

QCD ISR MONTE CARLO WITH $\alpha(k_T)^*$ **M. SKRZYPEK^a, K. GOLEC-BIERNAT^{a,b}, S. JADACH^a, W. PŁACZEK^c^aThe Henryk Niewodniczański Institute of Nuclear Physics
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We present a description of a Monte Carlo algorithm that solves the modified DGLAP-type evolution equation in QCD. The change with respect to normal DGLAP is in the form of the argument of the coupling constant. We consider primarily the CCFM-like form $\alpha(k_T)$, but also a simpler choice $\alpha(Q(1-z))$ is discussed. The evolution is performed in the rapidity space with strict rapidity ordering and no gaps in phase-space.

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Within last three years a project of constructing a new Monte Carlo (MC) QCD parton shower has been initiated by our group from IFJ-PAN in Cracow. To date the main emphasis has been put on the comprehensive analysis of the QCD evolution equations (EVEQs) and their implementations in the MC algorithms. Let me make here a brief overview of the results achieved so far. At first, as an introductory exercise, we constructed the standard forward Markovian algorithms solving the DGLAP-type EVEQs [1], both in the LO approximation [2] and later on at the NLO level as well [3, 4]. The important point of these papers was to show that with the modern computers it is feasible to use MC methods to solve EVEQs with high precision (of the order of 10^{-3}). In parallel to the Markovian algorithms we have developed a class of genuinely new algorithms, that we call Constrained MC

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(CMC). These algorithms allow for predefining, *i.e.* constraining, the final values of the evolution variables, both x and flavor type [5–7]. This way we have found a solution to the longstanding problem of QCD parton showers, so far solvable only by means of pretabulation and backward evolution. As a next step, or rather a whole direction, we turned into modified DGLAP and non-DGLAP EVEQs [8–10]. Some of these modifications, related to the argument of the running coupling constant, will be discussed in more detail in this presentation. In parallel with the analysis of the EVEQs and their solutions we initiated also other subjects, vital for the construction of the complete parton shower: the problem of smooth joining of the evolutions in two hemispheres (see contribution by S. Jadach in these proceedings) and the question of merging of our CMC parton shower with the NLO exact matrix element (see contribution by P. Stephens in these proceedings). Also, we made a first attempt [11] towards constructing a framework for fitting F_2 function obtained from our MC evolutions to the experimental data, which is a mandatory procedure to determine the shape of initial PDFs at low Q^2 . Finally, in the paper [12] we have presented a general framework of the hierarchical reorganization of the Markovian process. This simple but very powerful technique has been frequently used by us in the previous papers and therefore its solid mathematical foundation was necessary.

After this short “historical” overview let us get back to the main subject of this presentation, *i.e.* the modifications of the argument of the coupling constant in the DGLAP EVEQs. The original DGLAP uses $\alpha(Q)$, where Q is the evolution variable. However, by means of a clever replacement of the argument Q by $Q(1-z)$ one can resum some of the logarithmically enhanced higher order terms or include some of the low- x effects [13–15]. There is also another important choice of the argument — one can use the actual transverse momentum k_T^2 [16,17]. Construction of the Markovian MC algorithm based on this k_T -scheme is the main target of this presentation. In order to be able to express k_T in terms of the evolution variables, one has to define how the kinematical variables are related to the evolution variables and what kind of ordering do we want to use in the evolution. Let us briefly discuss the available options. There are three main types of orderings: in virtualities, where $Q^2 \sim m^2$ used by PYTHIA [18], in transverse momenta, $Q^2 \sim k_T^2$, used by ARIADNE [19] and in rapidity, $Q \sim \exp \eta$, used by HERWIG [20]. Each of these approaches has its pros and cons. For example the PYTHIA one (virtuality ordering) requires additional treatment to restore coherence effects whereas the HERWIG one (rapidity ordering) has gaps in the phase space coverage and requires complicate merging procedure with the matrix element. In our project we have decided to use the rapidity ordering. However, from the very beginning we insist on having the complete coverage of the phase space and do not allow for any gaps.

