TRANSVERSE MOMENTUM DISTRIBUTIONS FOR THE STANDARD MODEL BOSON PRODUCTION AT THE LHC*

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In this talk we discuss selected topics concerning calculations of transverse momentum distributions for the Standard Model boson production at the LHC.

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1. Introduction

The search for the Higgs boson is one of the highest priorities for the CERN Large Hadron Collider (LHC) physics program [1]. The dominant production channel in the low mass range, $m_H \lesssim 140\,\mathrm{GeV}$, is the gluon fusion, mediated at lowest order in the Standard Model (SM) by a heavy (mainly top) quark loop. In the considered mass range, experimental searches at the LHC will concentrate on the rare two-photon decay mode $H \to \gamma + \gamma$. In the absence of any constraints imposed on the events, the bulk of the cross section will be at relatively low transverse momenta of the photon pair, where the background is large. Therefore, one needs precise theoretical predictions for the transverse momentum (p_{T}) distribution of the produced Higgs boson. A possible way to improve the signal significance for Higgs discovery in the considered mass range is to study the $\gamma + \gamma +$ jet(s) final states [2]. This process will share many features, in particular regarding the importance of the higher order perturbative corrections, with the more inclusive reaction $pp \to H + X$ at large p_{T}, since in most cases a high-p_{T} Higgs will be accompanied by a recoiling jet. It is thus of interest to consider p_{T} distributions of Higgs produced in the gluon fusion process also at higher values of p_{T}.

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The core of the data taken at the LHC will be, however, coming from the Standard Model processes which have higher production rates than the Higgs production. Electroweak gauge boson production $pp \to V + X$ with $V = W^{\pm}, Z, \gamma$ is a good example of such process. In fact, measurement of the $p_{\rm T}$ distribution of the produced boson V is expected to be one of the early benchmarks established at the LHC. Due to its high rate and distinct experimental signature, W- and Z-boson production has even been proposed as a way to measure the LHC luminosity [3]. Apart from being a crucial measurement for the precise determination of the W-boson mass, M_W, the cross section for electroweak gauge boson production provides an important means to constrain information on the parton distribution functions.

Regarding the QCD corrections, the $p_{\rm T}$ distributions for the Drell–Yan type processes $pp \to V,\ H + X$ are known up to NLO, *i.e.* $\mathcal{O}(\alpha_{\rm S}^2)$ accuracy [4]. The corresponding inclusive total cross sections are known up to NNLO, which is also $\mathcal{O}(\alpha_{\rm S}^2)$ [5]. Due to the high complexity of the calculations, the results for the NNLO corrections to the $gg \to H + X$ process were obtained in the large-top-mass m_t limit, *i.e.* $m_t \to \infty$. In this limit the top quark loops may be replaced by point-like vertices, and the Feynman rules are given by an effective Lagrangian. This method is known to provide a very good approximation of the exact result for total cross section in the mass range $m_H < 2m_t$ [6].

It is a general feature of perturbative calculations in QCD that close to a phase space boundary partonic hard-scattering cross sections acquire large logarithmic corrections. These corrections are related to soft and collinear gluon emission and arise from cancellations between virtual and real contributions at each order in perturbation theory. When the transverse momentum carried by the produced boson is very small, $p_{\rm T} \ll Q$, the recoil corrections involving logarithms of ratio $p_{\rm T}^2/Q^2$, grow large. At large $p_{\rm T}$, in the limit when $\hat{s}$ approaches a kinematical boundary for a production of a particle with a given momentum $p_{\rm T}$, the threshold corrections can become important. In inclusive total cross sections, threshold corrections involve logarithms of $1 - z$ with $z = Q^2/\hat{s}$, $\hat{s}$ being the partonic center-of-mass energy and Q the invariant mass of the boson. Due to the presence of these logarithmic corrections, sufficiently close to the phase-space boundary, *i.e.* in the limit of soft and/or collinear radiation, fixed-order perturbation theory is bound to fail. A proper treatment of higher-order corrections in this limit requires resummation of logarithmic corrections to all orders.

