POLARIZATION EFFECTS IN TAU PRODUCTION BY NEUTRINOS*

Justyna Lagoda†, Danuta Kielczewska, Magdalena Posiadala

Institute for Experimental Physics, Warsaw University
Hoża 69, 00-681 Warsaw, Poland

Robert Sulej, Krzysztof Zaremba

Warsaw Technical University
Pl. Politechniki 1, 00-661 Warsaw, Poland

Tadeusz Kozlowski, Krzysztof Kurek, Piotr Mijakowski
Pawel Przewlocki, Ewa Rondio, Joanna Stepaniak
Maria Szeptycka‡

A. Soltan Institute for Nuclear Studies
Hoża 69, 00-681 Warsaw, Poland

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A direct proof of the existence of $\nu_\mu \to \nu_\tau$ oscillations is important. This proof can be obtained by an observation of the production of taons in charge current reactions $\nu_\tau + N \to \tau + X$. The influence of τ polarization on the characteristics of the CC events and on the efficiency of their selection is discussed. The neural network method is used to select τ leptons produced in ν_τ interactions.

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1. Introduction

The experimental observation of ν_μ disappearance (in atmospheric neutrinos in SK experiment and in K2K experiment in accelerator neutrinos) has led to the conclusion that they are transformed into (most likely) ν_τ in

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† Presently at LNGS (Laboratori Nazionale del Gran Sasso).

‡ Corresponding author: `szeptyck@fuw.edu.pl`

the process of oscillation. A direct proof of the $\nu_\mu \to \nu_\tau$ oscillations can be obtained by an observation of the production of taons in charge current reaction $\nu_\tau + N \to \tau + X$ (see Ref. [1]). The lifetime of taons is very short so its direct observation is impossible in most of the detectors. Taons must be reconstructed and identified from the decay products. The kinematics of the decay is sensitive to τ polarization. These effects were not consistently analyzed in previous studies.

Due to the oscillation processes the ν_τ should appear among the atmospheric neutrinos and in CNGS ν beam. Most of τ leptons would be of low energy close to the production threshold, therefore the polarization effects could be important. To study the influence of nuclear effects on the polarization of taons the interactions of ν_τ on nuclei were generated. The decays of polarized τ into leptonic and semileptonic channels were studied. The neural network was used for the selection of the interactions of ν_τ. Special care was given to study the influence of τ polarization and knowledge of the ν_τ direction on the efficiency of the selection. The detector effects were not included in the analysis, however cuts applied to the momenta of observed particles are realistic. The details of the analysis presented in this work can be found in Ref. [2].

2. Simulations of neutrino interactions

The neutrino interactions were generated using NUANCE (version 3.0) neutrino simulation software. For the details of physical models used in NUANCE see Ref. [3]. The generation of ν interactions is divided into two stages: first the calculations of the cross sections and rates and second the event generation. The generation includes primary interactions on independent nucleons and secondary interactions of hadrons inside nucleus. NUANCE can generate events produced by a ν beam, which flavor composition and energy profile can be modified. Interactions of produced particles in the detector material are not taken into account. All neutrino and antineutrino flavors Charge Current (CC) and Neutral Current (NC) processes are simulated. The following reactions were generated: neutrino scattering on electrons, nuclear elastic and quasielastic scattering (QE), resonant processes, dominated by Δ production, coherent and diffractive reactions and deep inelastic scattering.

The decays of leptons τ are handled by TAUOLA package [4], which can simulate following decay channels: $\tau^\pm \to \nu\nu e^\pm$, $\nu\nu\mu^\pm$, $\nu\pi^\pm$, $\nu\rho^\pm$, $\nu a_1^\pm$, $\nu K^\pm$, $\nu K^{*\pm}$, $\nu n\pi^{\pm 0}$ and multipion ($n > 3$) modes. In TAUOLA the effects of spin polarization of τ are taken into account (except for multipion mode) [5]. The decays of other short-lived particles which leave the nucleus are processed by

PYTHIA [6]. The calculations were performed for ^{16}O nucleus taking into account the Fermi motion of nucleons and a Pauli blocking. The nuclear effects and the secondary interactions inside the nucleus were included.

3. Polarization of taons produced in ν_τ interactions

In the original version of NUANCE the polarization of taons was calculated, only for those produced in deep inelastic scattering process. The formulas described in Ref. [7] were used. To include the processes which give the dominant contributions in the region of low energy of neutrino interactions (quasi-elastic scattering and resonance production) the approach proposed in Ref. [8] was used. In this approach the polarization of taons is calculated in the laboratory frame with the z-axis along the direction of the momentum of incoming neutrino. The momentum vector of τ defines the x–z plane.

The vector of spin polarization of taon is defined in the rest frame of τ. In this frame the z-axis is defined along the momentum of τ.

$$\vec{s} = (s_x, s_y, s_z) = \frac{P}{2}\left(\sin\theta_P\cos\varphi_P, \sin\theta_P\sin\varphi_P, \cos\theta_P\right), \tag{1}$$

where P denotes the degree of polarization and θ_P and φ_P are the polar and azimuthal angles of the spin vector, respectively. The vector of spin polarization is related to the spin density matrix $R_{\lambda\lambda'}$ by the following relation:

$$\frac{dR_{\lambda\lambda'}}{dE_\tau d\cos\theta} = \frac{1}{2}\begin{pmatrix} 1+P\cos\theta_P & P\sin\theta_P e^{i\phi_P} \\ P\sin\theta_P e^{-i\phi_P} & 1-P\cos\theta_P \end{pmatrix}\frac{d\sigma_{\mathrm{sum}}}{dE_\tau d\cos\theta}, \tag{2}$$

where E_τ is the energy of τ, θ is the production angle of τ in the laboratory frame, $\lambda/2$ is τ helicity, and $d\sigma_{\mathrm{sum}} = dR_{++} + dR_{--}$ is the cross section summed over spins.

