

TOWARDS NEW MONTE CARLO (QCD + EW) FOR W/Z PRODUCTION AT LHC* **

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An effort of formulating, implementing and testing new parton shower Monte Carlo for W/Z production at Large Hadron Collider (LHC) is presented. In particular, it is indicated how to construct a constrained Monte Carlo (CMC) parton shower algorithm implementing the CCFM-like evolution for a single hadron beam and how to combine two such CMCs into the single MC simulating initial state QCD radiation. Preliminary numerical results are presented.

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1. Introduction

The present contribution summarizes on selected aspects of the research effort lead at IFJ-PAN on the QCD Monte Carlo evolution and on the related project of developing new parton shower Monte Carlo (MC) program for W/Z boson production at the Large Hadron Collider (LHC). More details can be found in Refs. [1–7] and three other presentations at this Conference.

What is the main motivation driving the above efforts? First of all, experiments at Large Hadron Collider will offer a good-quality high-statistics data (millions of events) on the production of the W and Z bosons and their pairs. Hence, it will be highly nontrivial task to exploit these data fully in order to measure very precisely mass of W , anomalous couplings, parton

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luminosities and find indirect signals of the presence of new physics at multi-TeV scales. For this purpose one will need a new class of the Monte Carlo tools, which incorporate in a single package a high quality description of the beyond-the-leading-order QCD, QED and electroweak (EW) calculations. This kind of tools do not exist yet. I shall describe in the following new methods of MC modeling of QCD evolution and parton shower Monte Carlo, which will hopefully will lead us closer to a new combined QCD \otimes EW Monte Carlo precision calculations for LHC.

What are expectations concerning the future precision test of the electroweak sector of the Standard Model (and its extensions) at LHC, beyond what was achieved at LEP era? The main ingredients in the past tests of the EW theory, which has led to confirming it at the quantum level, were (apart from G_F) mass of the Z boson, the ratio of the vector and axial couplings deduced from the charge/spin asymmetry and mass of the t quark. Direct measurements of the W -boson mass did not play the leading role. In fact the indirect determination (through virtual corrections) of the W mass is still more precise (± 20 MeV) than its direct measurement (± 29 MeV) in LEP2 and TEVATRON experiments. The above situation may change dramatically at the LHC era. Preliminary estimates of the future W mass experimental error were in the range 15–25 MeV, see review [8]. Recently, series of ideas were forwarded which may potentially lead to experimental precision of 5–7 MeV, see [9]! Adding better measurements of the top mass and direct measurement of the Higgs mass, will boost significantly precision of the tests of the Standard Model (and its extensions) in the LHC era. It is also well known that LHC experiments are in excellent position to measure (or to eliminate) anomalous couplings of the vector bosons. On the other hand, it looks unlikely that the measurements of the charge asymmetries at LHC could supersede these of LEP. One also hopes, that the so-called parton luminosities and parton distribution functions (PDFs) can be put under $\sim 1\%$ control at LHC, leading to precision measurements of the parton-parton cross sections. All the above will require much better tools for calculating theoretical predictions within EW+QCD theory for the W/Z boson production process at LHC than available presently.

What are minimum requirements (specs) of the theoretical tools for W/Z boson production process at LHC? Generally they should: *(i)* include the first order EW corrections and the complete NLO-level QCD in a common programming environment (package), *(ii)* include QCD matrix elements at least at the NLO level, combined consistently with the parton shower *(iii)* provide unweighted, fully exclusive Monte Carlo events, *(iv)* include exponentiated QED final state radiation for leptonic final states.

Do we have at present MC-calculation tools which fulfill the above specs? For the moment there is no a single example, for any EW observable at LHC, which fulfills the above specs. Complete MC tools for the EW + QCD precision ($\sim 1\%$) predictions for LHC, accompanied with solid tests, are still not available. However, some partial solutions come close to specs. In particular, in recent years progress has been made for “semi-inclusive” QCD observables: NNLO PDFs [10, 11], NNLO distributions [12, 13], matching NLO with parton shower [14], *etc.* Summarizing, more effort is needed to implement the QCD + EW predictions in form of the *high-quality* exclusive MC tools (providing MC events), accompanied with a system of “numerical benchmarks”, better methods of evaluation/estimation of theoretical errors, *etc.*

In our effort, partly presented here, we focus on the QCD initial state radiation (ISR) for the purpose of W/Z production at LHC, believing that: *(i)* Combining resummed (exponentiated) calculations with the fixed order (NLO/NNLO) calculations is the most effective method of improving precision of perturbative QCD predictions. *(ii)* the aim is to improve parton shower MC formulation, before adding NLO corrections¹. *(iii)* the best method of adding the EW corrections is to combine them with the hard process matrix element in the MC implementation.

