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DYNAMICS OF THE MODEL OF THE CAENORHABDITIS ELEGANS NEURAL NETWORK

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The model of the neural network of nematode worm *C. elegans* resulting from the biological investigations and published in the literature, is proposed. In the model artificial neurons $S_i \in (-1, 1)$ are connected in the same way as in the *C. elegans* neural network. The dynamics of this network is investigated numerically for the case of simple external simulation, using the methods developed for the nonlinear systems. In the computations a number of different attractors, *e.g.* point, quasiperiodic and chaotic, as well as the range of their occurrence, were found. These properties are similar to the dynamical properties of a simple one dimensional neural network with comparable number of neurons investigated earlier.

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1. Introduction

Dynamics of artificial neural networks with different types of synaptic connections was extensively investigated starting from the smallest networks, for which analytical approach is successive (see *e.g.* [1, 2]), to the large networks, for which only numerical calculations are possible [3–5]. It is well known, however, that the topology of synaptic connections of such networks is much less simple than the topology of the neural networks of living organisms. Only simple animals (*e.g.* some kinds of nematodes) have neural networks consisting of relatively small number of neurons. For such animals investigations of the topology and the values of synaptic connections, as well as the neurons are possible using different methods *e.g.* application of microelectrodes registering action potentials generating by the firing neurons or different types of microscopes [6]. *C. elegans* is a nematode worm of the length of 1.2 mm, with almost transparent body and lives in soil. Its nervous system consists of 302 neurons, each of the size approx. 5 μm. For such small cells using microelectrodes for the investigations of an activity of neural network is not effective. For the observations of *C. elegans* (in particular its neural network) electron microscope combined with the optical microscope with differential interference contrast (DIC) can be used. It is important that this special optical microscope (owing to the transparency of the body) enables to observe living *C. elegans* and register an life activity of its neuron system [7].

Dynamical properties of the artificial neural network with the same structure as the living neural network of *C. elegans* are investigated in this network using the method of numerical simulations and analysis suitable for nonlinear systems.

2. The model

For the years interests in investigations of *C. elegans*, reported in scientific literature, is very large. Lately some sources with the databases containing description of neural network of *C. elegans* have appeared in the literature. In our work we have decided to use the data obtained in Cybernetic Caenorhabditis elegans Project (CCeP) performed at the Keio University in Japan and published in internet [8]; some informations were found also in [9, 10].

Sources of data in the database of synaptic connections are figures and tables in the two papers: Albertson and Thompson [9] — describe synaptic connections and structure in the pharynx and White *et al.* [10] — describe nervous system of the body except pharynx. Database of synaptic connection was constructed in CCeP program under the three policies: there should be one-to-one correspondence between information provided by the original papers and that described in various files in the database; the database should provide necessary information for reconstruction of the network's topology, as in the original paper; the database should provide sufficient information for any user to edit their own data file according to their peculiar request [8, 11]. Data in database are written in ASCII text files. Each synaptic connection is described by several items *e.g.*: the name of the reference neuron, the location and symbol of the synaptic contact, the name of the partner neuron, a comment on an ambiguous or questionable synaptic contact [8, 11].

In the construction of the model of artificial network following assumptions were made. All neurons ($N = 302$) are numbered starting from the beginning of the list in the database and to each name of neuron a sequent number is assigned. In description of one synaptic connection the names of reference and partner neuron are changed into numbers (another words: neuron i-th is connected with neuron j-th). This procedure is applied to the body and the pharynx of the worm. There were however some discrepancies in two databases presented in [11]. The data in these databases agrees just for the first $N_1 = 267$ neurons *i.e.* the part of the neural network located in the body of *C. elegans* without the pharynx. Therefore, we have performed most of calculations for the neural network of *C. elegans* without pharynx.

We assume that the current states of neuron S_i, in the network are as follows:

$$S_i(t+1) = \tanh[gh_i(t) + I(t)]\,, \tag{1}$$

where:

$$h_i(t) = \sum_{j=1}^{N} a_{ij} J_{ij} S_j \tag{2}$$

is the value of the local field, g is the gain parameter, a_{ij} is element of the adjacency matrix (equal 1 when neuron i and j are connected, 0 otherwise) and $I(t) = A\sin(2\pi t/T)$ is the external stimulation acting on each neuron. It was assumed in our calculations, that the synaptic connections J_{ij} between the neurons i and j have a constant value or the random values drawn from the uniform distribution in $[-1, 1]$. Synchronous updating of the states of neurons is used for the calculation of the time evolution of the network. As a starting configuration $\{S_i(0)\}$ random values of all neurons $S_i \in (-1, 1)$ were mostly assumed. In some cases, uniform initial conditions $\{S_i(0) = \text{const.}\}$ were also used. Assumptions for the present model are similar to those used for 1-dimensional model of neural network discussed in [12].