Let us now define the kinematics of our cascade and relate it to the evolution variables. We define k_i^μ to be the momenta of emitted partons, whereas q_i^μ denote the virtual partons along the ladder. The initial hadron carries $q_h^+ = 2E_h$. For each emitted parton we find

$$k_i^+ = q_{i-1}^+ - q_i^+ = 2E_h(x_{i-1} - x_i) = 2E_h x_{i-1}(1 - z_i); \quad \eta_i = \frac{1}{2} \ln \frac{k_i^+}{k_i^-}. \quad (1)$$

Consequently, the transverse momentum of emitted massless parton reads

$$k_{T_i} = \sqrt{k_i^+ k_i^-} = k_i^+ e^{-\eta_i} = x_{i-1}(1 - z_i) 2E_h e^{-\eta_i}. \quad (2)$$

This suggests the convenient definition of the rapidity-based evolution time $t = \log Q$ as

$$t_i = \log Q_i = -\eta_i + \ln(2E_h) \quad (3)$$

and the k_{T_i} becomes:

$$k_{T_i} = e^{t_i} x_i(1 - z_i)/z_i = e^{t_i} x_{i-1}(1 - z_i) = e^{t_i}(x_{i-1} - x_i). \quad (4)$$

Having defined the kinematics we are ready to write down the EVEQ for the evolution of the parton momentum distribution $x D(x, t)$ in the rapidity space with $\alpha(k_T)$. For the sake of simplicity we will present here the case of gluon emission only. The EVEQ reads

$$\begin{aligned} \partial_t x D(x, t) &= \int dudz \alpha_S(e^t x(1 - z)/z) z P(z) u D(u, t) \delta(x - zu) \\ &= \int du/u \alpha_S(e^t(u - x)) x P(x/u) D(u, t), \end{aligned} \quad (5)$$

where $P(z)$ denotes the standard Altarelli–Parisi kernel. It is composed of the real emission part $P^\theta(z)\theta(1 - \varepsilon - z)$ and its virtual partner $P^\delta(\varepsilon)\delta(1 - z)$. In standard DGLAP the infrared cut-off ε is just a small number. However, in the case of k_T -dependent α , in order to avoid the Landau pole we have to impose the finite cut-off λ such that $k_T > \lambda$. This condition translates into

$$\lambda < k_T = e^t x \frac{1 - z}{z} \Rightarrow z < \frac{e^t x}{e^t x + \lambda} \ll 1 \Rightarrow u > \lambda e^{-t} + zu. \quad (6)$$

The new cut-off is not infinitesimal anymore. In order to maintain the momentum sum rule in the evolution we have to ensure that

$$\int_0^1 dz P(z) = 0. \quad (7)$$

This can be achieved by introducing the same cut-off into the definition of the virtual part of the kernel as well. The modified definition reads now

$$P^\delta(u, t) = - \int_0^{1-\lambda e^{-t}/u} dz \alpha_S(e^t u(1-z)) z P^\theta(z). \tag{8}$$

We will skip the details of the derivation of the master formula which describes the solution of EVEQ in the form of the Markov process. Instead, we will only show its most important ingredients. The single emission step is described by a probability distribution of a step forward in t and z variables. The inner, z -distribution, reads

$$\frac{d\omega(z_i, t_i; i-1)}{dt_i dz_i} = \alpha_S(k_i^T) z_i P^\theta(z_i) \theta_{1-\lambda e^{-t_i}/x_{i-1} \geq z_i} e^{-\Phi(t_i, t_{i-1}; x_{i-1})} \theta_{t_i \geq t_{i-1}} \tag{9}$$

and probability of step in t is then given by the integral over dz_i of the above $d\omega$

$$\frac{d\omega(t_i; i-1)}{dt_i} = \theta_{t_i \geq t_{i-1}} (\partial_{t_i} \Phi(t_i, t_{i-1}; x_{i-1})) e^{-\Phi(t_i, t_{i-1}; x_{i-1})}. \tag{10}$$

The key ingredient of the above distributions, the Sudakov form factor Φ , is defined in a usual way as

$$\begin{aligned} \Phi(t_i, t_{i-1}; x_{i-1}) &= - \int_{t_{i-1}}^{t_i} dt' P^\delta(x_{i-1}, t') \\ &= \int_{t_{i-1}}^{t_i} dt' \int_0^{1-\lambda e^{-t'}/x_{i-1}} dz \alpha_S(e^{t'} x_{i-1}(1-z)) z P(z). \end{aligned} \tag{11}$$

