\tilde{\rho}_\mu\tilde{A}^\mu + \frac{1}{2}\left(\frac{\tilde{e}m_\rho}{g_\rho}\right)^2\tilde{A}_\mu\tilde{A}^\mu \,. \tag{5}$$

We will call the fields $\tilde{\rho}_\mu$ and $\tilde{A}_\mu$ the proto-rho and the proto-photon respectively. This Lagrangian is gauge invariant if the proto-photon and the proto-rho transform under $U(1)_{\rm EM}$ as[1]

$$\delta\tilde{A}_\mu = \frac{1}{\tilde{e}}\partial_\mu\Lambda \tag{6}$$

$$\delta\tilde{\rho}_\mu = \frac{1}{g_\rho}\partial_\mu\Lambda\,. \tag{7}$$

At this point it is necessary to remark that neither the proto-rho nor the proto-photon are mass eigenstate and hence they are not physical fields. The importance of using a physical basis has been recognized and advocated by other authors [4]. In fact, in a bit different context, the use of the physical basis has helped to enlighten the interaction between a color octet technirho and gluons [5]. In our case, electromagnetic gauge invariance enforces the mass matrix to have a null determinant and hence it implies that the physical photon is massless. When the mass matrix is diagonalized we find that the physical rho and photons fields are

$$A_\mu = \tilde{A}_\mu\cos\alpha + \tilde{\rho}_\mu\sin\alpha\,, \tag{8}$$

$$\rho_\mu = -\tilde{A}_\mu\sin\alpha + \tilde{\rho}_\mu\cos\alpha\,, \tag{9}$$

where

$$\cos\alpha = \frac{g_\rho}{\sqrt{\tilde{e}^2+g_\rho^2}}$$

and

$$\sin\alpha = \frac{\tilde{e}}{\sqrt{\tilde{e}^2+g_\rho^2}}\,.$$

Let now turn our attention to the charged pions. They are described by the Lagrangian

$$\mathcal{L}_\pi = D_\mu\pi^+ D^{\dagger\mu}\pi^- - m_\pi^2\pi^+\pi^-\,, \tag{10}$$

[1] When we wrote down the Lagrangian of the vector sector of VMD-2, we chose a representation where both, the proto-rho and the proto-photon, transform like gauge fields under $U(1)_{\rm EM}$. Nevertheless, in the physical basis, only the photon transforms as a gauge field while the physical rho transforms trivially because it is neutral. On the other hand, the charged rho's transform as

$$\rho^\pm \to e^{\pm i\Lambda}\rho^\pm\,.$$

Of course, the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(D_\mu\rho^+_\nu - D_\nu\rho^+_\mu)(D^\mu\rho^{-\nu} - D^\nu\rho^{-\mu}) + \tilde{M}_\rho\rho^+_\mu\rho^{-\mu}\,,$$

where D_μ is the usual covariant derivative, is gauge invariant. To this Lagrangian can be added all the interaction terms consistent with gauge invariance and the global symmetries we want to implement such as isospin invariance.

where

$$D_\mu = \partial_\mu + ix\tilde{e}\tilde{A}_\mu + i(1-x)g_\rho\tilde{\rho}_\mu \tag{11}$$

is the most general covariant derivative we can form with the fields $\tilde{A}_\mu$ and $\tilde{\rho}_\mu$ and x is a parameter.

Notice that here we slightly deviate from traditional VMD-2 because we include a direct coupling between the pions and the proto-photon. In this sense we are advocating for a partial vector meson dominance. It is this direct coupling with the proto-photon what will allow us to abandon universality. Some indications of a deviation from complete vector meson dominance were already communicated and discussed in [4]. Nevertheless, as far as we know this relation between universality and a direct coupling of the proto-photon to pions, have not been discussed before. On the other hand, the x variable plays a role similar to delocalization parameters in some deconstruction models [6].

In terms of the physical fields the covariant derivative can be written as

$$D_\mu = \partial_\mu + ieA_\mu + ig_{\rho\pi\pi}\rho_\mu\,, \tag{12}$$

where $e = \tilde{e}\cos\alpha = g_\rho \sin\alpha$ is the electric charge of the positron, and $g_{\rho\pi\pi}$ can be expressed as

$$g_{\rho\pi\pi} = g_\rho \cos\alpha \left(1 - \frac{x}{\cos^2\alpha}\right)\,. \tag{13}$$

In this context universality means $g_{\rho\pi\pi} = g_\rho\cos\alpha$ what only happens when $x = 0$.