The spin density matrix is calculated using the leptonic $L^{\mu\nu}_{\lambda\lambda'}$ and the hadronic $W_{\mu\nu}$ tensors:

$$\frac{dR_{\lambda\lambda'}}{dE_\tau d\cos\theta} = \frac{G_{\mathrm{F}}^2\kappa^2}{4\pi}\frac{p_\tau}{ME_\nu}L^{\mu\nu}_{\lambda\lambda'}W_{\mu\nu}, \tag{3}$$

where G_{F} is Fermi constant, p_τ is the momentum of τ, E_ν is the energy of the incoming neutrino, M is the nucleon mass and $\kappa = M_W^2/(Q^2+M_W^2)$ is the propagator factor where M_W is the mass of W boson and Q^2 is the four momentum transfer squared. The hadronic tensor is calculated for each production process: quasi elastic scattering, production of the resonance Δ and deep inelastic scattering. Details of calculations of the density matrix for a free nucleon can be found in Ref. [8]. Taons produced in ν_τ and $\bar{\nu}_\tau$

interactions are highly polarized for almost all production angles (except for the very small angles) and neutrino energies. High energy τ^-'s which are produced forward in the center of mass, are almost left-handed and the low-energy τ^-'s (produced backwards) are right-handed. For a given energy of the incoming neutrino, the taon can be produced either with low energy (backwards in CMS, low energy branch) or forward in the CMS (high energy branch). In Fig. 1 dark line shows low energy branch and light line shows high energy branch. At the neutrino energy close to the taon production threshold the two energy branches have the cross sections of the same order of magnitude. For higher energies of neutrinos, the contribution of the low energy branch can be neglected.

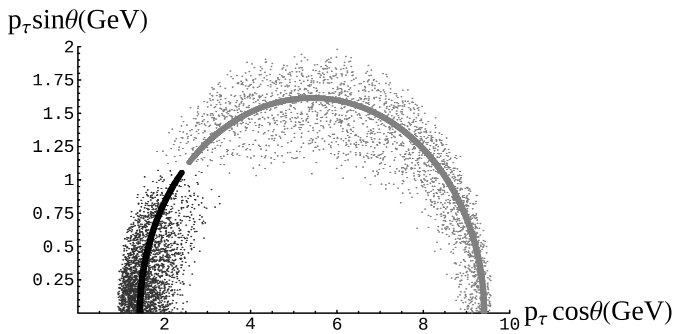

Fig. 1. Transverse momentum *vs* longitudinal momentum of τ in the laboratory frame. Taons are produced by 10 GeV neutrinos in $\nu_\tau N \to \tau\Delta$ interactions, where N is a quasi free nucleon. The dark and light shadow lines show the low and high energy solution respectively (the Fermi motion is not included). The dots show the effect of Fermi motion, and the effect of the smearing of the Δ resonance.

For a given neutrino energy the allowed kinematical region increases when the Fermi momentum is included in the calculations. Fig. 1 illustrates this effect for 10 GeV neutrinos interactions simulated with Fermi momenta generated according to simplified model in which the momenta are uniformly distributed from 0 up to 250 MeV. (In NUANCE the generation of the Fermi motion is more realistic). The extension of the kinematical boundaries is clearly visible. The smearing of the mass of the resonance Δ is included. The detailed description of the influence of Fermi motion on the polarization of τ can be found in Ref. [9].

The most striking polarization effects are expected in $\nu_\tau N \to \tau\Delta$ reaction near τ production threshold.

At higher energies the polarization vector is dominated by the s_z component. The dependence of the s_z on the incoming neutrino energy E_ν and the scattering angle θ is shown in Fig. 2 for the reaction $\nu_\tau N \to \tau\Delta$. The spin-flip effect due to the $\frac{3}{2}$ spin of Δ resonance is visible near the threshold energy, where s_z changes sign. NUANCE uses Fermi model for the generation of QE interactions and resonant processes. It was necessary to adjust and recalculate the polarization formulas from Ref. [8] for these processes to take into account the Fermi momentum and the binding energy of the target nucleon (see Ref. [9]). These formulas were implemented in NUANCE. For taons produced in deep inelastic processes, the polarization vectors calculated with the formula published in Ref. [8] were compared with polarization vectors calculated according to formula from Ref. [7] implemented in NUANCE. There is a very good agreement between two methods, which use different reference frames.

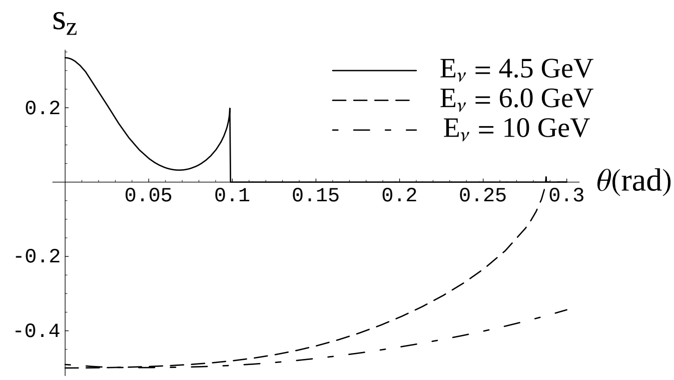

Fig. 2. The component s_z of polarization vector of τ as a function of the incoming neutrino energy E_ν (4.5 GeV solid line, 6 GeV dashed and 10 GeV — dashed-dotted line) and the τ production angle θ. Taons are produced in $\nu_\tau N \to \tau\Delta$ process. The effect of spin-flip near the threshold energy is visible. It is due to the $\frac{3}{2}$ spin of the resonance Δ.

4. Methods of taon selection

4.1. Neural network

For a separation of signal events with the production of τ from background events a multi-layered feed-forward network is used. The network consists of neurons organized in layers. The number of neurons of input layer corresponds to the number of variables used for selection of signal events. Output layer contains single neuron which is expected to give values close to 1 for signal events and close to 0 for background events. There are two hidden layers between input and output layers. The network is trained by supervised learning on Monte Carlo (training sample) where it is known which event belongs to the signal sample. Training procedure optimizes neuron interconnection coefficients (weights) to minimize an error function of a difference between desired and actual network output averaged over the whole training set. The number of hidden neurons is optimized during the training (see Ref. [10]). Prepared network is verified on a different MC testing sample.