A few comments are in order here. The two-dimensional integral defining the form factor Φ is not calculable analytically even in the case of the LO kernel P . However, it is possible to calculate it for the singular part of the LO kernel, *i.e.* $P_{1/(1-z)} = 1/(1-z)$. In this case we obtain

$$\begin{aligned} \Phi_{1/(1-z)}(t_i, t_{i-1}; u) &= (2/\beta_0) [\rho(t_i + \ln u) - \rho(t_{i-1} + t_u) \theta_{t_{i-1} + t_u > t_\lambda}] \\ \rho(t) &= \hat{t}(\ln \hat{t} - \ln \hat{t}_\lambda - 1) + \hat{t}_\lambda; \quad \hat{t} = t - \ln A_0, \quad t_\lambda = \ln \lambda, \quad t_u = \ln u. \end{aligned} \tag{12}$$

However, even $\Phi_{1/(1-z)}(t_i)$ cannot be inverted analytically, (to generate t_i). We have decided to perform this inversion numerically and a fast numerical routine has been written and used for this purpose. It turns out that generation of the $P_{1/(1-z)}$ part is enough to construct the MC algorithm, because

the leftover, finite part of the kernel $P_F = P - P_{1/(1-z)}$ is much smaller and can be re-introduced as a weight later on. The corresponding part Φ_F of the Sudakov form-factor can be expressed as a one-dimensional integral

$$\Phi_F(t_i, t_0; u) = \int_{t_\lambda - t_1}^{\min(t_u, t_\lambda - t_0)} dv F(v) \ln \frac{t_1 + v}{t_\lambda} + \theta_{t_0 + t_u > t_\lambda} \int_{t_\lambda - t_0}^{t_u} dv F(v) \ln \frac{t_1 + v}{t_0 + v} \quad (13)$$

which then needs to be evaluated numerically on an event-per-event-basis.

We can illustrate the above single emission in the Sudakov plane, shown in Fig. 1. The plane is defined by a pair of variables $(\log k^+, \log k^-)$ or equivalently $(\xi = \log \eta, \log k_T)$. The phase space for the first emission, which occurred at k_1 , is delimited by the yellow triangle limited from below by the $k_T = \lambda$ line. The available phase space for the next emission is delimited by the white trapezoid (from ξ_1 to ξ_*). The reduction of the phase space from above reflects the loss of k^+ in the first emission.

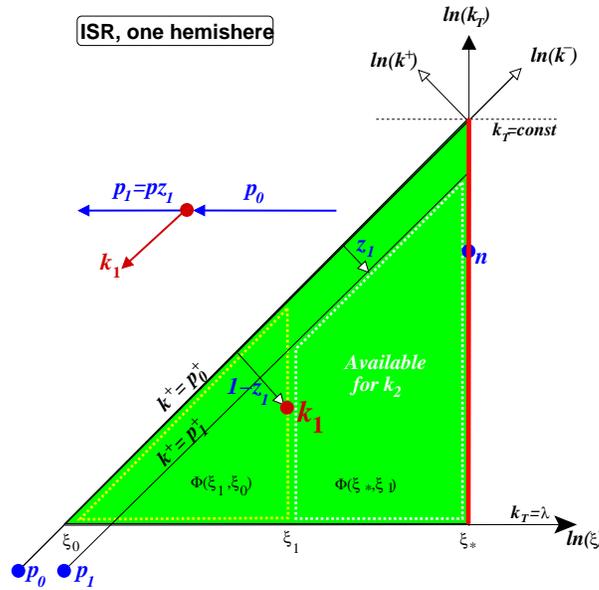


Fig. 1. Sudakov phase space of a single emission ordered in rapidity $\eta = \log \xi$ and with minimal cut-off λ on k_T .

We have also implemented the other argument in α , namely the $\alpha(e^t(1-z))$. From the technical point of view the implementation is simpler, because now the argument does not depend on x anymore. An interesting physical difference however, results from the different cut-off related to the

Landau pole. Namely, in this case the cut-off reads

$$\lambda \leq e^{t_i}(1 - z_i) = k_{T_i}/x_{i-1} \tag{14}$$

with an obvious consequence that in this case the k_T can drop below λ , down to

$$k_{T_i} \geq \lambda x_{i-1}. \tag{15}$$

We have illustrated this phenomenon in the Sudakov plane shown in Fig. 2. The trapezoid of the second emission (delimited by the blue line, between ξ_1 and ξ_*) overhangs now below the $k_T = \lambda$ line.