Numerical simulations of the dynamics of the system were performed for the control parameters: gain g and the amplitude of the external stimulation A. In the investigation of the local network dynamics, *i.e.*, dynamics of a single neuron embedded in the network, activities of all neurons $S_i(t_1)$ $(i = 1, 2, 3, \ldots, N)$ in a chosen time t_1 and return maps $S_i(t+T)[S_i(t)]$, showing the character of the Poincaré sections, were calculated, while for the observation of the global dynamical properties of the whole network average deviation

$$\sigma(t) = \frac{1}{N}\sum_{i=1}^{N}[S_i(0) - S_i(t)]^2 \tag{3}$$

and the return map $\sigma(t+T)[\sigma(t)]$ were used.

Numerical experiments were made for $N_1 = 267$ (the body of *C. elegans* without pharynx) and a part of calculations for $N = 302$ (the whole body). Period of external stimulation was $T = 24$, the behavior of the network for other values of T was also monitored.

3. Results and discussion

The statistical properties of the neural network was characterized using the quantities typical for the complex network (see *e.g.* [13, 14]). We have calculated that the average number of neighbors of each neuron is $\langle k\rangle = 13.7$ and the clustering coefficient $C = 0.23$. This value is close to the value $C = 0.28$ found for *C. elegans* by Kim [4].

Following the description of the neural network of *C. elegans* without the pharynx the number of neighbors of each neuron was calculated. Next, the neurons were delivered to the classes with respect to these numbers and the distribution of the neighbors of neurons $P(k)$ was calculated (Fig. 1). As results from Fig. 1, it seems that the neural network under investigation has no scale free properties. However, it should be noticed that calculations of $P(k)$ refer to only one neural network (without pharynx) described in the database [8]. Taking into account this additional part of the network the conclusion that the neural network has no scale free properties may be modified. The number of neighbors for all neurons in the body without and with the pharynx ($N_1 = 267$ and $N = 302$, respectively) are shown in Fig. 2. As we see, the neurons with a small (2, 3) and large (above 40) number of neighbors are located in the body, but in the pharynx number of neighbors does not exceed 18. It is worth while to notice that experimental investigations of the properties of a human brain show that the neural network has the scale free and small world properties [19]. However, results obtained in [19] were averaged over a number of experiments, while our calculations refer to the neural network of one nematode described in CCeP.

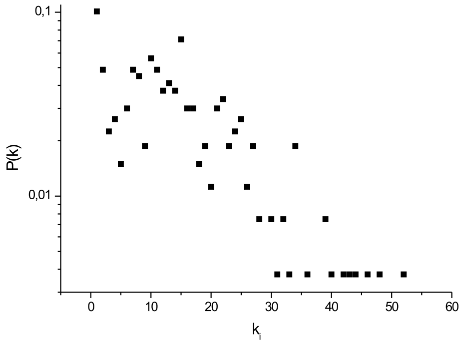

Fig. 1. The distribution $P(k)$ of the neighbors of neurons in the body of *C. elegans* without pharynx.

Let us examine the dynamic properties of the network. One of the properties of neural networks is pattern recognition, in which the network, as a result of its time evolution, reaches point attractor *i.e.* all neurons have certain states corresponding to one of the patterns memorized in the network.

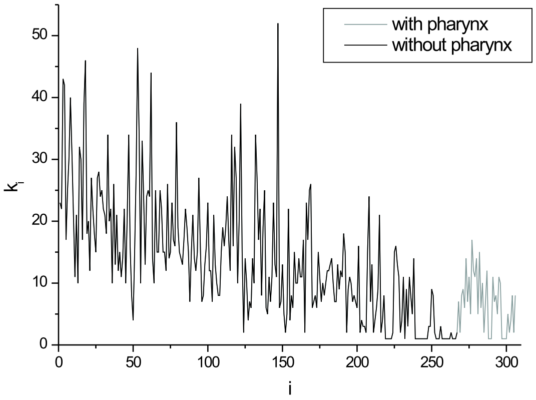

Fig. 2. The number k_1 of neighbors of the i-th neuron in the *C. elegans* without pharynx (black line) and with pharynx (grey line — the left part of the curve it is superimposed on the solid line).