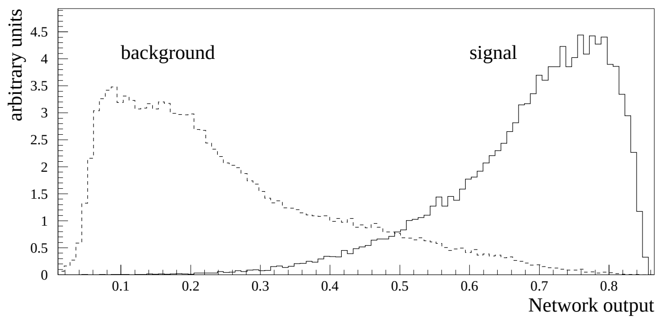

Fig. 3. Distribution of network output for the sample of 1E signal events and the sample of background events. (For the definition of classes see Sec. 4.3). In this figure the ratio of the number of signal events to background events is equal 1.

Fig. 3 shows the distribution of the network output for background and signal samples. An event is considered as a signal if the network output for this event is above a given threshold t. For a given testing set the value of threshold applied to the network outputs determines two variables that describe the quality of selection: purity $P_{\rm sel}$ and efficiency $E_{\rm sel}$:

$$P_{\rm sel}(t) = \frac{N_{\rm sig}(\mathrm{Output})}{N_{\rm bkg}(\mathrm{Output}) + N_{\rm sig}(\mathrm{Output})}\, 100\%\,, \tag{4}$$

$$E_{\rm sel}(t) = \frac{N_{\rm sig}(\mathrm{Output})}{N_{\rm sig}}\, 100\%\,, \tag{5}$$

where $N_{\rm sig}(\mathrm{Output})$ is the number of correctly recognized events of signal (network output is greater than the threshold t), $N_{\rm bkg}(\mathrm{Output})$ is the number of background events classified as signal, and $N_{\rm sig}$ is the total number of signal events. Varying the value of the threshold one obtains sequence of $P_{\rm sel}(t)$ and corresponding $E_{\rm sel}(t)$ values.

The training sample must contain large enough number of signal and background events to fill the multi-dimensional feature space of network input. In a case when the number of signal events is dominated by a much larger number of background events using the original signal and background ratios results in an unnecessary increase of training time. This may be avoided if the comparable numbers of signal and background events are used for training purposes. When the network is trained on such a sample to obtain the expected fraction of signal events and to calculate the purity, the numbers $N_{\rm sig}$ and $N_{\rm bkg}$ have to be rescaled:

$$P_{\rm sel} = \frac{a\, N_{\rm sig}(\text{Output})}{b\, N_{\rm bkg}(\text{Output}) + a\, N_{\rm sig}(\text{Output})}\, 100\%\,, \tag{6}$$

where a is the ratio of the fraction of the events of signal expected in the real data to the fraction of the events of signal in the input set, and b — the corresponding ratio for the background events. Such a procedure changes neither event probability distributions nor the sensitivity of the network for the regions in the feature space.

In the described case the main goal is to select the signal sample as pure as possible, which means that the network output should be close to 1, where very few events pass the selection. Mean squared error function (MSE), which is commonly used for training, does not emphasize sufficiently the influence of background events with network output close to 1, which are most injurious. Therefore the error function was changed to:

$$E_{\rm net} = \frac{1}{N}\sum_{i=1}^{N}(d_i - o_i)^4\,, \tag{7}$$

where N is the number of training events, index i runs over all training events, d_i is desired network output, o_i is actual network output. Chosen error function forces the network to be more focused on the events with high difference between the desired and the obtained network output. These events become more important during the training than in case of MSE function. Due to this modification the decrease of the selection efficiency in the range of intermediate purities is observed, but this effect is small. The details of this method are discussed in Ref. [11].

4.2. Generated samples of events

The analysis was carried out for two neutrino sources with different energy spectra and different content of neutrino flavors. The first was the CERN muon neutrino beam pointing to underground laboratory in Gran Sasso (CNGS) [12].

The second case concerns the atmospheric neutrinos with fluxes calculated according to [14–16] and [17]. The contributions of ν_μ and ν_e and their antineutrinos are of the same order.

The energy spectrum of ν_τ ($\bar\nu_\tau$) was calculated from ν_μ ($\bar\nu_\mu$) energy spectra using two-neutrino oscillation formula:

$$P(\nu_\mu \to \nu_\tau) = \sin^2(2\theta)\sin^2\left(\frac{1.27\Delta m^2[\text{eV}^2]L[\text{km}]}{E[\text{GeV}]}\right) \tag{8}$$

with the following oscillation parameters: $\sin^2 2\theta = 1$, $\Delta m^2 = 2.5\times10^{-3}$ eV^2, where E is neutrino energy and L is the distance from neutrino source to the detector (for the historical paper on neutrino oscillations see Ref. [18]).

The signal sample consists of the charge current interactions (CC) of ν_τ ($\bar{\nu}_\tau$) neutrinos which would be produced by the oscillations of muon neutrinos (antineutrinos). The background sample consists of neutral current interactions (NC) of ν_τ and $\bar{\nu}_\tau$, and of CC and NC interactions of ν_e, $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$.

For each source of neutrinos a sample of background events and three samples of signal events were generated with NUANCE. The samples of signal events differed by the assumed polarization of τ leptons. Taons were assumed to be either fully polarized, unpolarized or the polarization was calculated according to [8] and [9].

It was assumed that the momenta of the observed particles (electrons, muons, charged kaons, gammas from π^0 decays, charged pions and protons) were measured in an ideal detector. The momenta of charged pions were required to be greater than 50 MeV and the momenta of protons greater than 250 MeV. These values were set to ensure the possibility of an identification of particles (in liquid Argon for example) and of a reconstruction of their momenta. Neutrinos and neutrons were neglected.