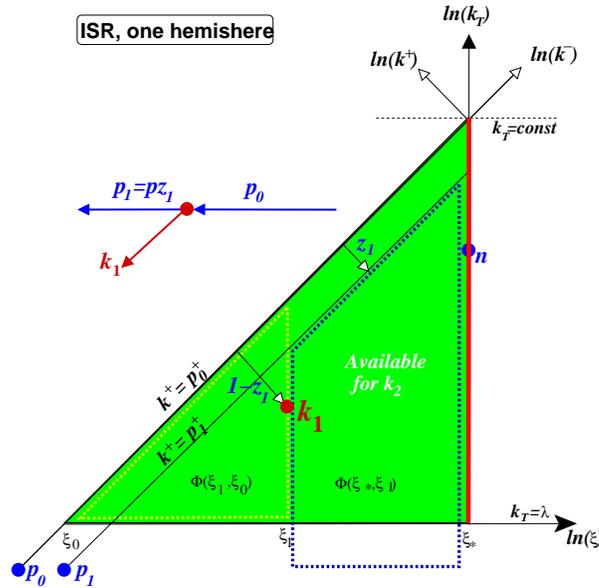


Fig. 2. Sudakov phase space of a single emission ordered in rapidity $\eta = \log \xi$ and with minimal cut-off λ on k_{T_i}/x_{i-1} .

Let us present some numerical results. We have obtained them from two independent implementations of the $\alpha(k_T)$ scheme, in the Markovian algorithm described here and in the Constrained algorithm of Ref. [21]. In both cases the same change of argument has been applied also for the quark-gluon transitions.

In Figs. 3 and 4 we show the case of gluon emissions only (no quark-gluon transitions), for gluon and quark PDFs, respectively. One can see an excellent agreement between MMC and CMC results, down to the precision of 10^{-3} , limited by the statistics.

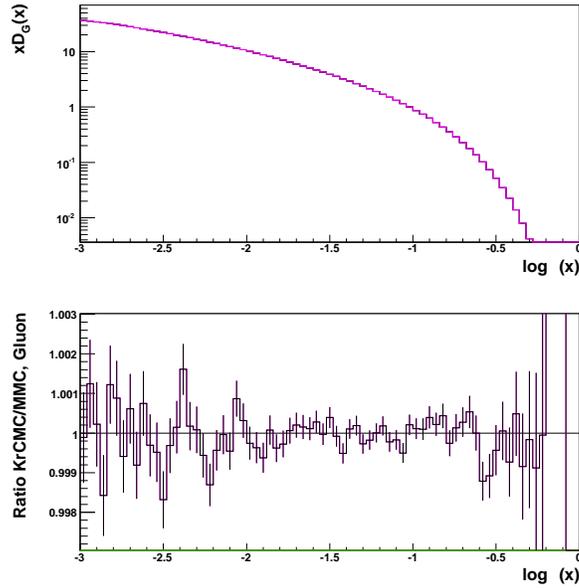


Fig. 3. Evolution of the gluon PDF from 1 GeV to 1000 GeV with $\alpha(k_T)$, rapidity ordering and minimal cut-off λ on k_T . No quark–gluon transitions.

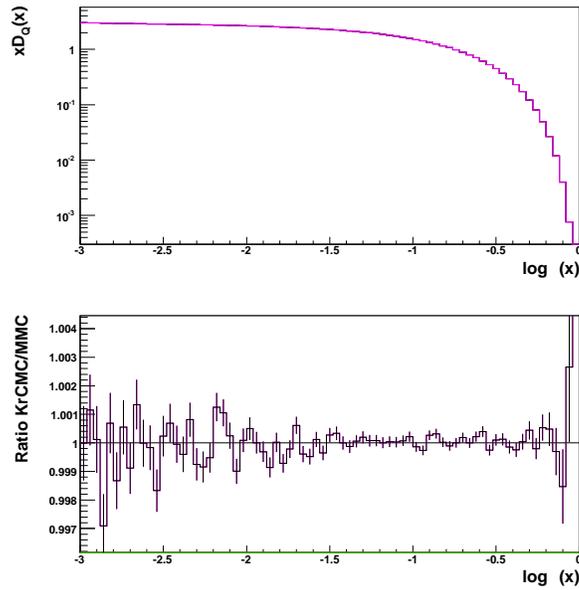


Fig. 4. Evolution of the quark PDF from 1 GeV to 1000 GeV with $\alpha(k_T)$, rapidity ordering and minimal cut-off λ on k_T . No quark–gluon transitions.

