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# HIGGS AMPLITUDES FROM TWISTOR INSPIRED METHODS*

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We illustrate the use of new on-shell methods, 4-dimensional unitarity cuts combined with on-shell recursions relations by computing the $A_4^{(1)}$ $(\phi, 1^-, 2^-, 3^+, 4^+)$ amplitude in the large top mass limit where the Higgs boson couples to gluons through an effective interaction.

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## 1. Introduction

The time for experiments at the LHC is approaching fast and it will be extremely important to have accurate predictions of Standard Model processes if we hope to find signals of new physics. Recent advances in on-shell techniques [1–3] have made it possible to calculate compact analytic expressions for multiparticle scattering amplitudes at one loop. Here we consider the application of these so called "twistor" inspired methods to one loop amplitudes with a massive, colourless scalar (the Higgs boson) two negative helicity gluons and two positive helicity gluons[1]. This builds on work for amplitudes with simpler helicity configurations [5, 6].

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[1] Analytic expressions for the $\phi$-MHV amplitude an arbitrary number of positive helicity gluons can be found in [4].

We consider the leading colour contribution to the one-loop amplitudes and, following [3], we split them into evaluating the "pure" 4-dimensional cut-constructible $C_4$ and rational $R_4$.

$$A_4^{(1)}(\phi,1^-,2^-,3^+,4^+) = C_4(\phi,1^-,2^-,3^+,4^+) + R_4(\phi,1^-,2^-,3^+,4^+)\,. \quad (1)$$

The pure cut piece contains all (poly)logarithmic terms (log,$\mathrm{Li}_2,\pi^2$) which can be found by computing the unitarity cuts in 4 dimensions [7, 8]. The remaining rational terms can then be evaluated using on-shell recursion relations.

## 2. The model

In the Standard Model the Higgs boson couples to gluons through a fermion loop where the dominant contribution is from the top quark. It is well known that for large $m_{\mathrm{t}}$, the top quark loop can be integrated out leading to the effective interaction,

$$\mathcal{L}_H^{\mathrm{int}} = \frac{C}{2} H\,\mathrm{tr}\,G_{\mu\nu}\,G^{\mu\nu}\,. \quad (2)$$

In the Standard Model, and to leading order in $\alpha_{\mathrm{s}}$, the strength of the interaction is given by $C = \frac{\alpha_{\mathrm{s}}}{6\pi v} + \mathcal{O}(\alpha_{\mathrm{s}}^2)$, with $v = 246$ GeV. $C$ has been calculated up to order $\mathcal{O}(\alpha_s^4)$ [9].

The MHV structure of the Higgs-plus-gluons amplitudes is best elucidated [10] by considering $H$ to be the real part of a complex field $\phi = \frac{1}{2}(H + iA)$, so that

$$\begin{aligned} \mathcal{L}_{H,A}^{\mathrm{int}} &= \frac{C}{2}\left[H\,\mathrm{tr}\,G_{\mu\nu}\,G^{\mu\nu} + iA\,\mathrm{tr}\,G_{\mu\nu}\,{}^*G^{\mu\nu}\right] \\ &= C\left[\phi\,\mathrm{tr}\,G_{\mathrm{SD}\,\mu\nu}\,G_{\mathrm{SD}}^{\mu\nu} + \phi^\dagger\,\mathrm{tr}\,G_{\mathrm{ASD}\,\mu\nu}\,G_{\mathrm{ASD}}^{\mu\nu}\right]. \end{aligned} \quad (3)$$

The amplitudes of the $\phi$ and $\phi^\dagger$ turn out to be much simpler than the corresponding $H$ and $A$ fields and so we proceed by calculating helicity amplitudes for gluons coupling to the $\phi$ and then construct the $\phi^\dagger$ amplitudes using parity symmetry. The full Higgs amplitudes are then made from the sum of $\phi$ and $\phi^\dagger$ amplitudes.

## 3. Cut constructible contributions

The unitarity method relies on sewing together tree-level amplitudes with on-shell propagators. Here we use the method of Brandhuber, Spence and Travaglini [11] which uses the off-shell continuation used for the tree-level MHV rules [12] to sew together tree MHV amplitudes. The subsequent integration over the continuation parameter, $z$, reconstructs only the specific parts of the integral functions which have cuts in the considered channel. The cut integrals that are encountered have been considered previously by Van Neerven [13].