Due to the high center-of-mass energy and design luminosity, the LHC will also offer a unique possibility to explore production of gauge bosons with very large $p_{\rm T}$. Although on the basis of simple arguments regarding the strength of coupling constants one might expect electroweak (EW) corrections to be in general much smaller than QCD corrections, for these classes of processes EW corrections from virtual boson exchange also become important. Owing to the finite weak-boson masses, the real emission of a soft/collinear Z or W boson can be observed as a separate process and hence does not need to be included in the definition of physical observables. Thus, in contrast to mass singularities in massless gauge theories such as QED or QCD, the EW mass singularities of virtual origin are not necessarily compensated by corresponding mass singularities from real weak-boson radiation. The dominant contribution to the EW correction is given by the logarithms of the ratio $(\hat{s}/M_W^2)$. Typically, at $\sqrt{\hat{s}} \simeq 1$ TeV these corrections, also known as electroweak Sudakov logarithms, are estimated to yield one-loop corrections of tens of per cent and two-loop corrections of a few per cent and need to be included in the analysis.

2. Resummation approaches at small and large transverse momentum

2.1. Recoil and joint resummation at small p_{T}

A formalism to resume all terms of the perturbation series which are at least as singular as $1/p_{\mathrm{T}}^2$ when $p_{\mathrm{T}} \to 0$ in the p_{T} distribution for the Drell–Yan process $pp \to V + X$ has been proposed by Collins, Soper and Sterman (CSS) [8]. The resummation is performed in the Fourier conjugate of $\boldsymbol{p}_{\mathrm{T}}$-space, $\boldsymbol{b}$-space, what allows to built the transverse momentum conservation condition into the formalism [9]. At the parton level the resummed part of the cross section is of the form

$$\frac{d\hat{\sigma}^{\mathrm{res}}}{dq_{\mathrm{T}}^2} = \frac{\sigma_0}{2} \int\limits_0^\infty bdb\, J_0(q_{\mathrm{T}}b)\, e^{S(b,Q^2)}\,. \tag{1}$$

The Sudakov factor $S(b, Q^2)$ in Eq. (1) reads

$$S(b,Q^2) = -\int\limits_{\frac{b_0^2}{b^2}}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\Big[\ln\Big(\frac{Q^2}{\bar{\mu}^2}\Big)A(\alpha_S(\bar{\mu}^2)) + B(\alpha_S(\bar{\mu}^2))\Big]\,, \tag{2}$$

$$A(\alpha_S) = \sum_{i=1}^{\infty}\Big(\frac{\alpha_S}{2\pi}\Big)^i A^{(i)} \quad B(\alpha_S) = \sum_{i=1}^{\infty}\Big(\frac{\alpha_S}{2\pi}\Big)^i B^{(i)} \tag{3}$$

with $b_0 = 2\exp(-\gamma_{\mathrm{E}})$. The first few coefficients of the perturbative series (3) can be obtained, both for quark and gluon initial states [11], from the exact fixed-order perturbative calculation in the high p_{T} region by comparing the logarithmic terms therein with the corresponding logarithms generated by the first few terms of the expansion of $\exp(S(b,Q^2))$ in (1). At larger values of p_{T}, $p_{\mathrm{T}} \sim Q$, the logarithmic pieces will no longer dominate and to obtain correct predictions for the full p_{T} spectrum one has to match the resummed expression with the full fixed-order result [8, 10]. The numerical predictions for the massive gauge boson production at the LHC obtained using the CSS formalism can be found in [12], whereas Higgs boson production via gluon fusion at the LHC was studied in [13].