2. Single hadron QCD evolution using Monte Carlo

Having all the above long term priorities in mind, our march towards a high quality MC for the QCD ISR (+EW) for the W/Z production at LHC has been started with the series of the exercises on the QCD evolution of the integrated and unintegrated PDFs using Monte Carlo methods, translating evolution variables into four-momenta, wherever possible. This was done keeping also in the scope deep inelastic lepton–hadron scattering, as the primary source of our knowledge of the PDFs. In the above activity the main emphasis was on the so-called Constrained Monte Carlo (CMC) [2, 16], in which the energy and type of the outgoing parton in the evolution (shower) is predefined — it is not based on the Markovian algorithm. This new MC technology was developed and used for the first time for Monte Carlos for QCD processes. It will be briefly described in the following section.

On the other hand, the more traditional well know methodology based on the Markovian algorithm [1, 5] was also developed and used for testing newly developed CMCs. The more traditional works on the Markovian MCs (MMCs) include a lot of novel elements, such as pushing MMC numerical calculations to the unheard (for MC) precision level of 0.1%. MMC-programs implement presently the following QCD evolution variants:

¹ Contrary to strategy at MC@NLO project [15], which takes parton shower MC as it is.

- DGLAP LL/NLL cross-checked with QCDNum16 and APCHEB to within 0.2%.
- CCFM-like $\alpha_S(q(1-z))$, $\epsilon_{\text{IR}} = q_0/q$, with quark–gluon (Q–G) transitions.
- CCFM all-loop, $\alpha_S(k^T)$, $k^T > \lambda$, with Q–G transitions.

CMCs available presently feature the following QCD evolution variants:

- DGLAP LL cross-checked with MMC and QCDnum16, including quark–gluon transitions.
- CCFM-like $\alpha_S(q(1-z))$, $\epsilon_{\text{IR}} = q_0/q$, with Q–G transitions, tested with MMC.
- full CCFM all-loop, $\alpha_S(k^T)$, $k^T > \lambda$, agrees with MMC, Q–G transitions already available at the time of writing.

In the prototype of the parton shower MC described below the single hadron shower (evolution) follows CCFM evolution with rapidity ordering and $\alpha_S(k^T)$. The rapidity boundary (maximum) in the parton shower of one hadron beam matches perfectly the rapidity boundary (minimum) of the second showering hadron. The infrared boundary is set by the $k^T > \lambda$ condition, with $\lambda \sim 1\text{GeV}$.

3. New parton shower MC

3.1. Factorization formula for parton shower MC

The parton shower MC is usually formulated not within some well defined factorization framework derived rigorously in perturbative QCD, but is rather based on some kind of a recipe, in which many elements of the common knowledge on the structure (resummation) of the infrared and collinear singularities are combined, resulting in the exclusive multi-parton distribution to be used in the MC. Our basic formula for the QCD ISR in the parton shower reads as follows

$$\sigma = \int_0^1 \frac{dx}{x} \frac{d\bar{x}}{\bar{x}} \sum_{f\bar{f}f_0\bar{f}_0} \int dx_0 \int d\bar{x}_0 \tilde{D}_{f_0}(t_\lambda, x_0) \tilde{D}_{\bar{f}_0}(t_\lambda, \bar{x}_0) \times \mathcal{U}_{ff_0}(t_F, x|t_\lambda, x_0) \mathcal{U}_{\bar{f}\bar{f}_0}(t_B, \bar{x}|t_\lambda, \bar{x}_0) \theta_{\hat{s}>0} \sigma_{f\bar{f}}^{\text{Born}}(\hat{s}).$$

We shall explain step by step all ingredients in the above formula. The following notation was employed:

- $\tilde{D}_f(t, x) = xD_f(t, x)$ is PDF for $f =$ parton type (quark, antiquark, gluon).
- $K = \sum_F k_i$ and $\bar{K} = \sum_B \bar{k}_i$ are total 4-momenta of emitted gluons in the forward (F) or backward (B) hemispheres.
- $\hat{s} = (q_0 + \bar{q}_0 - K - \bar{K})^2 = (px_0 + \bar{p}\bar{x}_0 - K - \bar{K})^2$.
- $q_0 = x_0q$, $\bar{q}_0 = \bar{x}_0\bar{q}$ are 4-momenta of primordial partons at the scale $q_0 = \lambda \simeq 1$ GeV,
- $\mathcal{U}_{ff_0}(t_F, x|t_\lambda, x_0)$ is the *fully exclusive* evolution operator, see below.
- $t_F(x, \bar{x}) = \ln \sqrt{s} + \eta^*(x, \bar{x})$ and $t_B(x, \bar{x}) = \ln \sqrt{s} - \eta^*(x, \bar{x})$, where $\eta^*(x, \bar{x}) = \frac{1}{2} \ln(x/\bar{x})$ is the rapidity boundary between the F/B hemispheres.

The construction of the evolution operator will be described in the following.

3.2. Evolution equation and its solution

The parton distribution in Eq. (3.1) obeys CCFM/DGLAP evolution equation

$$\begin{aligned} \partial_t D_f(t, x) &= \sum_{f'} \int_x^1 du \mathcal{K}_{ff'}(t, x, u) D_{f'}(t, u), \\ \mathcal{K}_{ff'}(t, x, u) &= \frac{\alpha_S(k^\text{T}(t, x, u))}{\pi} \frac{1}{u} P_{ff'}\left(t, \frac{x}{u}\right) \\ &= -\mathcal{K}_{ff}^v(t, u) \delta_{ff'} \delta_{x=u} + \mathcal{K}_{ff'}^\theta(t, x, u) \theta_{u \geq x + \lambda e^{-t}}, \end{aligned} \quad (3.1)$$

where $k^\text{T}(t, x, u) = (u - x)e^t$ and $P_{ij}(t, z)$ are the LL DGLAP kernels. Its solution

$$\tilde{D}_f(t, x) = xD_f(t, x) = \sum_{f_0} \int_x^1 dx_0 \mathcal{U}_{ff_0}(t, x|t_0, x_0) \tilde{D}_{f_0}(t_0, x_0) \quad (3.2)$$

can be conveniently expressed in terms of the evolution operator \mathcal{U} and the initial condition at t_0 . The evolution operator is *the time-ordered exponential* in the combined space of f and x (we use $y_i \equiv x_i - x_{i-1}$)

$$\begin{aligned} \mathcal{U}_{ff_0}(t, x|t_0, x_0) &= e^{-\Phi_f(t, t_0|x)} \delta_{x_0-x} \delta_{ff_0} \\ &+ \sum_{n=1}^{\infty} \sum_{f_0, f_1, \dots, f_{n-1}} \left[\prod_{i=1}^n \int_{t_0}^t dt_i \theta_{t_i > t_{i-1}} \int_{\lambda e^{-t_i}}^{x_0-x} dy_i \right] e^{-\Phi_f(t, t_n|x)} \\ &\times \left[\prod_{i=1}^n \frac{x_i}{x_{i-1}} \mathcal{K}_{f_i f_{i-1}}^\theta(t_i, x_i|x_{i-1}) e^{-\Phi_{f_{i-1}}(t_i, t_{i-1}|x_{i-1})} \right] \delta_{x_0-x=\sum y_j}. \end{aligned} \quad (3.3)$$

It obeys the Chapman–Kolmogorov–Smoluchowski–Einstein identity

$$\sum_{f'} \int dx' \mathcal{U}_{ff'}(t, x|t', x') \mathcal{U}_{f'f_0}(t', x'|t_0, x_0) = \mathcal{U}_{ff_0}(t, x|t_0, x_0)$$

and the normalization rules:

$$\sum_f \int dx \mathcal{U}_{ff_0}(t, x|t_0, x_0) = 1, \quad \mathcal{U}_{ff_0}(t, x|t, x_0) = \delta_{x_0-x} \delta_{ff_0}.$$

The Sudakov form-factor $\Phi_f(t_{i+1}, t_i|x) = \int_{t_{i-1}}^{t_i} dt \mathcal{K}_{ff}^v(t, x)$ enters as usual.