The tree-level QCD amplitudes have been known for some time [14] and the relevant $n$-point tree-level $\phi$-amplitudes have been recently computed using the CSW method [10,15]. The topologies obtained by sewing together all possible configurations of joining a tree-level $\phi$-MHV amplitude with a pure QCD amplitude are shown in figure 1 where both gluons and fermions are allowed to circulate in the loop [4]. The final result is given by [4]:

$$\begin{aligned}
C_n(\phi, 1^-, 2^-, 3^+, 4^+) &= c_\Gamma A_n^{(0)}(\phi, 1^-, 2^-, 3^+, 4^+)\Bigg[U_4 \\
&+\Bigg(\frac{N_P}{3}\frac{\langle 1432\rangle^3}{\langle 12\rangle^3}L_3(s_{341}, s_{41}) + \frac{N_P}{3}\frac{\langle 2341\rangle^3}{\langle 21\rangle^3}L_3(s_{234}, s_{23}) \\
&-\frac{N_P}{2}\frac{\langle 1432\rangle^2}{\langle 12\rangle^2}L_2(s_{341}, s_{41}) - \frac{N_P}{2}\frac{\langle 2341\rangle^2}{\langle 21\rangle^2}L_2(s_{234}, s_{23}) \\
&+\frac{N_P}{6}\frac{\langle 1432\rangle}{\langle 12\rangle}L_1(s_{341}, s_{41}) + \frac{N_P}{6}\frac{\langle 2341\rangle}{\langle 21\rangle}L_1(s_{234}, s_{23}) \\
&+\frac{\beta_0}{N}\frac{\langle 1432\rangle}{\langle 12\rangle}L_1(s_{341}, s_{41}) + \frac{\beta_0}{N}\frac{\langle 2341\rangle}{\langle 21\rangle}L_1(s_{234}, s_{23})\Bigg)\Bigg], \qquad (4)
\end{aligned}$$

where for convenience, we have introduced

$$\beta_0 = \frac{11N - 2N_F}{3}, \qquad N_P = 2\left(1 - \frac{N_F}{N}\right), \qquad L_k(s,t) = \frac{\log(s/t)}{(s-t)^k} \qquad (5)$$

and

$$\begin{aligned}
U_4 &= \sum_{i=1}^{4}\left(\mathrm{F}_3^{1\mathrm{m}}(s_{i,i+2}) - \mathrm{F}_3^{1\mathrm{m}}(s_{i,i+3})\right) - \frac{1}{2}\sum_{i=1}^{4}\mathrm{F}_4^{1\mathrm{m}}(s_{i,i+2}; s_{i,i+1}, s_{i+1,i+2}) \\
&-\frac{1}{2}\sum_{i=1}^{4}\mathrm{F}_4^{2\mathrm{me}}(s_{i,i+3}, s_{i+1,i+2}; s_{i,i+4}, s_{i+1,i+3})\,. \qquad (6)
\end{aligned}$$

The one-mass triangle $\mathrm{F}_3^{1\mathrm{m}}$ and box functions $\mathrm{F}_4$ can be found in [7].

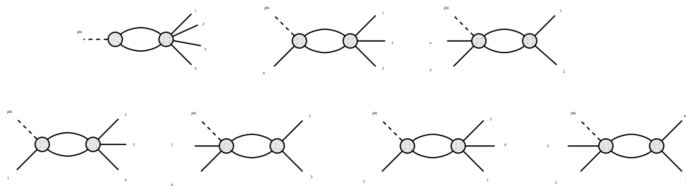

Fig. 1. The topologies for the cut-constructible part of the four gluon $\phi$-MHV amplitude.

## 4. Rational contributions

We calculate the rational part using the unitarity bootstrap proposed by Bern, Dixon and Kosower [3] which generalised the tree level recursion of Britto, Cachazo and Feng [16]. The method relies on simple complex analysis and the factorisation properties of one-loop amplitudes [17]. In order to use this method it is very important to first remove all spurious singularities from the pure cut terms. Once this is achieved the remaining rational terms can be calculated using a recursion relation.