Although the $\boldsymbol{b}$-space method succeeds in recovering a finite, positive result in the $p_{\mathrm{T}} \to 0$ limit, it suffers from several drawbacks. For example, since the integration in (1) extends from 0 to ∞, it is impossible to make predictions for any p_{T} without having a prescription for how to deal with the non-perturbative regime of large b. One prescription is to artificially prevent b from reaching large values by replacing it with a new variable $b_* = b/\sqrt{1+b^2/b_{\mathrm{lim}}^2}$, and by parametrising the non-perturbative large-b region in terms of the form factor F^{NP} [8], which is generally a Gaussian in b. The result of the most up-to-date fits for F^{NP} can be found in [14]. A recent proposal to include an additional function of the partonic momentum fractions x in the F^{NP} [12], resulting in a significant broadening of the p_{T} spectrum for electroweak gauge boson production at the LHC, awaits further investigation [15].

To deal with the technical drawbacks of the $\boldsymbol{b}$-space method, it has been proposed to resum the logarithms directly in the $\boldsymbol{p}_{\mathrm{T}}$-space. This method relies on deriving an approximation of the $\boldsymbol{b}$-space formalism and various techniques, each selecting and resumming different subsets of logarithmic terms, have been developed [16]. The numerical predictions obtained using the p_{T}-space method for the Higgs boson production via gluon fusion at the LHC can be found in [17].

The single SM boson production at hadron colliders has been also described in the framework of unintegrated parton distribution functions [18]. It is then possible to show the correspondence between this approach and the standard CSS $\boldsymbol{b}$-space resummed formula for the leading contributions *i.e.* including the first-order coefficients in the expansion of the A, B functions, *cf.* Eq. (3).

The soft and collinear gluon emission is also responsible for another class of contributions in the theoretical expressions in production processes of the Drell–Yan type. In the limit of partonic center-of-mass energy approaching the invariant mass Q of the produced boson, *i.e.* the ratio $z = Q^2/\hat{s} \to 1$ the threshold corrections contribute the dominant part of the cross-section. At $\mathcal{O}(\alpha_{\mathrm{S}}^n)$, the leading logarithmic (LL) contributions are of the form $\alpha_{\mathrm{S}}^n \ln^{2n-1}(1-z)$, the next-to-leading (NLL) are of the form $\alpha_{\mathrm{S}}^n \ln^{2n-2}(1-z)$ *etc.* A proper treatment of higher-order corrections in this limit requires resummation of logarithmic corrections to all orders and resummation techniques for threshold corrections are well established in this case [19]. However, resummation of recoil and threshold effects is known to lead to opposite effects: suppression and enhancements of the partonic cross-section, respectively. A full analysis of soft gluon effects in transverse momentum distribution should, therefore, if possible, take both types of corrections simultaneously into account. A joint, simultaneous treatment of the threshold and recoil corrections was first introduced in [20]. It relies on a novel refactorization of short-distance and long-distance physics at fixed transverse momentum and energy [20]. Similarly to standard threshold and recoil resummation, exponentiation of logarithmic corrections occurs in the impact parameter $\boldsymbol{b}$-space [8], Fourier-conjugate to transverse momentum $\boldsymbol{p}_{\mathrm{T}}$-space as well as in the Mellin-N moment space [19], conjugate to z-space. The resulting expression respects energy and transverse momentum conservation.

In the framework of joint resummation, the Sudakov exponent at the NLL accuracy has a classic form known from recoil resummation

$$S^{\mathrm{joint}}(N,b,Q,\mu) = -\int\limits_{Q^2/\chi^2}^{Q^2} \frac{dk_{\mathrm{T}}^2}{k_{\mathrm{T}}^2} \left[A(\alpha_{\mathrm{S}}(k_{\mathrm{T}}))\, \ln\left(\frac{Q^2}{k_{\mathrm{T}}^2}\right) + B(\alpha_{\mathrm{S}}(k_{\mathrm{T}}))\right] . \quad (4)$$