3.3. Relation to unintegrated PDF

The above PDF (solution of the evolution equation) is formulated in a manifest fully exclusive form, hence, a straightforward relation to the so-called unintegrated PDF (uPDF) is defined in a natural way, simply by inserting into the evolution operator an extra δ -function defining the total accumulated transverse momentum of the emitter parton:

$$\begin{aligned} \mathcal{U}_{ff_0}(t, x, \vec{k}^T|t_\lambda, x_0, \vec{0}) &= \sum_{n=0}^{\infty} \frac{1}{n!} \sum_{f_0, f_1, \dots, f_{n-1}} \left[\int \prod_{i=1}^n \frac{dk_i^+ dk_i^- d\varphi_i}{k_i^+ k_i^-} \theta_{\eta_i > \eta^*} \right] \\ &\times \omega_{f_0, \dots, f_n}^n(t, x; t_0, x_0 | k_1^\mu, \dots, k_n^\mu) \delta_{(x_0-x)p^+ = \sum_j k_j^+} \delta_{\vec{k}^T = \sum_i \vec{k}_i^T}, \end{aligned} \quad (3.4)$$

$$\tilde{D}_f(t, x, \vec{q}^T) = \sum_{f_0} \int_x^1 dx_0 \mathcal{U}_{ff_0}(t, x, \vec{q}^T|t_0, x_0, \vec{0}) \tilde{D}_{f_0}(t_0, x_0). \quad (3.5)$$

In other words, uPDF and PDF are related (by construction) in a simple and elegant way

$$\tilde{D}_f(t, x) = \int d^2 \vec{q}^T \tilde{D}_f(t, x, \vec{q}^T). \quad (3.6)$$

It is the so-called *strong factorization* scheme, see Ref. [17, 18].

The above uPDF obeys evolution equation of its own

$$\partial_t D_k(t, x, \vec{q}^T) = \sum_j \int_x^1 du \int d^2 k^T \mathcal{K}_{kj}(t, x, u) D_j(t, u, \vec{q}^T - \vec{k}^T) \delta_{|\vec{k}^T| = (u-x)e^t}. \quad (3.7)$$

Of course, the fully exclusive MC model of \mathcal{U} does even a better job than the above still fairly inclusive uPDF!

3.4. Relation between evolution variables and 4-momenta

In the above formulation of the parton shower MC we use implicitly the following translation of the evolution parameters into four-momenta. More precisely 4-momenta of the emitted particles, their light-cone \pm variables and rapidities are related as follows:

$$k_i = (k_i^+, k_i^-, \vec{k}_{T_i}), \quad \vec{k}_{T_i}^2 = k_i^+ k_i^-, \quad e^{2\eta_i} = \xi_i = \frac{k_i^-}{k_i^+} = \frac{\vec{k}_{T_i}^2}{s k_i^{+2}}. \quad (3.8)$$

Moreover, we use the following parametrization of the “eikonal phase space element”:

$$\frac{d^3 k_i}{2k_i^0} \frac{1}{k_i^- k_i^+} = \frac{d\xi_i dk_i^+ d\varphi_i}{\xi_i k_i^+}.$$

The IR boundary is set on k_i^T , which defined as follows:

$$k_{T_i}^2 = k_i^+ k_i^- = k_i^{+2} \xi_i > \lambda^2, \quad k_i^+ = p_0^+ (1 - z_i) x_{i-1} > \frac{\lambda}{\sqrt{\xi_i}}.$$

We choose the rapidity as the evolution time (CCFM), that is: $q_i = p_0^+ \sqrt{\xi_i}$, where $p_0 = (p_0^+, 0, 0, 0)$ is the primary emitter four-momentum. One can also check that scalar variable q_i is the maximum k_T of the next emission.

3.5. CMC in a nutshell

A short description of the CMC algorithm:

- Mapping of the evolution time $t_i \rightarrow s_i$ and $u_i = x_i - x_{i-1} \rightarrow y_i$, such that Jacobian eliminates completely the (simplified) kernel $zP_{ff}(z, t)$.
- Ordering in $s_i(t_i, y_i)$ temporarily removed (compensated by $1/n!$)
- The constraint $\delta(x - \sum u_i)$ is eliminated/fulfilled by means of the parallel shift $y_i \rightarrow y_i - Y$, see also Fig. 1.
- Quark–gluon transitions modeled with a “brute force” method using general purpose MC simulator FOAM [19].
- Appropriate correcting MC weights is applied at the end.
- For more details see ref. [20] and contribution to 2005 HERA-LHC Conference [21]²
- Such an algorithm is now implemented in CMC (and tested using MMC) for the all-loop CCFM [7], including quark–gluon transitions.

