MATCHING CONSTRAINED MONTE CARLO TO NLO MATRIX ELEMENT*

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We present a new method of matching the Constrained Monte Carlo (CMC) algorithm to next-to-leading order matrix elements. In such matching, one must simultaneously treat the infra-red singularities of the matrix element and double counting issues between the two contributions. In the proposed method these two issues are dealt with by a specially defined counterterm. This counterterm is analyzed in detail and the connection of the CMC algorithm and formal factorization theorems is given. This counterterm also allows many new processes to be quickly integrated into the algorithm as the application of the counterterm can be done at the matrix element generation level and all residual issues in the parton shower are universal.

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1. Introduction

The most predictive tool we have at our disposal for analyzing high energy hadron–hadron data is perturbative Quantum Chromodynamics (pQCD). With this we can compute order-by-order in powers of the coupling constant, $\alpha_{\rm S}$, the contributions to a given process. The drawback with this order-by-order approach is that this becomes computationally prohibitive beyond next-to-next-to-leading order (NNLO), and for many processes even beyond next-to-leading order (NLO). This order, and higher, is necessary, however, to connect the many particle final state events from experiments with the underlying theory. Fortunately, the theoretical framework to incorporate the most relevant aspects of the higher-order contributions has

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been developed and is well known [1–4]. The result is an evolution equation, for example the DGLAP equation [1], which *resumes* these leading contributions.

In general, however, this framework to include the higher-order effects is only strictly valid in the *inclusive* sense, where all extra particles have been integrated out. The experiments are capable of studying the *exclusive* properties of the particles as well and a connection of these exclusive states to the theory is necessary. This is achieved by means of a *parton shower* [5,6]. This method solves the evolution equation by means of a Monte Carlo algorithm, where each sample of the Monte Carlo can be related to the fully exclusive four momenta of the particles. To date, this relationship between the evolution equation and the exclusive states has only been possible to the leading logarithmic (LL) approximation.

In order to obtain the most precise prediction of the theory we want to combine the LL parton shower with the NLO perturbative calculation. This matching is non-trivial due to the singularity structure of the NLO result and the double counting issues between the two contributions. The double counting is related directly to the specific implementation of the parton shower. In this paper we report the current status of matching a NLO calculation with the LL parton shower generated according to the new constrained Monte Carlo (CMC) algorithm [7–10].

The paper is structured as follows: In Section 2 we define the CMC algorithm and particular features which are relevant for the proposed matching scheme. In Section 3 we propose an algorithm which provides the full NLO result. In Section 4 we present a new counterterm to use in the NLO calculation to remove the singularities. This new counterterm is particularly attractive for use in our matching prescription. In Section 5 we study the relation to the standard factorization schemes and suggest the proper treatment. Last we conclude these results and present our future plans.

2. Constrained Monte Carlo algorithm

The Monte Carlo method is used to solve an evolution equation of the form

$$Q^2 \frac{\partial D(x,Q^2)}{\partial Q^2} = \int\limits_x dz P(z,Q^2) D(x/z,Q^2)\,, \tag{1}$$

in order to resume the large logarithms in the soft and collinear regions. The upper bound of the integral is not specified as this corresponds to different treatments of the "unresolvable" emissions. $P(z)$ is the kernel of the evolution and $D(x,Q^2)$ is the distribution of the longitudinal momentum fraction x at scale Q^2. For simplicity we have dropped the explicit flavor indices.

We denote the solution of the constrained evolution [7–10] as

$$D(x,q) = \mathcal{G}(x,q;x_0,q_0)D_0(x_0,q_0)\,, \tag{2}$$

where $D_0(x_0,q_0)$ is the non-perturbative structure of the proton and q is some evolution time. In this representation it is implicit that the full four momentum of the emissions are contained in $\mathcal{G}$. The form of $\mathcal{G}$ which satisfies the evolution equation is given as

$$\mathcal{G}\left(x,q^2;x_0,q_0^2\right) = \sum_{n=0}^{\infty} \Delta\left(q^2,q_n^2\right) \left[\prod_{i=1}^{n} \int_{q_{i-1}^2}^{q^2} \frac{dq_i^2}{q_i^2} \int dz_i \Delta\left(q_i^2,q_{i-1}^2\right) P\left(z_i,q_i^2\right)\right]$$