The quantity $\chi(N,b)$ appearing in the lower limit of integration in (4) organises the logarithms in N and b. The LL and NLL logarithmic terms in the threshold limit, $N \to \infty$ (at fixed b), and in the recoil limit $b \to \infty$ (at fixed N) are correctly reproduced with the following choice of the form of $\chi(\bar{N},\bar{b}) = \bar{b} + \bar{N}/(1+\eta\,\bar{b}/\bar{N})$, where η is a constant and we define $\bar{N} = N\mathrm{e}^{\gamma_{\mathrm{E}}}, \bar{b} \equiv bQ\mathrm{e}^{\gamma_{\mathrm{E}}}/2$ with γ_{E} the Euler constant. The coefficients in the expansions of the functions A and B in (4) are the same as in the standard recoil resummation. By incorporating the full evolution of parton densities [21], the jointly resummed cross section correctly includes also the leading $\alpha_{\mathrm{S}}^n \ln^{2n-1}(\bar{N})/N$ terms to all orders. More details on the joint resummation method, in particular definition of the inverse Mellin and Fourier required to avoid singularities associated with the Landau pole, and matching with the fixed-order p_{T} distributions can be found in Ref. [21]. A full phenomenological analysis of Z boson production at the Tevatron in the framework of joint resummation can be found in [21], whereas Higgs boson production at the LHC was studied in [22]. Direct-γ production for fixed-target and pp ($p\bar{p}$) scattering experiments has been analysed in [20, 23]. A comparison of resummed predictions for Higgs production, described in more details in Refs. [24], including also predictions provided by the Monte Carlo generators, is shown in Fig. 1.

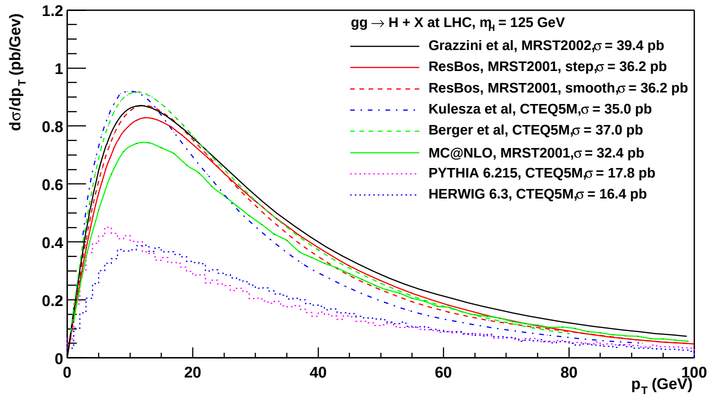

Fig. 1. The predictions for the production a 125 GeV mass Higgs boson at the LHC. Figure taken from [24].

2.2. Threshold resummation at large $p_{\rm T}$

At large $p_{\rm T}$, threshold corrections to the $p_{\rm T}$ distribution can be resummed on their own. As in the fully inclusive case, threshold logarithms also dominate the cross section when the transverse momentum of the produced boson is large, even though they are of a somewhat different form. In the $p_{\rm T}$ distribution, when the cross section is integrated over all rapidities of the boson, they occur in the partonic cross sections as $\alpha_{\rm S}^k \ln^m(1-\hat{y}_T^2)$, $m \leq 2k$, where $\hat{y}_T = (p_{\rm T} + m_{\rm T})/\sqrt{\hat{s}}$ with $m_{\rm T} = \sqrt{p_{\rm T}^2 + m_{\rm T}^2}$. Also, unlike the fully-inclusive case, for a boson produced at large $p_{\rm T}$ there need to be a recoiling parton already in the Born process, whose color charge plays a role for the structure of the resummed expression. The resummation of threshold logarithms for high-$p_{\rm T}$ Higgs boson production was considered in [25], whereas Ref. [26] is concerned with high-$p_{\rm T}$ W production in hadronic collisions. Besides the obvious differences related to the different final state considered, the two calculations also differ in the technical treatment of the resummed formulas. In Ref. [26] a NNLO expansion of the resummed expression is obtained and used, while in [25] the full NLL-resummed expression was kept. In the massless limit the structure of the resummed expression is similar to that for prompt-photon production in hadronic collisions [27].