² See also <http://jadach.web.cern.ch/jadach/>

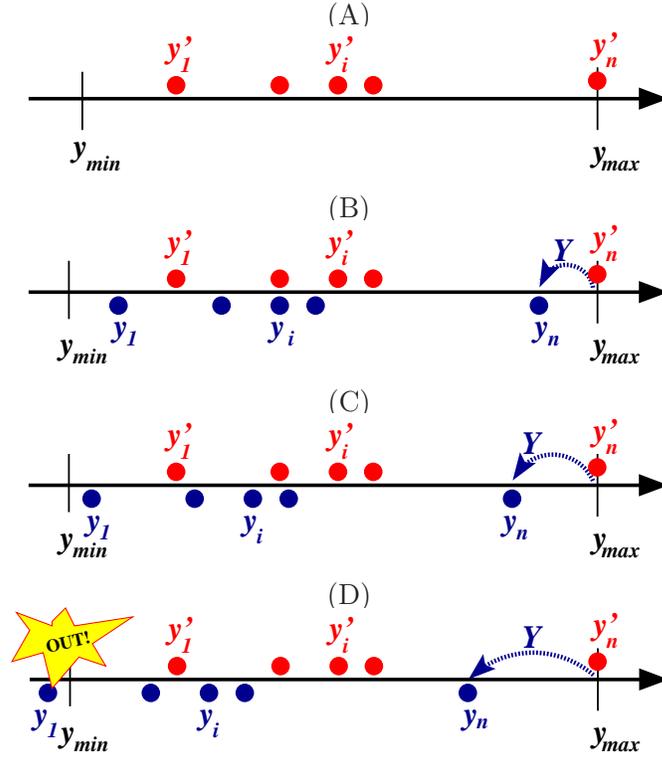


Fig. 1. The linear shift $y'_i \rightarrow y_i = y'_i - Y(y'_1, y'_2, \dots, y'_n)$ in the CMC algorithm illustrated in four steps: (A) begin with y'_i such that one of them $y_n \equiv y_{max}$, (B), (C) shift $y'_i \rightarrow y_i$ by Y , where Y solves the constraint condition $x = \sum u_i(y_i)$; Y is therefore a complicated function of all y'_i . (D) Sometimes the smallest y'_i is shifted out of the phase space, below the IR limit y_{min} . Such an event gets zero MC weight.

3.6. CMC for single proton

In the new parton shower we exploit PDFs evolved using the all-loop CCFM evolution equation³. In the CCFM evolution we implement $\alpha(p^T) = \alpha(e^t x(1-z)/z)$ dependence. No gluons are emitted below $p^T = p_{min}^T = 1$ GeV. Just for illustration let us show explicitly the distribution of the

³ In our numerical exercises the non-Sudakov CCFM form-factor is usually switched off.

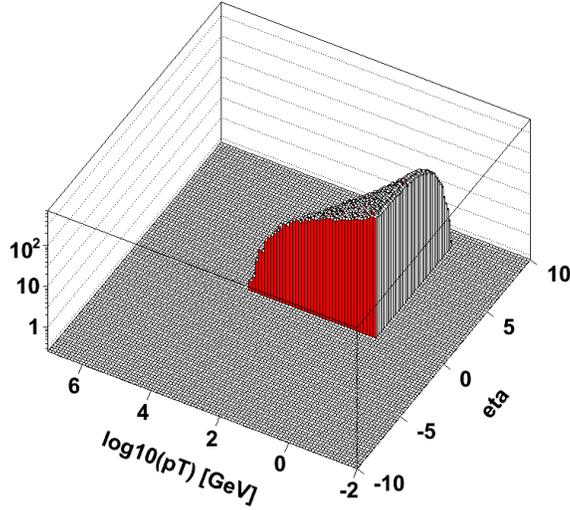


Fig. 3. The distribution of all gluons emitted from a single proton from CMC. The sharp boundaries of rapidity and minimum k^T are clearly visible.