4.3. Variables used for classification of events

For the selection of events based on the neural network two data samples are needed, namely the signal sample and the background sample. The events in each sample were divided into three classes: 1E — events with at least one electron among produced particles, 1M — events with at least one muon among produced particles, 0L — events without charged leptons.

In the testing samples the numbers of signal (or background) events for each class were of the order of 100 000 events, whereas in the training samples they were of the order of 50 000 events.

For each class of events a set of variables, being sensitive to the selection of taons, was calculated from the momenta of observed particles. These variables and the multiplicities of produced particles (see Table I) are used as an input for a neural network. The networks were trained separately for each class (1E, 1M, 0L) and for each polarization of taons.

The set of the kinematical variables from Table I is sufficient for a wide class of neutrino detectors. However, in the case of emulsion chambers, the topological variables like the particle impact parameter are also important.

TABLE I

Description of variables used in neural network analysis.
The transverse momenta were calculated with respect to the ν direction. The variable ρ_l was defined as: $\rho_l = \frac{p_{\rm t}^{\rm lep}}{p_{\rm t}^{\rm lep}+p_{\rm t}^{\rm had}+p_{\rm t}^{\rm miss}}$, where $p_{\rm t}$ denotes transverse momenta of leptons (lep), hadrons (had) and missing particles (miss), respectively.

Description	Name	Classes
Total momentum($\lvert\sum_i \vec{p}_i\rvert$)	$p_{\rm tot}$	all
Scalar sum of momenta ($\sum_i \lvert\vec{p}_i\rvert$)	$p_{\rm scal}$	all
Missing transverse momentum	$p_{\rm t}$	all
Ratio of scalar sum of momenta to total momentum	$p_{\rm scal}/p_{\rm tot}$	all
Number of electrons and photons	N_{el}	all
Number of charged pions	N_{π}	all
Number of protons	N_p	all
Transverse momentum of the most energetic lepton	$p_{\rm t}^{\rm lep}$	1E, 1M
Normalized $p_{\rm t}^{\rm lep}$	ρ_l	1E, 1M
Largest momentum	$p^{\rm max}$	0L
Transverse momentum of the particle with the largest momentum	$p_{\rm t}^{\rm max}$	0L

5. Analysis of τ production in CNGS beam

The average neutrino energy of CNGS beam is about 17 GeV. It is dominated by the ν_μ component with small admixture of other flavors (0.8% of ν_e, 2.1% of $\bar{\nu}_\mu$, and 0.07% of $\bar{\nu}_e$, see Ref. [12]). Because of the mass of τ (1.77 GeV) the neutrino threshold energy for its production on a free nucleon is 3.46 GeV. In the CNGS beam the energy of most of the ν_τ which would appear in the oscillations of $\nu_\mu \to \nu_\tau$ would be above threshold for the production of τ.

The baseline of CERN–GranSasso beam is $L = 730$ km. The CNGS neutrino energy spectra at Gran Sasso are shown in Fig. 4. The left panel shows the beam composition as produced at CERN. At Gran Sasso it is very slightly affected by oscillations. The right panel shows tau neutrinos (antineutrinos) fluences produced by oscillations. The expected fluence of ν_τ ($\bar{\nu}_\tau$) is very small and the spectrum is dominated by the low energy region.

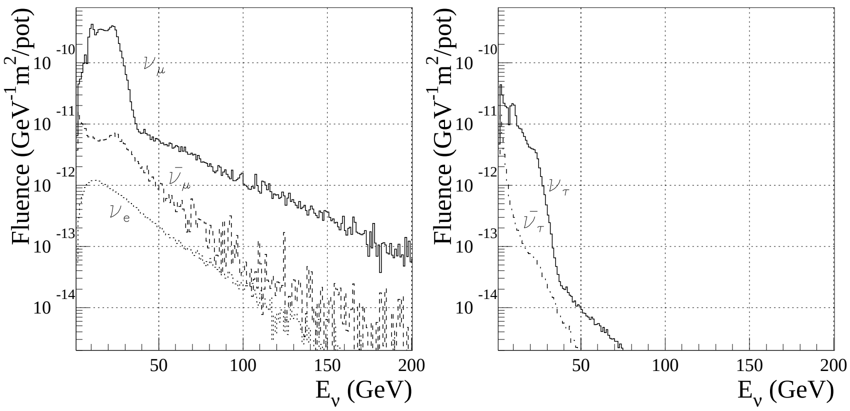

Fig. 4. Fluences of ν_e,ν_μ and $\bar{\nu}_\mu$ (left panel [12]), ν_τ and $\bar{\nu}_\tau$ fluences resulting from oscillations expected at Gran Sasso (right panel).

5.1. Effects of the polarization of τ on the distributions of the kinematical variables

The taons produced by ν_τ in CNGS are expected to be highly polarized. The mean polarization equals to 0.93 for τ^- and 0.95 for τ^+ therefore small differences between samples with calculated polarization and fully polarized samples are expected. A sample with unpolarized taons is used for comparison.

The distributions of the kinematical variables for the events with one charged lepton (classes 1E and 1M) are very similar. The effects of polarization are seen in the average values and in the shapes of the distributions of the variables which depend on the total and transverse momenta. They are clearly visible in the ratio of the distributions for samples with different polarizations. An increase of the average lepton momenta with increasing τ polarization is observed for 1E and 1M classes. This effect can be explained by a fact that the charged leptons are mostly emitted in the direction close to that of the parent τ's when they are polarized. The same effect is observed for the variable $p_{\rm tot}$ where the contribution from lepton momentum is dominant. For the fully polarized sample the mean value of the $p_{\rm tot}$ increases by about 6% as compared to unpolarized τ's. This effect is also visible for the sample with calculated polarization (see Fig. 5).

