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THE STUDY OF FUSION OF DIFFERENT ISOTOPES/ISOTONES LEADING TO THE SAME COMPOUND NUCLEUS

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We present the fusion of series of reactions of isotopes/isotones leading to the same compound nucleus. This is achieved by transferring either neutrons or protons from one to the other colliding nucleus. Our findings for the normalized barrier heights, positions as well as fusion cross-sections suggest that such fusion of transfer reactions can be parameterized in terms of an asymmetry power law with power factor close to two. This power factor is nearly the same as has been reported for the asymmetric term in mass formula.

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1. Introduction

Recently, large efforts are reported for the fusion of heavy-ions in the neutron–proton plane along the drip line [1–5]. This domain is also exploring several new dimensions in nuclear physics. The recent efforts in this direction are concentrated on the fusion of nuclei leading to compound nuclear mass $A \approx 100$, where a large enhancement is reported in the corresponding fusion cross-sections for neutron-rich nuclei [1–6]. Interestingly, these experimental findings can be categorized into two domains.

In the first case, the projectile is kept fixed and targets are chosen as different isotopes. For example, ^{27}Al+^{70,72,73,74,76}Ge [7], ^{40}Ar+^{112,116,122}Sn, ^{144,148,154}Sm [8], ^{16}O+^{148,150,152,154}Sm [9], ^{16}O/^{32}S+^{112,116,120}Sn [10], *etc.* This leads to the dynamics of asymmetric colliding nuclei. In the second category, both the target and projectile are varied as different isotopes of the same combinations. For example, ^{32,34}S+^{24,26}Mg [11], ^{16,18}O+^{24,26}Mg [12], $^{16-18}$O+$^{112,116-120,122,124}$Sm [13], ^{16,18}O+^{58,60,64}Ni [14], ^{28,30}Si+^{58,62,64}Ni, ^{32,34,36}S+^{58,64}Ni [15], ^{32,36}S+^{92,94,96,98,100}Mo,$^{100-102,104}$Ru, $^{104-106,108,110}$Pd

[16], ^{46,50}Ti+^{60,64}Ni [17], *etc.* Note that the above listed reactions do not lead to the same compound nucleus [1–17].

One of the interesting questions would be how the fusion dynamics depends on the isotopic/isotonic nature of the colliding pair forming the same compound nucleus. Before that, let us first define the different asymmetries: *(i)* the neutron-asymmetry: $(\eta_N)_{AZ} = \left[\frac{N_1-N_2}{N_1+N_2}\right]$ and *(ii)* charge-asymmetry: $(\eta_Z)_{AZ} = \left[\frac{Z_1-Z_2}{Z_1+Z_2}\right]$ [18]. The (AZ) represents the formation of the same compound nucleus. In the literature, several reports can be seen with contrary effects of the above kind. For example, the variation of η_N from 0.07 to 0.20 in ^{A_1}Mg + ^{A_2}S $[(A,Z) = (58,28)]$ reactions decreases the barrier height and increases the barrier position, whereas η_N variation from 0.36 to 0.44 in ^{A_1}Si+^{A_2}Ni reactions reports just reverse trends with increase in the barrier height and a decrease in its position [15]. Large number of such cases can be seen in references [7–17].

These contradictory claims in the literature demand a systematic study of the effect of the isotopic/isotonic ratio on the fusion leading to the same compound nucleus.

2. The theoretical framework

For the theoretical study of fusion dynamics, the starting point is to define a realistic nuclear potential $V_{\rm N}(R)$. By adding the Coulomb potential $V_{\rm C}(R)$, one can define the total potential $V_{\rm T}(R)$ as

$$V_{\rm T}(R) = V_{\rm N}(R) + V_{\rm C}(R)\,. \tag{1}$$

Since, we want to study the normalized quantities, the contribution of angular momentum potential will have no effect. In other words, the study is limited to head on collisions only.