4. CMC for proton–proton collision

4.1. Joining smoothly two evolutions of two hemispheres

Before combining two CMCs described above into a single CMC for the W/Z production at LHC an important problem has to be solved: In the existing CMC for single evolution the constraint is on $\sum_F p_i^+$ of all partons in the forward hemisphere and separately on $\sum_B p_i^-$ in the backward one, while in reality we need the constraint on the effective mass \hat{s} of the W/Z boson involving also $\sum_F p_i^-$, $\sum_B p_i^+$ and all transverse momenta.

$$\sigma = \int_0^1 \frac{dx}{x} \frac{d\bar{x}}{\bar{x}} \sum_{f\bar{f}f_0\bar{f}_0} \int_0^1 d\hat{x} \sigma_{f\bar{f}}^{\text{Born}}(s\hat{x}) \int dx_0 \int d\bar{x}_0 \tilde{D}_{f_0}(t_\lambda, x_0) \tilde{D}_{\bar{f}_0}(t_\lambda, \bar{x}_0) \times \mathcal{U}_{f f_0}(t_F, x|t_\lambda, x_0) \mathcal{U}_{\bar{f} \bar{f}_0}(t_B, \bar{x}|t_\lambda, \bar{x}_0) \theta_{\hat{s}>0} \delta_{\hat{x}=\hat{s}/s}. \quad (4.1)$$

The question is: Can we impose in the MC the constraint on \hat{s} , which is a nontrivial function of 4-momenta of all emitted partons? The answer is yes and we are describing briefly the method in the following.

4.2. Imposing constraint on \hat{s}

How can we impose the constraint on \hat{s} which is a nontrivial function of all emitted 4-momenta? We are going to explain the method, without

going into details. The solution based on the rescaling of 4-momenta is the following: (i) Replace complicated constraint on \hat{s} with a simplified one. (ii) Keep the total control on the overall normalization corrected rigorously with the help of the special compensating MC weight W_{MC} :

$$\delta(sx - (p_{0\text{F}} + p_{0\text{B}} - K_{\text{F}} - K_{\text{B}})^2) \longrightarrow \delta(sx - s_0 \hat{Z}_{\text{F}} \hat{Z}_{\text{B}}) W_{\text{MC}},$$

where $K_{\text{F}} = \sum_{\text{F}} k_{i\text{F}}$ and $K_{\text{B}} = \sum_{\text{B}} k_{i\text{B}}$ are total momenta of emitted partons in the F/B hemispheres and $\hat{Z}_{\text{F}} = 1 - \sum_{\text{F}} x_i^+$, $\hat{Z}_{\text{B}} = 1 - \sum_{\text{B}} x_i^-$ are total light-cone variables restricted to a single F/B hemisphere. More details were given in the HERA-LHC workshop presentation, June 2006⁴.

4.3. Preliminary numerical results

The above scheme of joining two CMC for showering every beam into single parton shower is already implemented in a prototype parton shower for W production at LHC. In Fig. 4 the distributions of the ISR gluons emitted from the initial proton beams is shown. In Fig. 5 very preliminary (unrealistic) distributions of the rapidity and the transverse momentum of the produced W -boson is also shown. (The above results are from the presentation at HERA-LHC workshop, June 2006.)

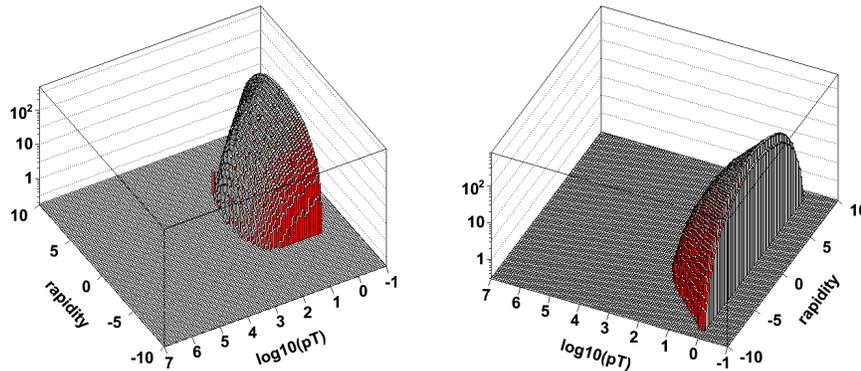


Fig. 4. The CMC results for the distribution of all gluons emitted from two proton beams. In the left-hand side plot rapidity of the W boson is fixed to -3 while in the right-hand side plot it is unrestricted.