The spurious poles in Eq. 4 appear in the functions $L_2$ and $L_3$, which can be removed by replacing these functions with new functions $\widehat{L}_2$ and $\widehat{L}_3$

$$L_i(s,t) = \widehat{L}_i(s,t) + \frac{1}{2(s-t)^{i-1}}\left(\frac{1}{t}+\frac{1}{s}\right), \qquad i = 2,3\,. \tag{7}$$

The additional rational terms must then be subtracted off again such that $C_4 = \widehat{C}_4 + CR_4$ with $CR_4$ given by:

$$CR_n(\phi, 1^-, 2^-, 3^+, 4^+) = C_4[L_1, L_2, L_3] - C_4[\widehat{L}_1, \widehat{L}_2, \widehat{L}_3]\,. \tag{8}$$

The direct recursive terms are calculated by making a shift into complex momenta. In the case of the $\phi$-MHV amplitudes on can avoid all boundary terms and non-factorising 3-point loop amplitudes by choosing $\widehat{p}_1 = p_1 + z|2\rangle[1|$ and $\widehat{p}_2 = p_2 - z|2\rangle[1|$. This shift gives us a recursion relation in terms of lower point $\phi$-MHV amplitudes, known finite $\phi$-amplitudes [5] and QCD amplitudes leading to a rational term $R^D$. The recursive part of the rational contribution is defined by

$$R_4^D = \sum_i \frac{A_L^{(0)}(z)R_R(z) + R_L(z)A_R^{(0)}(z)}{P_i^2}\,. \tag{9}$$

For the $|1|\langle 2|$ shift the contributing diagrams are shown in Fig. 2.

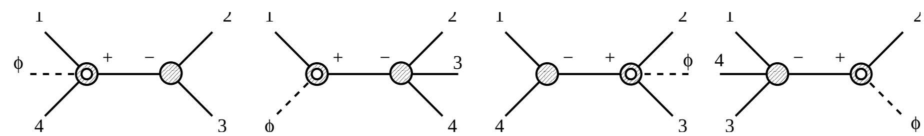

Fig. 2. The direct recursive diagrams contributing to $R_4(\phi, 1^-, 2^-, 3^+, 4^+)$ with a $|1|\langle 2|$ shift.

The final step is to remove overlap terms which appear due to a double counting of poles in $CR_4$. These are computed by evaluating $CR_4(z)/z$ at the poles in recursion:

$$O_4 = \sum_\alpha \frac{CR_4(z_\alpha) P_\alpha(z_\alpha)}{P_\alpha(0)} \,. \tag{10}$$

Collecting results for the four gluon case [4] and constructing the Higgs amplitude yields

$$\begin{aligned} R_4(H; 1^-, 2^-, 3^+, 4^+) &= G(1,2,3,4,\langle\ \rangle,[\ ]) + G(2,1,4,3,\langle\ \rangle,[\ ]) \\ &+ G(3,4,1,2,[\ ],\langle\ \rangle) + G(4,3,2,1,[\ ],\langle\ \rangle)\,, \end{aligned} \tag{11}$$

where

$$\begin{aligned} &G(1,2,3,4,\langle\ \rangle,[\ ]) = \\ &\frac{N_P}{96\pi^2}\Bigg[2\frac{\langle 23\rangle^2[34]^3\langle 41\rangle}{\langle 34\rangle[14]\langle 3|41|3]^2} - \frac{\langle 23\rangle^2[34]^3}{\langle 34\rangle[14]^2\langle 3|41|3]} + 3\frac{\langle 12\rangle\langle 23\rangle[34]^2}{\langle 34\rangle[14]\langle 3|41|3]} \\ &- \frac{\langle 4|13|4]\langle 2|13|4]^2}{s_{341}[41]^2\langle 34\rangle^2} + \frac{s_{412}[34]\langle 23\rangle}{[41][12]\langle 34\rangle^2} + \frac{\langle 2|13|4]\langle 12\rangle}{[41]\langle 34\rangle^2} - \frac{1}{4}\left(\frac{[34]}{[12]} - \frac{\langle 12\rangle}{\langle 34\rangle}\right)^2\Bigg] . \end{aligned} \tag{12}$$

## 5. Conclusions

We have employed four-dimensional unitarity and recursion relations to compute the one-loop corrections to a specific amplitude involving four gluons and a colourless scalar — the Higgs boson. The amplitude presented here may be useful in computing the gluon fusion contamination of the weak boson fusion signal for events containing a Higgs and two jets at the LHC.

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