The barrier height $\rm V_B$ and position $\rm R_B$ are, then, determined from the condition

$$\frac{dV_{\rm T}(R)}{dR}\Big|_{R=R_{\rm B}} = 0\,, \qquad \text{and} \qquad \frac{\rm d^2V_T(R)}{\rm dR^2}\Big|_{\rm R=R_B} \leq 0\,. \tag{2}$$

Once barrier parameters are defined, one can calculate the fusion cross-section as $\sigma_{\rm fus}\,(mb) = 10\pi R_{\rm B}^2\left[1-\frac{V_{\rm B}}{E_{\rm cm}}\right]$ with $E_{\rm cm}$ being the centre of mass energy. This is a simple sharp cut off fusion cross-section where coupled channel effects do not play a role. This formula is valid for the above Coulomb barrier energies only. For a systematic normalized study, this is a good approximation. We shall use the Skyrme Energy Density Model and Ngô–Ngô potential for the nuclear part of the potential.

In the Skyrme Energy Density Formalism (SEDF) [19], the $V_{\rm N}(R)$ is calculated as a difference of the energy expectation $E = \int H(\vec{r})\, d\vec{r}$ of colliding nuclei at a separation distance R and at complete isolation (*i.e.* at ∞)

$$\begin{aligned} V_{\rm N}(R) &= E(R) - E(\infty) \\ &= \int \left[H(\rho,\tau,J) - H_1(\rho_1,\tau_1,J_1) - H_2(\rho_2,\tau_2,J_1)\right] dr\,, \end{aligned} \tag{3}$$

where $H(\rho,\tau,J)$ is the Skyrme Hamiltonian density comprising of nucleonic density (ρ), kinetic energy density (τ) and spin density ($\vec{J}$). For details, reader is referred to Ref. [19]. The density parameters, namely the half density radius R and surface diffuseness a are taken from Ref. [19]. The central densities ρ_{0i} are then computed from the relation

$$\rho_{0i} = \frac{3A_i}{4\pi R_{0i}^3} \frac{1}{\left[1 + \frac{\pi^2 a_i^2}{R_{0i}^2}\right]}\,. \tag{4}$$

We shall use SIII force for present analysis.

In the second approach, Ngô–Ngô [20] parameterized the potential in a proximity fashion where nucleus–nucleus potential can be divided into a geometrical factor and universal function. The nuclear part of the parameterized potential is written as [20]

$$V_{\rm N}(R) = \overline{R}\Phi(s)\,, \tag{5}$$

where $\Phi(s)$ is the universal function given in Ref. [20]. The radius is calculated as

$$R_i = \frac{NR_{n_i} + ZR_{p_i}}{A_i}\,, \tag{6}$$

The sharp radii for protons and neutrons are given by

$$R_{p_i} = r_{0_{pi}} A_i^{1/3}\,, \qquad R_{n_i} = r_{0_{ni}} A_i^{1/3}\,, \tag{7}$$

with

$$r_{0_{pi}} = 1.128~{\rm fm}\,, \qquad r_{0_{ni}} = 1.1375 + 1.875 \times 10^{-4} A_i\,. \tag{8}$$

The different radii formula for the neutron and proton takes the isotopic dependence into account. The expressions for the central neutron and proton densities is then given by

$$\rho_n(0) = \frac{3}{4\pi}\frac{N}{A}\frac{1}{r_{0n}^3}\,, \qquad \rho_p(0) = \frac{3}{4\pi}\frac{Z}{A}\frac{1}{r_{0p}^3}\,. \tag{9}$$

3. Results and discussion

Using the above mentioned formalisms, we shall study the effect of isotopic/isotonic ratio on the fusion of nuclei leading to the same compound nucleus. In Fig. 1 we display the relative importance of the variation of η parameters (either η_Z or η_N) on the fusion of colliding nuclei leading to the same compound nucleus.