The effects of polarization are different for the samples with charged leptons and the sample without charge leptons (0L). For 0L class the modification of momenta goes in the opposite direction. The size of the effect on the mean values of $p^{\rm max}$ and $p_{\rm tot}$ is of the same order.

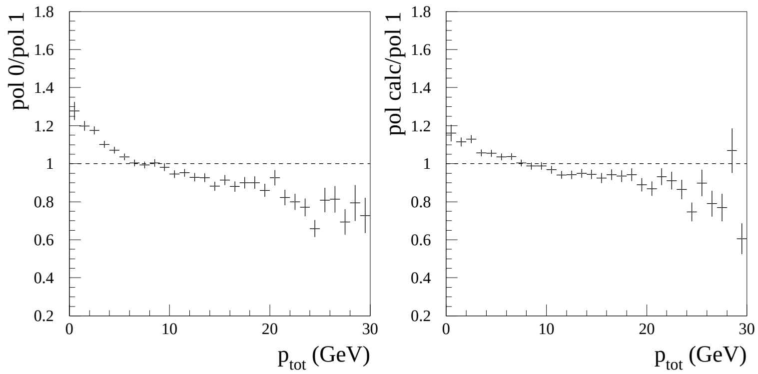

Fig. 5. The ratio of the distributions of the total momentum of the events of 1E class (CNGS beam). Left panel: ratio of the unpolarized sample to the fully polarized sample, right panel: ratio of the sample with calculated polarization to the fully polarized sample.

The events belonging to 0L class consist of the decays of τ into the neutrino and pions or kaons. The mesons have spin 0, therefore the spin of parent τ must be taken by a neutrino. Thus, for a fully polarized taon its direction will be followed by a neutrino. Because of the momentum conservation the other decay products must be emitted in the opposite direction in the τ rest frame. Due to the Lorenz boost this results in smaller momenta in the laboratory frame of backwards emitted particles (see Fig. 6). In particular this effect is strong for the 2 body decay: $\tau \to \nu_\tau + \pi$.

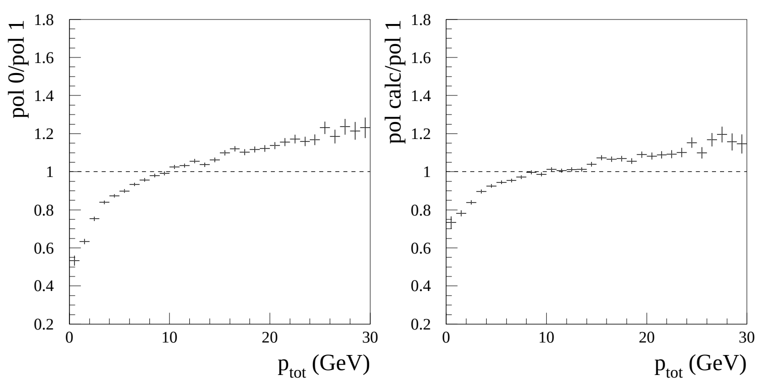

Fig. 6. The ratio of the distributions of the total momentum for the events of 0L class (CNGS beam): left panel: ratio of the unpolarized sample to fully polarized sample, right panel: ratio of sample with calculated polarization to fully polarized sample.

The effects of similar size and opposite for one lepton and 0L classes were observed for transverse momenta of fastest particles ($p_{\rm t}^{\rm lep}$ and $p_{\rm t}^{\rm max}$, respectively). For example, in 0L class the mean $p_{\rm t}^{\rm max}$ is smaller by 5% for sample with full polarization comparing to the sample with calculated polarization.

5.2. The influence of the polarization of τ on the results of the neural network classification

The results of the neural network classification will be presented on the purity *versus* efficiency plots. Classes 1E, 1M and 0L will be discussed separately. The neural networks were trained separately for each class of events and for a given class for each sample of events with different polarization of taons.

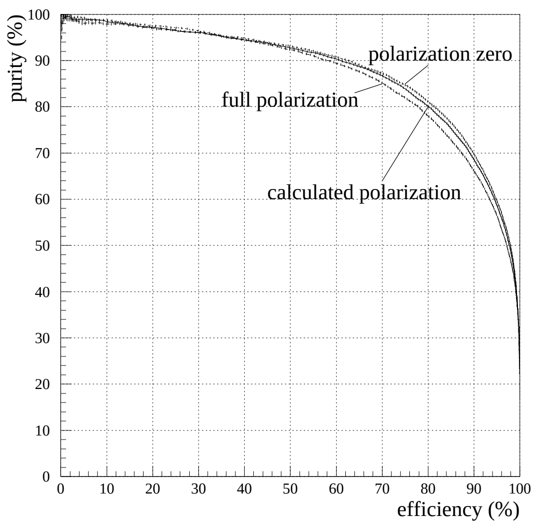

Fig. 7. Purity *vs* efficiency plot: the results are shown for 3 samples with different polarization of τ, for 1E class of events. The estimated signal to background ratio equals to 16.7%.

As it was shown for classes 1E and 1M the distributions of kinematical variables and τ polarizations effects are very similar. The results of classification for these classes (see Fig. 7 and Fig. 8) are however very different due to the different signal to background ratio. It equals 16.7% for 1E class and 0.13% for 1M class.

The results of separation of τ weakly depend on the value of the polarization. Because of the signal to background ratio the 1E class is the most

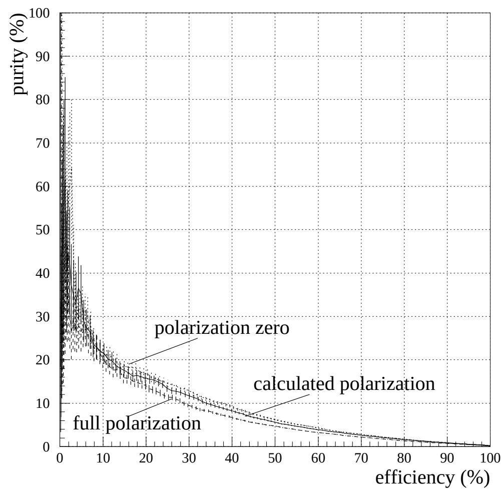

Fig. 8. Purity *vs* efficiency plot: the results are shown for 3 samples with different polarization of τ for 1M class of events. The estimated signal to background ratio equals to 0.13%.