During further time evolution the network remains in this point attractor. Thus, the number of point attractors of the network can be a measure of the memory capacity of the neural network. To study this property we have investigated the network of the nematode's body without pharynx for the case of not active external stimulation ($A = 0$) and the synaptic connections with a chosen constant values (-0.5 or -1.0). In such a case, the network settles on one of a number of coexisting point attractors. The position of each attractor in the space of all network states depends on the value of synaptic connections, the value of the gain parameter g and the initial values of neurons $S_i(0)$. The number of these attractors is very large (and is

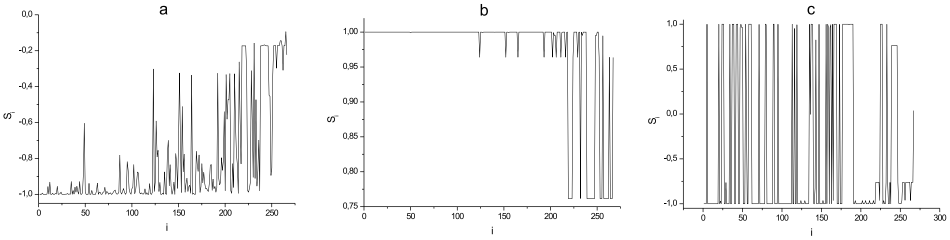

Fig. 3. The examples of the activities S_i of the neurons in the case of coexisting point attractors and no external stimulation $A = 0$. Different initial values of neurons $S_i(0)$, different and constant values of synaptic connections $J_{ij} = -0,5$, -1.0 and -1.0 and gain parameter $g = 0.5$ were used in the cases a, b and c, respectively. (The points corresponding to the states of the neurons were connected with line).

less than the number of the state of the network. Three examples of such point attractors are shown in Figs. 3(a), (b), (c), where the activities of neurons S_i (i=1,2,...,N_1) corresponding to three different point attractors are shown. Each attractor was obtained for different chosen value of synaptic connections J_{ij} and gain parameter g. Comparing Figs. 3(a) and (b) it can be seen that in the case (a) more neurons have the states close to minimal value $S = -1$, while in the case (b) more neurons have the values close to maximal value $S = 1$. In the case shown in Fig. 3(c) numbers of neurons with the values $S = -1$ and $S = 1$ are comparable (and this attractor seems to correspond to more complex pattern than those from Figs. 3(a) and (b)).

Dynamical properties of the network are different for the case of active external stimulation. For the case of external sinusoidal stimulation with the amplitude $A = 1$ and constant value of synaptic connections all neurons during their time evolution follow the stimulation, however, when the random initial conditions of the neurons $S_i(0)$ are applied, their activities $S_i(t)$, i = 1,2,3,...,N_1 are different. In Fig. 4 these activities are shown for times $t = T/3$; $2T$; $3T$, the period of stimulation $T = 24$ and $g = 0.35$. As we see, the curve for $t = 2T$ overlaps the curve for $t = 3T$ which means that the states of all neurons after $\Delta t = T$ have the same value. On the other hand, after $\Delta t \neq mT$ (where $m = 1,2,3,\ldots$) neurons have different values — as we see comparing the curve for $t = 2T$ with the curve for $t = T/3$. Thus we can state that for the values of control parameters the work oscillates periodically with the period equals to the period of external stimulation T.

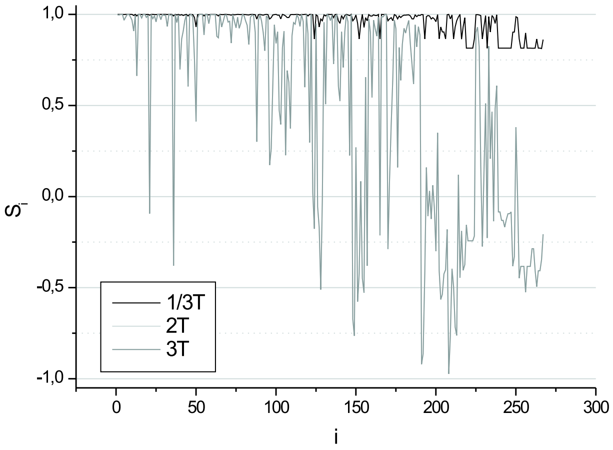

Fig. 4. The activities S_i of neurons for the case of a constant value of synaptic connections, random initial values of neurons, $g = 0.35$, $A = 1$ and three times $t = T/3$; 2T and 3T, as shown in the inset, where used. (The curves for $t = 2T$ and $t = 3T$ are superimposed).