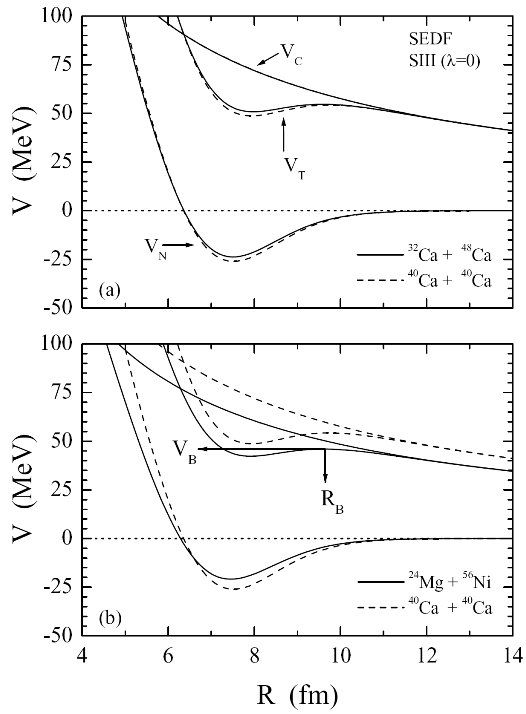

Fig. 1. The nuclear potential $V_{\mathrm{N}}(R)$, Coulomb potential $V_{\mathrm{C}}(R)$ and total potential $V_{\mathrm{T}}(R)$ (in MeV) as a function of the inter-nuclear distance R(fm). In part (a) we display ^{40}Ca+^{40}Ca and ^{32}Ca+^{48}Ca, whereas in part (b) we display ^{40}Ca+^{40}Ca and ^{24}Mg+^{56}Ni, respectively.

In the upper part, we display ^{32}Ca+^{48}Ca with $\eta_N = 0.4$, whereas in the lower part, we display the collision of $^{24}_{12}$Mg+$^{56}_{28}$Ni with $\eta_Z = 0.4$. In both the cases, the content of η variation (*i.e.* either η_N or η_Z) is the same (equal to 0.40). We also show the symmetric reaction of ^{40}Ca+^{40}Ca with $\eta_N = \eta_Z = 0$ for comparison . We notice that, the nuclear part $V_{\mathrm{N}}(R)$ is quite similar in both the cases, whereas due to the stronger variation in the Coulomb potential, a significant variation can be seen for the barrier with η_Z variation. In contrary, the change in the barrier for $^{32}_{20}$Ca+$^{48}_{20}$Ca reaction over $\eta_N = 0$ (^{40}Ca+^{40}Ca) is insignificant. Similar nuclear potential in both the cases is not surprising because if one looks in terms of proximity concept (where nuclear potential $V_{\rm N}$ can be written in terms of a geometrical factor ($2\pi\overline{R}$) and an universal constant ($\Phi(s)$), the change in the nuclear potential is proportional to the $\overline{R}$ variation which, in both the cases, is same leading to same nuclear potential. Further, due to strong Coulomb variation, η_Z variation should have significant impact compared to η_N variation.

As stated in the introduction, no experiments analyze the barrier parameters ($V_{\rm B}$ and $R_{\rm B}$) directly. Instead, some theoretical models are employed to extract these informations. Due to different approaches for extractions, no unique pattern exists in literature for the study of fusion of different isotopes/isotones leading to the same compound nucleus. As noted by many authors different extraction techniques can lead to large variation in the barrier results.

For the present study, we take two series of reactions leading to compound masses $A = 80$ and 120. We here start with symmetric $A_1 = A_2$ reaction and then transferred only neutrons in the case of η_N and only protons in the case of η_Z. In the first case, the proton number is kept fixed as that for $A_1 = A_2$ pair. Similarly, for the second case, neutron number is kept fixed. For example, for $A = 80$, we started with ^{40}Ca+^{40}Ca and then transferred neutrons to either of the colliding nuclei so that one has ^{38}Ca+^{42}Ca, ^{36}Ca+^{44}Ca, ^{34}Ca+^{46}Ca, ^{32}Ca+^{48}Ca, ^{30}Ca+^{50}Ca and so on. On the other hand, for η_Z study, we started with ^{40}Ca+^{40}Ca and then transferred protons so that one has ^{38}Ar+^{42}Ti, ^{36}S+^{44}Cr, ^{34}Si+^{46}Fe, ^{32}Mg+^{48}Ni, ^{30}Ne+^{50}Zn, ^{28}O+^{52}Ge, ^{26}C+^{54}Se and ^{24}Be+^{56}Kr.