At large $p_{\rm T}$, $p_{\rm T} \gtrsim m_t$, the large m_t approximation is known to deteriorate [28], and a full calculation that includes all effects from the top quark loop is required. Fortunately, the large logarithms are insensitive to the structure of the Higgs-gluon coupling since they are associated only with emission of soft and collinear gluons from the external lines. Therefore, even though the predictions for the cross sections might not be entirely applicable at large $p_{\rm T}$, one can be confident that K-factors, defined as ratios

$K^{\mathcal{A}/\mathcal{B}} \equiv \frac{d\sigma^{\mathcal{A}}/dp_{\rm T}}{d\sigma^{\mathcal{B}}/dp_{\rm T}}$, generally will be. The ratios of the NLO and NLL distribution to the LO distribution, along with NLL to NLO ratio, are shown in Fig. 2. As can be seen from the dotted line for $K^{\rm NLL/NLO}$, resummation predicts an increase of about 10% of the cross section beyond NLO. The results presented in Fig. 2 should be taken into account in the analysis of future LHC data.

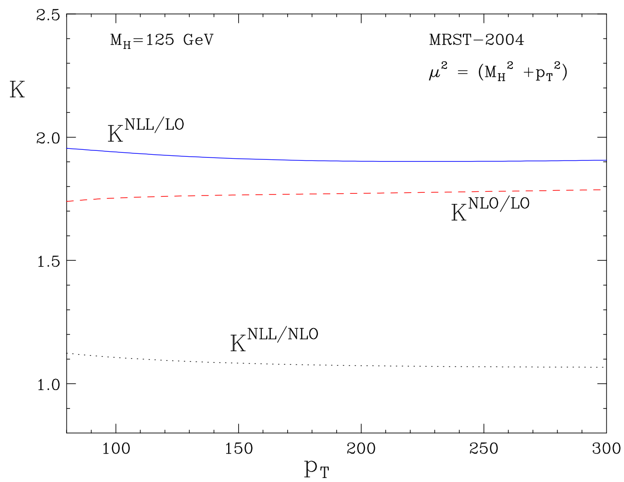

Fig. 2. K-factors, as defined in the text, for the Higgs boson $p_{\rm T}$ distribution at the LHC. Figure taken from [25].

3. Electroweak corrections to transverse momentum distribution of gauge bosons

In the region $\sqrt{\hat{s}} \gg M_W \simeq M_Z$ the EW corrections are strongly enhanced by logarithmic mass singularities. At $\mathcal{O}(\alpha^L)$ the LL corrections are of the form $\alpha^L \ln^{2L}(\hat{s}/M_W^2)$ while the NLL corrections are of the form $\alpha^L \ln^{2L-1}(\hat{s}/M_W^2)$, *etc.* These EW logarithms originate from soft/collinear emission of virtual EW gauge bosons off initial- or final-state particles.

In the calculation of the full one-loop corrections to the hadronic production of Z bosons [29] and the hadronic production of photons [30] at large $p_{\rm T}$ only weak virtual corrections are considered. For the W boson production [32] also the electromagnetic (virtual and real) contributions need to be calculated together with the weak corrections. Of course, to achieve reliable predictions at high $p_{\rm T}$, the NLO QCD corrections need to be taken into account as they can amount to several tens of per cent correction for these processes. Numerical results for the one-loop corrections to the Z-boson and photon production processes can be also found in Ref. [33].

The NLL approximation of the full result, valid in the high energy region, is obtained from the Born result by multiplying it with a global factor. In particular, for the unpolarised squared amplitude for $q\bar{q} \to Zg$ we have [31][1]

$$\overline{\sum_{\mathrm{pol}}}|\mathcal{M}^{q\bar{q}}|^2 = 64\pi^2\alpha\alpha_{\mathrm{S}}\frac{\hat{t}^2+\hat{u}^2+2M_Z^2\hat{s}}{\hat{t}\hat{u}}\left[A_0+\left(\frac{\alpha}{2\pi}\right)A_1+\left(\frac{\alpha}{2\pi}\right)^2A_2\right]. \quad (5)$$