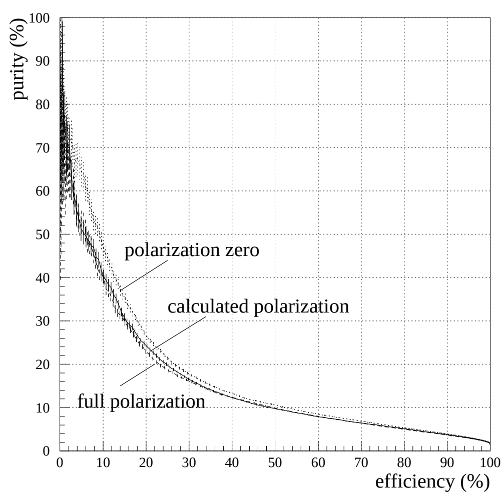

Fig. 9. Purity *vs* efficiency plot: the results are shown for 3 samples with different polarization of τ for 0L class. The estimated signal to background ratio equals to 1.4%.

promising for the τ search. For the 1M class this ratio is very small. These conclusions are specific for the τ searches in ν_μ beam and the taons selection based on the kinematical variables.

The results for 0L class are shown in Fig. 9. The analysis of this class was not considered at the early stage of the search for the oscillation signal in CNGS beam. The analysis of signal selection in the decay channel $\tau \rightarrow \nu_\tau + \rho$ was presented in Ref. [13].

The difference between the purity for fully polarized and unpolarized samples can reach almost 10%. The sample with calculated polarization gives the results similar to the fully polarized sample.

The class 0L may be considered promising as it can provide an additional check on the results obtained for class 1E, which enables better signal from background separation.

6. Analysis of the production of taons by the atmospheric neutrinos

The ν_τ can be observed in the oscillations of atmospheric neutrinos. The distance between the production and the detection points of neutrino depends on the its incident zenith angle. Thus, the flux of appearing ν_τ changes with the zenith angle. In this analysis the detector was assumed to be at the depth of 1 km and the neutrinos were produced 15 km above Earth's surface. Flux of ν_τ and $\bar{\nu}_\tau$ (see Fig. 10) was calculated according to the

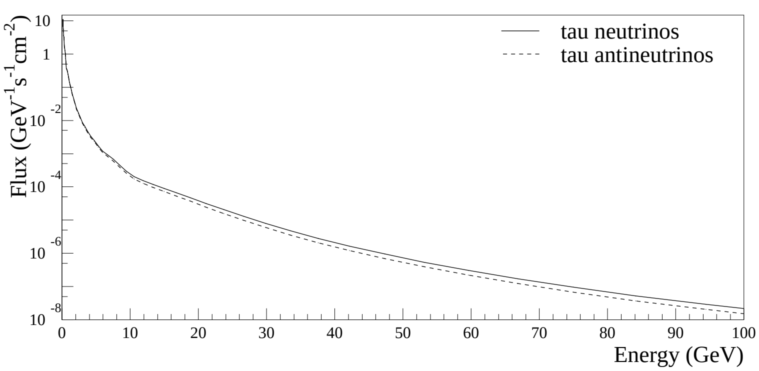

Fig. 10. Flux of ν_τ and $\bar{\nu}_\tau$ resulting from the oscillations of ν_μ and $\bar{\nu}_\mu$. The flux is integrated over the zenith angles $> 90°$.

two-neutrino oscillation probability formula. Only the neutrinos coming at zenith angles $\theta > 90°$ were taken for the analysis. Therefore the baseline for atmospheric neutrinos varies from 450 km up to 13000 km, approximately. Using this flux the signal to background ratio obtained for 1E, 1M and 0L classes was 0.66%, 0.65% and 1.89%, respectively.

6.1. The effect of τ polarization on the distributions of the kinematical variables

The energy spectrum of atmospheric neutrinos is much softer than the CNGS spectrum. In spite of this the leptons τ produced by atmospheric neutrinos are also highly polarized. The mean calculated polarization is 0.90 for τ^- and 0.94 for τ^+. The most important difference between neutrinos in CNGS beam and atmospheric neutrinos is that the true directions of the atmospheric neutrinos are unknown, but have to be reconstructed from the momenta of the produced particles. The differences between the real and reconstructed directions can be large (see Fig. 11). The classification based on the variables calculated with respect to the neutrino direction, will then be worsened. The effect of neutrino direction on the transverse mo-

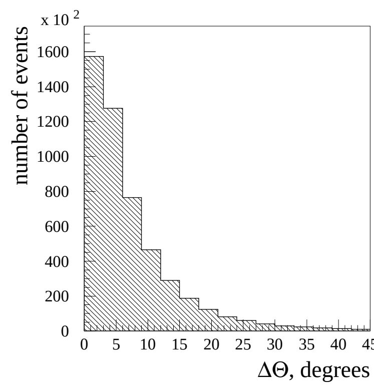

Fig. 11. The angle between true direction of interacting neutrino and the direction reconstructed from the momenta of produced particles for events 1E, 1M and 0L. (Neutrons and secondary neutrinos are neglected in the reconstruction of the direction of the initial neutrino).

mentum $p_{\mathrm{t}}^{\mathrm{max}}$ is quite strong. As an example of this effect the distributions of transverse momenta for 0L class for known and reconstructed directions of neutrinos are shown in Fig. 12. For the 1E and 1M classes the effects of the polarization of taons are observed in the distributions of vector and scalar sum of momenta (p_{tot} and p_{scal}, respectively). This is similar to the effect observed for ν_τ interactions in CNGS beam (see Fig. 5). Comparing to fully polarized sample the mean values of p_{tot} and p_{scal} decrease by about 4% for the sample with calculated polarization and by more than 6% for the unpolarized sample.