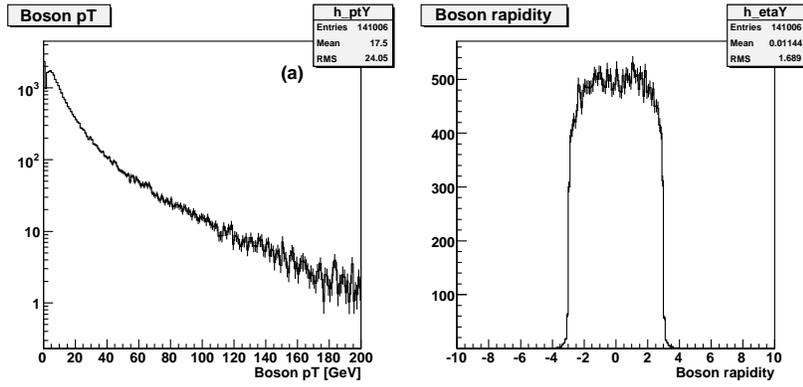


Fig. 5. The transverse momentum and rapidity distribution of the electroweak boson from the double CMC prototype program.

4.4. What about MMC/CMC for DIS?

One may ask a natural question: Can one construct a similar CMC/MMC family of programs for modeling ep deep-inelastic scattering (DIS), with the aim of calculating the CCFM-like evolution of F_2 and simulating the parton

⁴ See <http://jadach.web.cern.ch/jadach/>

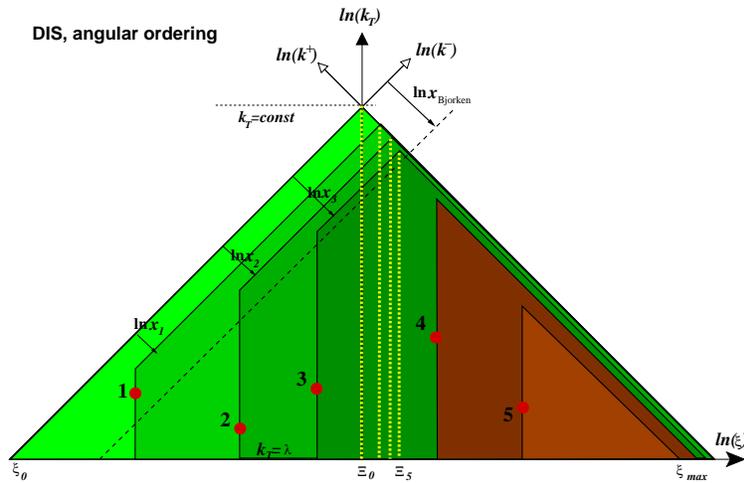


Fig. 6. The phase space of MMC/CMC with the angular ordering parton shower based on CCFM-like evolution. The entire triangle represents the so-called “current region”. The proton fragmentation region is to the left of the triangle. The right-hand side part of the triangle is populated by the final state gluonstrahlung.

shower at the same time? In Fig. 6 we present in a schematic way the generation distributions of the ISR and FSR gluons in the rapidity and $\ln k^T$ defined in the Breit frame of the incoming/outgoing quarks. In our opinion this type of the parton shower MC should be developed in parallel to MC for W/Z production at LHC. The twin MC programs for DIS at HERA and W/Z production at LHC would provide the best means for exploiting maximally data from HERA for the purpose of LHC data analysis. It looks that only MMC-type program is needed for describing DIS data. (It is good news, as CMC is much harder to develop and test.)

5. Summary and outlook

Let us summarize on the recent activity of the development of the CMC for W/Z production at LHC and related works:

- Combining two single CMC evolutions into one MC for W/Z production at LHC is in progress. More testing is needed.
- Getting more realistic distributions of the W/Z rapidity and k^T .
- Quark–gluon transitions in CMC and double CMC.
- The non-Sudakov form-factor for the full CCFM compatibility.
- QCD NLO in the hard process and evolution; See also parallel talk by Phil Stephens.

Further plans are the following:

- Better EW and QED FSR matrix elements (from WINHAC and SANC).
- Cross-checks with uPDFs from CASCADE and SMALLX.
- CMC/MMC for DIS process, fitting F_2 .
- Establishing the relation to the k^T -ordering and CSS [22] in b -space.