The amplitude for the other partonic subprocess contributing to the hadronic cross sections are easily obtained from Eq. (5) using crossing symmetry and CP transformations. The tree-level contribution A_0 to Eq. (5) reads

$$A_0 = \sum_{\lambda=\mathrm{L}\,,\mathrm{R}}\left(I^Z_{q_\lambda}\right)^2 \qquad \text{with} \qquad I^Z_{q_\lambda} = \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}T^3_{q_\lambda} - \frac{s_{\mathrm{W}}}{c_{\mathrm{W}}}\frac{Y_{q_\lambda}}{2}\,, \quad (6)$$

where $T^3_{q_\lambda}$ and Y_{q_λ} are the weak isospin and hypercharge for left- ($\lambda = \mathrm{L}$) and right-handed ($\lambda = \mathrm{R}$) quarks, and we use the shorthands $c_{\mathrm{W}} = \cos\theta_{\mathrm{w}}$ and $s_{\mathrm{W}} = \sin\theta_{\mathrm{w}}$ for the weak mixing angle θ_{w}. The $\mathcal{O}(\alpha)$ contribution reads

$$A_1 = -\sum_{\lambda=\mathrm{L,R}} I^Z_{q_\lambda}\left[I^Z_{q_\lambda}C^{\mathrm{ew}}_{q_\lambda}\left(L^2_{\hat{s}}-3L_{\hat{s}}\right)+\frac{c_{\mathrm{W}}}{{s_{\mathrm{W}}}^3}T^3_{q_\lambda}\left(L^2_{\hat{t}}+L^2_{\hat{u}}-L^2_{\hat{s}}\right)\right], \quad (7)$$

where $L_{\hat{r}} \equiv \ln(|\hat{r}|/M_W^2)$ and $C^{\mathrm{ew}}_{q_\lambda} = Y^2_{q_\lambda}/(4{c_{\mathrm{W}}}^2)+C_{q_\lambda}/{s_{\mathrm{W}}}^2$ are the eigenvalues of the electroweak Casimir operator for quarks, with $C_{q_{\mathrm{L}}} = 3/4$ and $C_{q_{\mathrm{R}}} = 0$.

The size of the logarithmically enhanced contributions grows with energy and for transverse momenta of hundreds of GeV also the two-loop logarithms become important. At the NLL accuracy we have

$$\begin{aligned} A_2 \; = \; & \sum_{\lambda=\mathrm{L,R}}\left\{\frac{1}{2}\left(I^Z_{q_\lambda}C^{\mathrm{ew}}_{q_\lambda}+\frac{c_{\mathrm{W}}}{{s_{\mathrm{W}}}^3}T^3_{q_\lambda}\right)\left[I^Z_{q_\lambda}C^{\mathrm{ew}}_{q_\lambda}\left(L^4_{\hat{s}}-6L^3_{\hat{s}}\right)\right.\right. \\ & \left. +\frac{c_{\mathrm{W}}}{{s_{\mathrm{W}}}^3}T^3_{q_\lambda}\left(L^4_{\hat{t}}+L^4_{\hat{u}}-L^4_{\hat{s}}\right)\right]-\frac{T^3_{q_\lambda}Y_{q_\lambda}}{8{s_{\mathrm{W}}}^4}\left(L^4_{\hat{t}}+L^4_{\hat{u}}-L^4_{\hat{s}}\right) \\ & \left. +\frac{1}{6}I^Z_{q_\lambda}\left[I^Z_{q_\lambda}\left(\frac{b_1}{{c_{\mathrm{W}}}^2}\left(\frac{Y_{q_\lambda}}{2}\right)^2+\frac{b_2}{{s_{\mathrm{W}}}^2}C_{q_\lambda}\right)+\frac{c_{\mathrm{W}}}{{s_{\mathrm{W}}}^3}T^3_{q_\lambda}b_2\right]L^3_{\hat{s}}\right\}, \quad (8) \end{aligned}$$