In the 0L class, as in CNGS case, the increase of τ polarization softens the momenta distributions for final state particles (p_{tot} and $p_{\mathrm{t}}^{\mathrm{max}}$). With respect to fully polarized sample the mean value of p_{tot} increases by about 6% for the unpolarized sample and by 4% for the sample with calculated

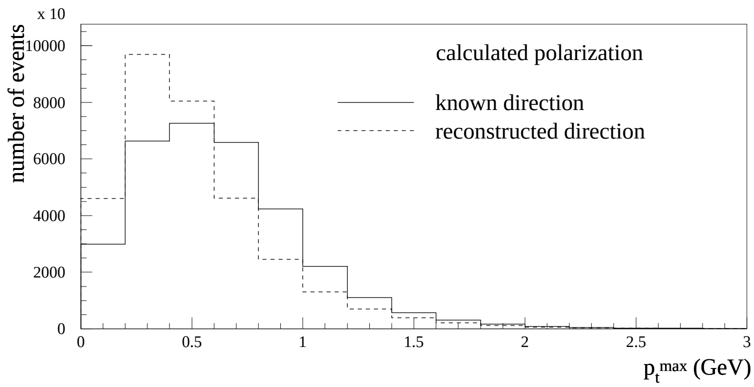

Fig. 12. The distribution of the transverse momentum of the particle with the largest momentum appearing in the event of 0L class for atmospheric ν_τ. The polarization of taons is calculated. The solid and dashed lines denote the results obtained for true and reconstructed direction of the initial neutrino.

polarization. Fig. 13 shows the dependence of the largest momentum ($p_{\rm max}$) in 0L events on the polarization. The mean value of $p_{\rm max}$ increases by 7% for sample with calculated polarization and by 11% for unpolarized τ's with respect to the value for fully polarized sample. In the two-body $\tau \to \nu_\tau + \pi$ decay this increase is almost 12% for the sample with calculated polarization and over 20% for the unpolarized sample. The effects of polarization found in all classes are almost the same as in the CNGS beam. The small differences are due to the different shapes of neutrino energy spectra.

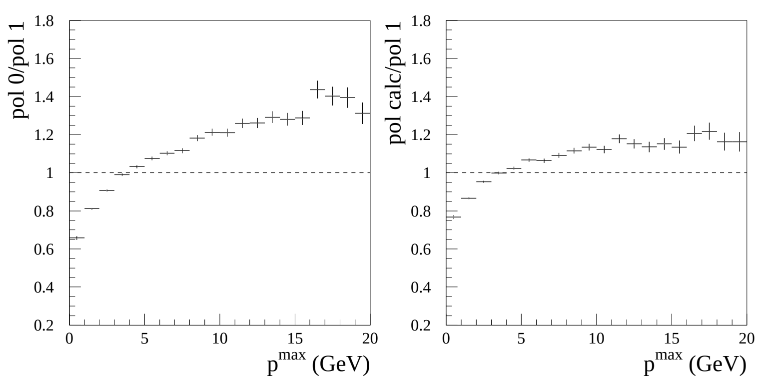

Fig. 13. The ratios of the distributions of the largest momentum appearing in the 0L class event produced by atmospheric ν_τ. Left panel shows the ratio of the distributions of unpolarized to fully polarized samples, right panel for sample with calculated polarization to fully polarized sample.

6.2. The influence of the polarization of taons on the results of the neural network classification

The missing transverse momentum, $p_{\rm t}$, allowed quite effective rejection of the background events in the CNGS case. For the atmospheric ν_τ, the results of the selection of taons obtained with the neural network will be worse than in CNGS beam. The use of the reconstructed direction of the primary neutrino instead of the true direction makes $p_{\rm t}$ variable unusable (it is always 0). The results of the selection of taons are presented on the purity versus efficiency plots for the real and reconstructed direction of the neutrino. The neural networks were trained separately for each case.

In the case of atmospheric neutrinos the signal to background ratio in classes 1E and 1M is the same (0.66%) and the obtained results are very similar. The efficiency–purity plot for class 1E (see Fig. 14) shows the results for three differently polarized samples with either true or reconstructed direction of neutrino.

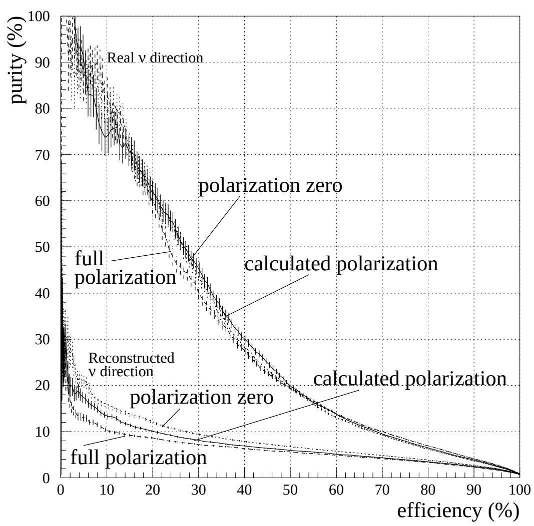

Fig. 14. Purity *vs* efficiency plot for three samples of τ with different polarizations for 1E class of events. Each sample is analyzed assuming either true or reconstructed direction of atmospheric ν. The signal to background ratio equals to 0.66%.

In the classes 1E and 1M the selection ability of the network is affected by the lack of knowledge of real neutrino direction. When the reconstructed direction is used, as in a real experiment, the purity of selected sample is below 10% for 30% efficiency, and the results weakly depend on τ polarization.