In Figs. 5 and 6 we present the full evolution with the quark–gluon transitions for gluon and quark PDFs. The lines in the top frame correspond respectively (from upper to lower) to total distribution, and its components with 0,1,2,3 and 4 transitions. One can see that four transitions contribute well below the required 10^{-3} precision level and can be safely neglected. In the bottom frames the comparison of MMC and CMC results is shown for all components. As before, the agreement at the level of 10^{-3} is achieved.

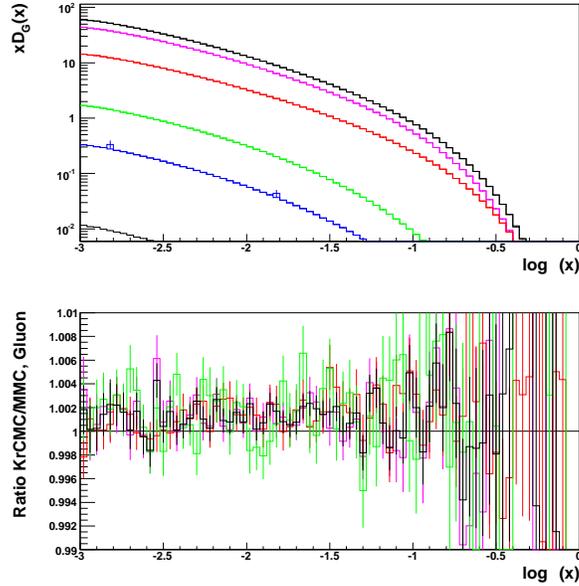


Fig. 5. Evolution of the gluon PDF from 1 GeV to 1000 GeV with $\alpha(k_T)$, rapidity ordering, minimal cut-off λ on k_T and quark–gluon transitions. Curves in the top frames correspond respectively (from upper to lower) to total distribution, and its components with 0,1,2,3 and 4 transitions.

To summarize, in this presentation we gave a brief report on one of our recent activities within the developed project of constructing a new QCD parton shower MC program. We showed how we implemented the modified running coupling constants in the DGLAP-type Markovian evolution algorithm. We presented the cases of $\alpha(k_T)$ as well as the simpler $\alpha(Q(1-z))$ one. Numerical comparisons between two implementations (MMC and CMC) confirmed the precision of implementations at the level of 10^{-3} , both for the pure gluonstrahlung as well as for the full evolution with quark–gluon transitions.

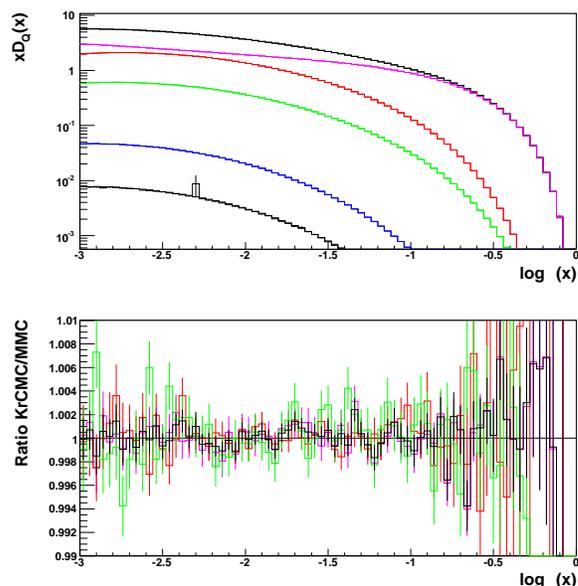


Fig. 6. Evolution of the quark PDF from 1 GeV to 1000 GeV with $\alpha(k_T)$, rapidity ordering, minimal cut-off λ on k_T and quark–gluon transitions. Curves in the top frame correspond respectively (from upper to lower) to total distribution, and its components with 0,1,2,3 and 4 transitions.

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