In Fig. 2 we display the variation in the barrier position

$$\Delta R_{\rm B}\% \left[= \frac{R_{\rm B} - R_{\rm B}^{\rm sym}}{R_{\rm B}^{\rm sym}}\%\right] ,$$

barrier height $\Delta V_{\rm B}\%$ and fusion cross-section $\Delta\sigma_{\rm fus}\%$ over the symmetric colliding nuclei leading to $A = 120$ units for η_N. The $\Delta\sigma_{\rm fus}\%$ is calculated at $E_{\rm cm}$ =1.25V$_{\rm B}^0$. Here we display three different calculations: (a) the potential calculated within Skyrme energy density model with H. de Vries density [21] is labelled as SDV, (b) the above mentioned SEDF potential with density parameters taken from the works of Ngô–Ngô (labelled as SN), and (c) the potential parameterized by Ngô–Ngô. We see only a marginal decrease in the barrier positions throughout the η_N variation. As a result, marginal increase in the barrier heights can be noticed. These variations results in small decrease in $\sigma_{\rm fus}$ at $E_{\rm cm}$ =1.25V$_{\rm B}^0$. All the variations in $\Delta R_{\rm B}\%$, $\Delta V_{\rm B}\%$ and $\Delta\sigma_{\rm fus}\%$ can be nicely parameterized in terms of asymmetric power law proportional to η_N. One also notices that the power factor is close to two. The effect of η_N tends to be stronger for $\eta_N \geq 0.5$.

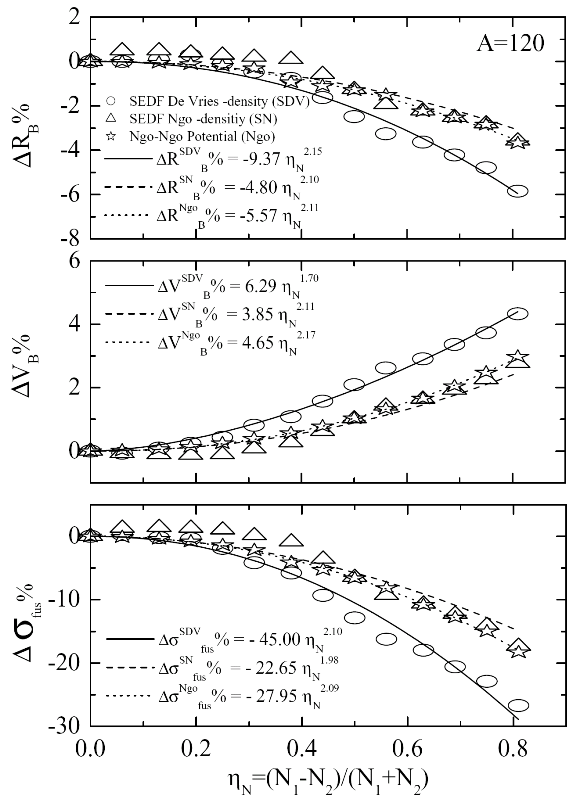

Fig. 2. The $\Delta R_{\rm B}\%$, $\Delta V_{\rm B}\%$ and $\Delta\sigma_{\rm fus}\%$ as a function of η_N. We display the results using SDV, SN and Ngô potential (see text) for the compound masses $(A,Z)=(120,56)$. The lines are the power law fits made using χ^2 minimization.

Further, we also see that the SN and Ngô potentials are quite close whereas SDV potential differs quite significantly. This points towards the dependence of barrier study on technical parameters like the density profile of the colliding nuclei. However, in all the cases, the power factor remains close to two. This can be understood in terms of asymmetric term of mass formula which also has similar dependence.