Last but not least, we would like also to mention related project of calculating the one-loop electroweak corrections by SANC group [23], which we are planning to interfaced to our MCs. The other related MC event generators: TAUOLA [24], PHOTOS [25] and WINHAC [26], are also supported by the same Krakow group of theorists, for the future LHC experiments.

REFERENCES

- [1] S. Jadach, M. Skrzypek, *Acta Phys. Pol. B* **35**, 745 (2004) [[hep-ph/0312355](#)].
- [2] S. Jadach, M. Skrzypek, *Nucl. Phys. Proc. Suppl.* **135**, 338 (2004), Presented at the Conference “Loops and Legs”, April 2004, Zinnowitz, Germany.
- [3] S. Jadach, M. Skrzypek, *Acta Phys. Pol. B* **36**, 2979 (2005) [[hep-ph/0504205](#)].
- [4] S. Jadach, M. Skrzypek, Report CERN-PH-TH/2005-146, IFJAN-V-05-09, Contribution to the HERA-LHC Workshop, CERN-DESY, 2004-2005, <http://www.desy.de/~heralhc/>, [[hep-ph/0509178](#)].
- [5] K. Golec-Biernat, S. Jadach, W. Placzek, M. Skrzypek, *Acta Phys. Pol. B* **37**, 1785 (2006) [[hep-ph/0603031](#)].
- [6] S. Jadach, M. Skrzypek, *Nucl. Phys. Proc. Suppl.* **157**, 241 (2006).
- [7] S. Jadach, W. Placzek, M. Skrzypek, P. Stephens, Z. Was, [[hep-ph/0703281](#)].
- [8] S. Haywood *et al.*, [[hep-ph/0003275](#)].
- [9] M. Boonekamp, *Acta Phys. Pol. B* **38**, 339 (2007).
- [10] S. Moch, J.A.M. Vermaseren, A. Vogt, *Nucl. Phys.* **B688**, 101 (2004). [[hep-ph/0403192](#)].
- [11] S. Moch, A. Vogt, *Phys. Lett.* **B631**, 48 (2005) [[hep-ph/0508265](#)].
- [12] C. Anastasiou, L.J. Dixon, K. Melnikov, F. Petriello, *Phys. Rev.* **D69**, 094008 (2004) [[hep-ph/0312266](#)].
- [13] C. Anastasiou, K. Melnikov, F. Petriello, *Nucl. Phys.* **B724**, 197 (2005) [[hep-ph/0501130](#)].
- [14] S. Frixione, B.R. Webber, *J. High Energy Phys.* **0206**, 029 (2002) [[hep-ph/0204244](#)].
- [15] S. Frixione, B.R. Webber, [hep-ph/0601192](#).
- [16] S. Jadach, M. Skrzypek, *Comput. Phys. Commun.* **175**, 511 (2006) [[hep-ph/0504263](#)].
- [17] B. Andersson *et al.* [Small x Collaboration], *Eur. Phys. J.* **C25**, 77 (2002) [[hep-ph/0204115](#)].
- [18] J.R. Andersen *et al.* [Small x Collaboration], *Eur. Phys. J.* **C35**, 67 (2004) [[hep-ph/0312333](#)].
- [19] S. Jadach, *Comput. Phys. Commun.* **152**, 55 (2003) [[physics/0203033](#)].
- [20] S. Jadach, M. Skrzypek, *Comput. Phys. Commun.* **175**, 511 (2006) [[hep-ph/0504263](#)].
- [21] S. Alekhin *et al.*, [hep-ph/0601013](#).
- [22] J.C. Collins, D.E. Soper, G. Sterman, *Nucl. Phys.* **B250**, 199 (1985).
- [23] A. Arbuzov *et al.*, [hep-ph/0703043](#).
- [24] S. Jadach, Z. Was, R. Decker, J.H. Kühn, *Comput. Phys. Commun.* **76**, 361 (1993).
- [25] E. Barberio, Z. Was, *Comput. Phys. Commun.* **79**, 291 (1994).
- [26] C.M. Carloni Calame, S. Jadach, G. Montagna, O. Nicrosini, W. Placzek, *Acta Phys. Pol. B* **35**, 1643 (2004) [[hep-ph/0402235](#)].