On the other hand, we must avoid a direct coupling of the proto-rho with leptons (leptons do not interact strongly) and hence they will be described by the Lagrangian

$$\begin{aligned} \mathcal{L} &= \bar{\psi}\gamma^\mu\left(i\partial_\mu + \tilde{e}\tilde{A}_\mu\right)\psi \\ &= \bar{\psi}\gamma^\mu\left(i\partial_\mu + eA_\mu - e\tan\alpha\rho_\mu\right)\psi\,. \end{aligned} \tag{14}$$

With these ingredients, we obtain the following expression for the amplitude of the process $e^+e^- \to \pi^+\pi^-$

$$\mathcal{M} = -ie^2\bar{v}\gamma_\mu u(k_1 - k_2)^\mu\left[1 - \frac{q^2}{q^2 - M_\rho^2}\left(1 - \frac{x}{\cos^2\alpha}\right)\right]\,. \tag{15}$$

Hence, the pion form factor may be written as

$$F(q^2) = \left[1 - \frac{q^2}{q^2 - M_\rho^2}\left(1 - \frac{x}{\cos^2\alpha}\right)\right] \tag{16}$$

or, in a more familiar way

$$F(q^2) = \left[1 - \frac{q^2}{q^2 - M_\rho^2}\frac{g_{\rho\pi\pi}}{g_\rho \cos\alpha}\right]. \tag{17}$$

The pion form factor we obtained is similar to the one obtained using VMD-1: it has a correct behavior for $q^2 = 0$ and it does not depend on universality. The first feature is a consequence of having used physical fields while the second has its roots in the direct coupling of the proto-photon with pions.

The values of the mixing angle α and the x parameter can be obtained from experiment. The partial decay widths $\Gamma(\rho \to e^+e^-)$ and $\Gamma(\rho \to \pi^+\pi^-)$ can be written as

$$\Gamma(\rho \to e^+e^-) = \frac{1}{3}\alpha_{\rm EM}\tan^2\alpha M_\rho\,, \tag{18}$$

$$\Gamma(\rho \to \pi^+\pi^-) = \frac{1}{12}\alpha_{\rm EM}\left(1 - \frac{x}{\cos^2\alpha}\right)^2 \frac{M_\rho}{\tan^2\alpha}\left(1 - \frac{4m_\pi^2}{M_\rho^2}\right)^{3/2}, \tag{19}$$

where $\alpha_{\rm EM}$ is the electromagnetic fine-structure constant. Using the experimental values $\Gamma(\rho \to e^+e^-) = 6.85$ keV and $\Gamma(\rho \to \pi^+\pi^-) = 146.4$ MeV we obtain $\tan\alpha = 0.0603$, $x = -0.177$, $g_{\rho\pi\pi} = 5.92$ and $g_\rho = 5.03$. These values of $g_{\rho\pi\pi}$ and g_ρ are in agreement with those obtained by more sophisticated fittings based on VMD-1 [2].

In conclusion, we have constructed a VMD-2-like Lagrangian which correctly describes the pion form factor without needing the universality hypotheses. The main ingredients of our approach are a strong use of electromagnetic gauge invariance, the use of physical fields and the allowance for a direct coupling of the proto-photon with the pions which leads to a partial-only dominance of the rho vector meson.

The author received support from Universidad Austral de Chile
(DID grant S-2006-28).

REFERENCES

[1] J.J. Sakurai, *Ann. Phys.* **11**, 1 (1960).
[2] H.B. O'Connell, B.C. Pearce, A.W. Thomas, A.G. Williams, *Prog. Part. Nucl. Phys.* **39**, 201 (1997) [hep-ph/9501251].
[3] N.M. Kroll, T.D. Lee, B. Zumino, *Phys. Rev.* **157**, 1376 (1967).
[4] J. Schechter, *Phys. Rev.* **D34**, 868 (1986).
[5] A.R. Zerwekh, R. Rosenfeld, *Phys. Lett.* **B503**, 325 (2001) [hep-ph/0103159].
[6] R.S. Chivukula, E.H. Simmons, H.J. He, M. Kurachi, M. Tanabashi, *Phys. Rev.* **D71**, 115001 (2005) [hep-ph/0502162].