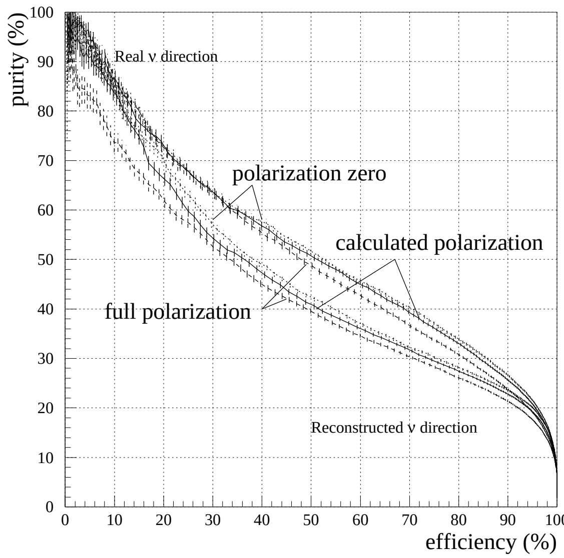

Fig. 15. Purity *vs* efficiency plot for three samples of τ with different polarizations for 0L class of events. Each sample is analyzed assuming either true or reconstructed direction of atmospheric ν. The signal to background ratio equals 1.88%.

On the contrary, in 0L class the classification ability, (shown in Fig. 15), for samples with reconstructed neutrino direction is only a little worse than for samples with real neutrino direction. The reason is that in this class in the τ decay only one neutrino is produced and takes out the energy and momentum. In class 0L the difference in the purity obtained for the fully polarized and unpolarized sample can reach 10%. The results for the sample with calculated polarization are as expected between those obtained for the unpolarized and fully polarized τ's. The results obtained in the neural network classifications for 0L class are promising.

7. Selection of taons decaying without charged lepton

As it was discussed in previous sections taon polarization has little influence on the efficiency of taon selection. This is seen from efficiency–purity plots (see for example Fig. 14). There are however two observations which demand some explanation. In the interactions of atmospheric neutrinos, where the direction of the initial particle is unknown, the efficiency of selection of taons in the events belonging to 0L class (no charge lepton is produced) is higher than in 1E class (an electron is produced).

This can be understood as a result of the difference between the distributions of the total momentum of particles for background samples in 0L and 1E classes. In 0L class background is dominated by neutrino NC interactions, in which the unobserved secondary neutrino takes most of the momentum. Therefore the total momentum of observed particles is small. It is not the case for 1E events where the background is dominated by CC electron neutrino interactions, and the momentum of the secondary electron is measurable. Taon production, because of its mass, has an energy threshold. Because of this for signal events in class 0L and 1E the distributions of the total observed momentum are similar.

The discussed distributions are shown in Fig. 16 (top and middle panels). Therefore the differences in background distributions explain the differences in efficiency–purity plots for 0L and 1E classes. It should be stressed that this conclusion is valid for low energy beam, atmospheric neutrinos in this case. The initial neutrino energy is much higher in CNGS beam than in

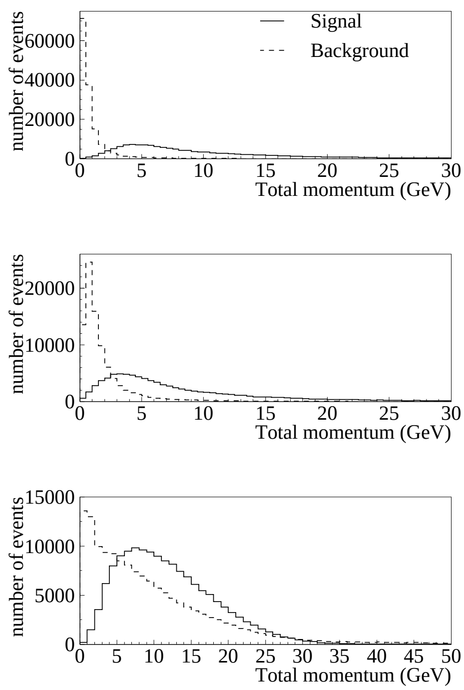

Fig. 16. Distributions of the total momentum for background (dashed line) and signal (solid line): — class 1E, atmospheric neutrinos, bottom panel — class 0L, CNGS neutrinos.

atmospheric neutrinos. The effects of energy threshold for taon productions are less important and the differences between the total momentum distributions for signal and background events in 0L class are much smaller than in a case of atmospheric neutrinos (see Fig. 16, bottom panel). Therefore in the case of 0L class events produced by CNGS neutrino beam the calorimetric variable — total momentum distribution — is much less effective in the signal — background separation than in low energy neutrino interactions.

8. Summary and conclusions

The reaction $\nu_\tau + A \to \tau + X$ was studied. The energy spectra of the interacting ν_τ were obtained by applying two neutrino oscillation formula to the energy spectrum of CNGS ν_μ beam and to the flux of atmospheric neutrinos.

The events of interactions of neutrinos were generated with NUANCE code. The analysis was performed for events with the production of unpolarized taons, fully polarized taons and taons with calculated polarization. The events with 1 electron or 1 muon or without charged leptons were considered.

The analysis was done using either true or reconstructed direction of the atmospheric ν_τ. The knowledge of the true direction of the incoming neutrinos is important for the selection of taons among the events with charged leptons.

The influence of the taon polarization on the distributions of kinematical variables of ν_τ CC events was studied. Few percent differences between the distributions of kinematical variables were found for the unpolarized sample and the sample with full polarization. The results of correctly treated polarization of taons and the correctly treated nuclear effects confirm this observation — the calculated polarization has few percent influence on the distribution of the kinematical variables in all classes.

The method of neural network was used for the separation of signal events (CC interactions of ν_τ) from background composed of NC interactions of all ν flavors and CC interactions of all neutrinos except ν_τ.

The separation of CC ν_τ interactions using neural network looks very promising. It was performed in multi dimensional space. To reduce the dominant background influence the non standard error function was used in the neural network teaching procedure.

The results of separation of signal from background weakly depend on the polarization of taons. The obtained purity and the efficiency of signal sample depends drastically on signal to background ratio. For low energy neutrino beam the results of taon selection obtained for events without charge leptons can be used as a cross check for the results obtained for events with one charged lepton.

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