For the case of random values of synaptic connections it is also possible to obtain quasiperiodic evolution of the network. In such a case the oscillations of the network are more complex and some additional characteristic frequencies (besides the frequency of the stimulation) appear in the oscillations. This type of time evolution of the network can be easily found using return maps $S_i(t+T)[S_i(t)]$ or $\sigma(t+T)[\sigma(t)]$, because different kind of loops (sometimes very complex) correspond to such evolution.

Quasiperiodic evolution for the case of no external stimulation ($A = 0$), is presented in Figs. 5(a) and (b), where the return maps: $S_i(t+T)[S_i(t)]$ for a chosen neuron ($i = 45$) and $\sigma(t+T)[\sigma(t)]$, have the form of loops [13]. When the external stimulation is present quasiperiodic evolution also appears. In Fig. 6 the return map $\sigma(t+T)[\sigma(t)]$ for the amplitude of external stimulation $A = 2.2$ and gain parameter $g = 0.8$ is shown. More complex loop than in the previous case shows that in this case the number of characteristic frequencies is grater. For the case of constant value of synaptic connections quasiperiodic evolution of the network was not found.

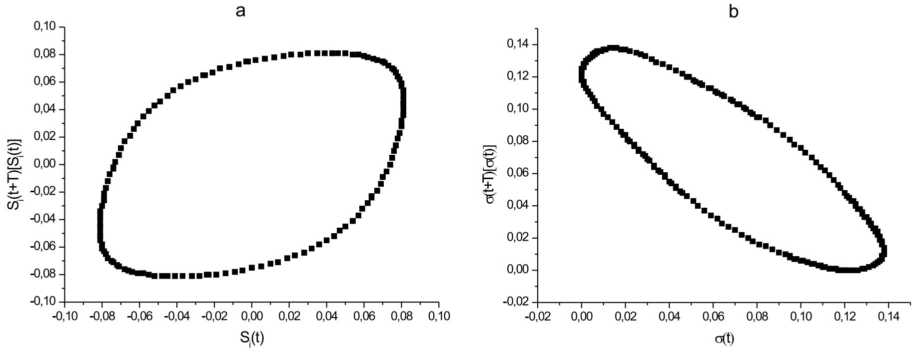

Fig. 5. Quasiperiodic evolution of the network for the case of random values of synaptic connections and $A = 0$; a — the return map $S_i(t+T)[S_i(t)]$ for a chosen neuron ($i = 45$); b — the return map $\sigma(t+T)[\sigma(t)]$.

It is well known that normal dynamics of parts of human brain have chaotic character (*e.g.* neurons connected with the smell sensors) [15]. In the neural network of *C. elegans* chaotic evolution can be observed also, however, similarly to the case of quasiperiodic evolution, only for the case of random values of synaptic connections. This type of evolution is confirmed in Figs. 7(a) and (b), where the return maps $\sigma(t+T)[\sigma(t)]$ (registered in some, sufficiently long time interval), consist of the randomly located points. This form of return map is typical for chaotic dynamics. In Figs. 7(a) and (b) the values of the amplitude of external stimulation were $A = 0$ and $A = 1$, respectively. For other values of A also the chaotic evolution of the network was observed. This type of evolution depends also on the values of the gain parameter.

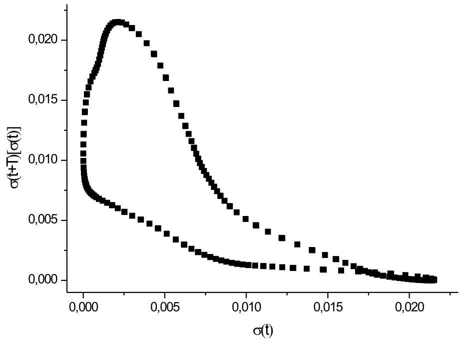

Fig. 6. The return map $\sigma(t+T)[\sigma(t)]$ for the quasiperiodic evolution of the network. The random values of synaptic connections, external stimulation $A = 2.2$ and $g = 0.8$, were used.