[1] Since at two-loop level the purely weak corrections cannot be isolated from the complete electroweak corrections in a gauge-invariant way, we have to consider the combination of weak and electromagnetic virtual corrections. The latter are regularised by means of a fictitious photon mass $\lambda = M_W$.

where $b_1 = -41/(6{c_{\rm W}}^2)$ and $b_2 = 19/(6{s_{\rm W}}^2)$ are the one-loop β-function coefficients associated with the U(1) and SU(2) couplings, respectively. The expressions presented here have been obtained using results of Refs. [34]. For analogous expressions for the W and photon production the reader is referred to Ref. [35] and Ref. [30], respectively.

The relevance of the EW effects for the transverse momentum distributions of the gauge bosons produced at the LHC is demonstrated in Fig. 2, where the relative NLO (full EW correction) and the NNLO corrections (in the NLL approximation), integrated over $p_{\rm T}$ starting from $p_{\rm T} = p_{\rm T}^{\rm cut}$, are presented as a function of $p_{\rm T}^{\rm cut}$. This is compared with the statistical error,

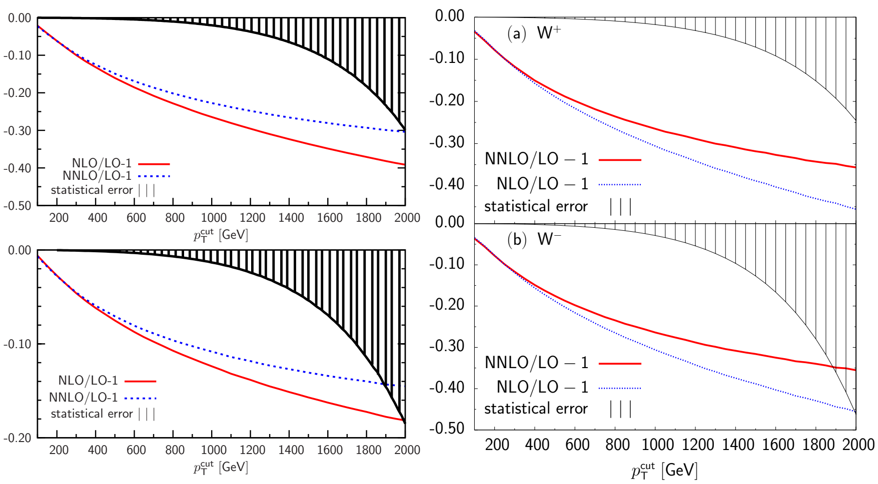

Fig. 3. Relative NLO (solid) and NNLO (dotted) corrections w.r.t. the LO prediction and statistical error (shaded area) for the unpolarised integrated cross section for (left upper) $pp \to Zj$, (left lower) $pp \to \gamma j$ and (right) $pp \to Wj$,at $\sqrt{s} = 14\,{\rm TeV}$ as a function of $p_{\rm T}^{\rm cut}$. Figures taken from [29, 30, 35].

defined as $\Delta\sigma_{\rm stat}/\sigma = 1/\sqrt{N}$ with $N = \mathcal{L} \times \sigma_{\rm LO}$. A total integrated luminosity $\mathcal{L} = 300\ {\rm fb}^{-1}$ for the LHC is assumed. It is clear from Fig. 3, that the size of the one-loop (two-loop logarithmic) corrections is much bigger than (comparable to) the statistical error for both the Z-, W-boson and the γ production. These results do not include contributions coming from real radiation of the massive gauge bosons (as well as photons in the case of Z and γ production). Such contributions have been calculated in Ref. [36] where their effect on the $p_{\rm T}$ distributions of Z-bosons and photons at the LHC was investigated. Although the contributions are positive, the net corrections remain of order of tens of percents, underlining the importance of the EW effects at hadron colliders in general, and specifically their impact on the $p_{\rm T}$ distributions of the EW gauge bosons.

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