In Fig. 3 we display the results for η_Z variation using SEDF only. Here we also present the mass dependence by taking compound masses equal to 80 and 120 units. The displayed quantities are the $\Delta R_{\rm B}\%$, $\Delta V_{\rm B}\%$ and $\Delta\sigma_{\rm fus}\%$ as a function of η_Z. As mentioned above, the difference between η_N and η_Z variation is that in the former case, the product of charges $Z_1 \cdot Z_2$ is fixed whereas in the latter one, there is a drastic change in the product. We see that the $\Delta R_{\rm B}\%$ shows again a mild dependence whereas $\Delta V_{\rm B}\%$ can be seen with a massive dependence, as a result $\Delta\sigma_{\rm fus}\%$ has also drastic influence of η_Z, compared to η_N where the effect was insignificant. As reported in the case of η_N, we here again notice a power law in terms of η_Z. Due to different trends in $\Delta V_B\%$ and $\Delta R_{\rm B}\%$ variations, a sharp increase in the fusion probabilities can be seen. This also hints towards the formation of super-heavy elements with such reactions. The above conclusion is not affected by the change in the nuclear potential or by using different density profiles. Again, we found the power law factor is ≈ 2 pointing towards power law of asymmetric term. In both cases, the mass dependence of compound nucleus does not play any role. This also points toward universal nature of our present study.

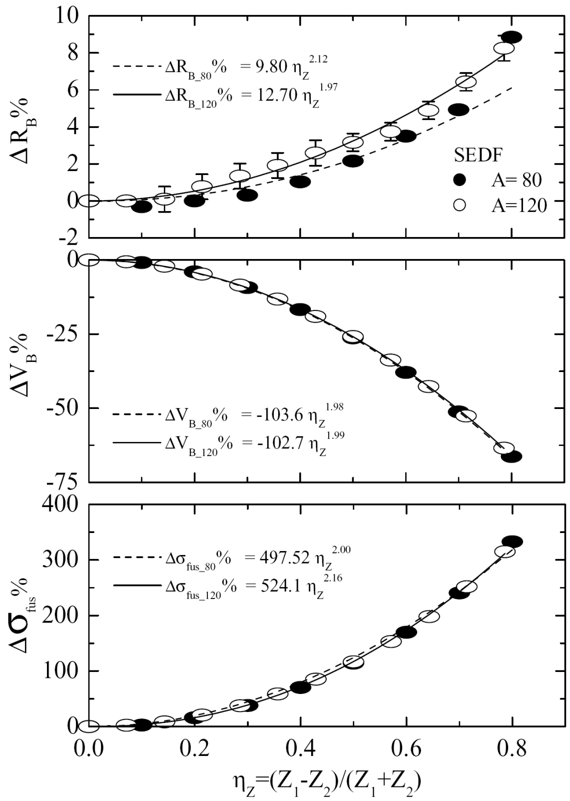

Fig. 3. Same as Fig. 2, but for η_Z variation using SEDF. Here compound masses chosen are $(A, Z) = (80, 40)$ and $(120, 56)$, respectively.

In case of η_Z variation, the major contributor, the Coulomb potential has a different story. Now, its variation is dominated by the two competing factors: (a) the reduction in the Coulomb potential due to increase in the barrier position ($\propto \frac{1}{R_{\rm B}}$) with η_Z and (b) the reduction in the Coulomb contribution with η_Z due to product Z_1Z_2. As η_Z increases, the product (Z_1Z_2) tends to decrease drastically. The dominance of the later factor is stronger, therefore, we see a overall steep reduction in the Coulomb contribution at the barrier and also in the barrier heights with increase in η_Z parameter. In both cases, we see that additional change of barrier parameters over the symmetric colliding nuclei has a similar asymmetric dependences as known for the asymmetric term of the mass formula.

4. Summary

Summarizing, within the framework of microscopic energy density formalisms, we presented a systematic study of the effects of isotopic/isotonic variations in the fusion dynamics leading to the same compound nucleus. This study was performed by transferring neutrons from one nucleus to another, in one case, whereas transferring the protons in the other case. In other words, in the former case, the proton number is conserved whereas neutron number is conserved in the second case, thus keeping the same compound nucleus. We find that the isotopic variation does not have significant impact whereas isotonic effects are stronger in fusion studies. Both effects can be parameterized in terms of asymmetric power law with power factor close to two. The stronger effect in isotones is due to variation in the Coulomb strength. The additional variation of the fusion due to neutron/charge asymmetry yields a asymmetric power law with power factor $\propto 2$. This has similar dependence as has been obtained for the asymmetric term in mass formula.

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