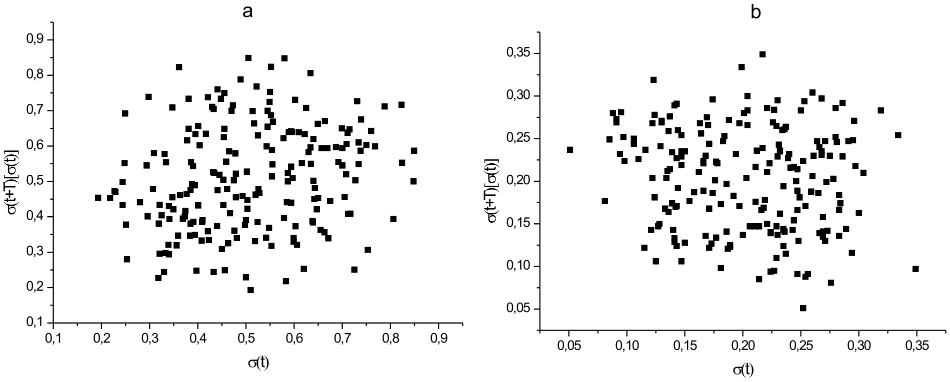

Fig. 7. Return maps $\sigma(t+T)[\sigma(t)]$ for the case of chaotic evolution; (a) — no external stimulation ($A = 0$); (b) — external stimulation $A = 1.0$. The random values of synaptic connections and gain parameter $g = 0.8$, were used.

The ranges of the different types of network dynamics, as a function of the control parameters A and g (and random values of synaptic connections), are shown in Fig. 8. The limits between the ranges corresponding to different types of dynamics are not sharp. (For the defined value of g and increasing values of A the limiting point — triangle — was located if more then 20% computer runs belong to the intermediate range, and square was located if more then 80% computer runs belong to the periodic dynamic). It can be seen that for the gain parameter g approx. greater than 0.37 the chaotic motion occurs when the value of the amplitude of external stimulation A is sufficiently small. For the larger values of A, in the intermediate region between periodic and chaotic evolution, quasiperiodic evolution as well as the evolution with the period $3T$ (where T is the period of external stimulation) are observed. When the amplitude A is sufficiently large the periodic oscillations of the network are observed and they obey the external stimulation.

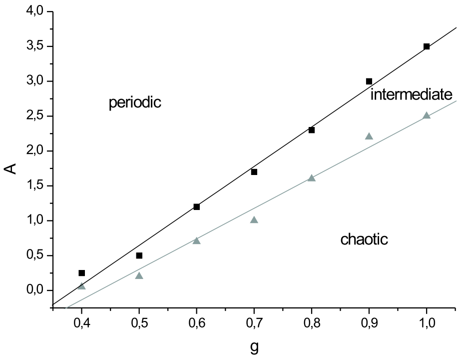

Fig. 8. The ranges of the different types of the network dynamics as a function of the control parameters A and g and the random values of synaptic connections. In the intermediate range quasiperiodic or periodic evolution with the period $3T$, occur. Straight lines were fitted to the calculated points to show the approximate limits of the ranges.

It can be seen from Fig. 8 that for some value of A, with the increasing values of g, the region of quasiperiodic evolution is the transition region between the periodic and chaotic regions. It is worth while to notice that the route to chaos via quasiperiodic motion is one of the main routes to chaos as observed earlier in a number of dynamical systems. In particular the route to chaos by period doubling and bifurcations appearing for a specific values of the gain parameter, was observed in a simplest neural network with $N = 2$ neurons by Wang [1]. Different routes to chaos (including the route via quasiperiodic motion [2]) were found in larger neural network [13,16,17]. In our investigations, however, instead of the period doubling, the route to chaos by the increase of the period of time evolution of certain neurons from T to the value $3T$, was found. This behavior corresponds to Li–Yorke theorem known as "period 3 implies chaos" [12,18].

It is interesting that the results of investigations of the dynamics of the model of neural network with the same topology of synaptic connections as in *C. elegans*, is similar to the dynamics of a simple one dimensional network described in [12]. In particular, point, periodic, quasiperiodic attractors, as well as the chaotic evolution of the network, were found in both cases. The route to chaos has a mixed character — by quasiperiodic evolution or by increase of the period of oscillations to $3T$. The current realization of the route to chaos is determined by the choice of random initial state of the network and the choice of the distribution of random synaptic connections.

In conclusion it can be noticed that the occurrence of the four types of attractors — point, periodic, quasiperiodic and chaotic — shows rather rich dynamics of the neural network with the topology of the neural network of *C. elegans*. It should be stressed however, that we used very simple model of a single neuron and external stimulation. Our results may deliver only preliminary information about real dynamics of *C. elegans* and future investigations in this field are necessary. In particular finding the relations between the attractors of the neural network and the life functions of *C. elegans* (*e.g.* motion of the body or pattern recognition) seems to be very interesting.

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