$$\times\, \delta\Big(x - x_0 \sum_j z_j\Big)\,, \tag{3}$$

with $\Delta(Q^2,q^2)$ the Sudakov form factor

$$\Delta\left(q^2,q_0^2\right) = \exp\Big(-\int_{q_0^2}^{q^2} \frac{dq'^2}{q'^2} \int dz' P(z',q'^2)\Big)\,, \tag{4}$$

defined such that $\int dx\, xD(x,Q^2) = 1$. Again the upper bound on the z integration has been omitted as several choices are possible and each corresponds to a slightly different evolution. Details can be found in [11]. In Eq. (3) the constraint can be seen explicitly by the δ function.

The evolution operator $\mathcal{G}$ satisfies the relation

$$\mathcal{G}(x,q^2;x_0,q_0^2) = \int dx' \mathcal{G}(x,q^2;x',q'^2)\mathcal{G}(x',q'^2;x_0,q_0^2)\,. \tag{5}$$

Thus

$$D(x,q^2) = \int dx' \mathcal{G}(x,q^2;x',q'^2)D(x',q'^2)\,. \tag{6}$$

There are many choices of the ordering variable q which satisfies the evolution equation given in Eq. (1). If we write the momentum of an emitted gluon as

$$k_i = y_i p + \bar{y}_i \bar{p} + k_{\perp i}\,, \tag{7}$$

with p and $\bar{p}$ the momentum of the hadrons, we define the evolution variable and momentum fraction to be

$$q_i \;=\; \sqrt{S\frac{\bar{y}_i}{y_i}}\,, \qquad z_i = 1 - \frac{y_i}{x_{i-1}}\,, \tag{8}$$

$$\bar{q}_i \;=\; \sqrt{S\frac{y_i}{\bar{y}_i}}\,, \qquad \bar{z}_i = 1 - \frac{\bar{y}_i}{\bar{x}_{i-1}}\,, \tag{9}$$

in the forward hemisphere and backward hemispheres respectively; additionally $S = (p + \bar{p})^2$. These evolution variables are related to the rapidity in the lab frame. It is important to note that this solution to the evolution equation has full coverage of phase space (modulus the "unresolved" soft emissions).

In the constrained evolution the final x value can be given and the evolution proceeds such that the constraint is matched. This means that the evolution can proceed forward from the non-perturbative proton structure to the perturbative scale of the hard process, while retaining important features of the hard process, *e.g.* resonances.

This is quite different from the standard backward evolution algorithm [6] where the evolution is guided by a tabulated collinear pdf. In this case the hard process sets the initial conditions and the evolution proceeds backwards toward the proton. In order to conserve momentum of the event it is necessary to boost the incoming quark lines independently and preserve a set of invariants of the hard process. This procedure, however, destroys other invariants. For example, one may choose to preserve the rapidity and mass of the s-channel, but by doing so the t-channel is changed.

Thus far we have only discussed the evolution of one proton. For LHC predictions we of course must consider two proton evolutions simultaneously. The two hemispheres can be divided in many different ways. As the evolutions are ordered in rapidity, the most obvious choice is to divide the two hemispheres by some constant rapidity, η^*. By construction the phase space for each evolution is filled completely and the choice of the rapidity separation is free. In the CMC implementation `KrCMC` [12] the choice is made to fix the rapidity at the rapidity of the s-channel. This gives

$$\eta^* = \frac{1}{2} \log \frac{x}{\bar{x}} \,. \tag{10}$$

It has been shown [12] how to match the two hemispheres according to a division by a fixed rapidity and how to constrain the evolution such that s is conserved and is given by the true momentum, not just the rescaled hadronic center of mass energy. Using this we now have the hadronic cross section at leading order, plus resummation

$$\begin{aligned} d\sigma_{\mathrm{HH}} \;=\; & \int dx d\bar{x} \int dx_0 d\bar{x}_0 \, \mathcal{G}(x, q^2; x_0, q_0^2) \, D_0(x_0, q_0^2) \\ & \times \mathcal{G}(\bar{x}, \bar{q}^2; \bar{x}_0, \bar{q}_0^2,) D_0(\bar{x}_0, \bar{q}_0^2) d\sigma_{\mathrm{B}}(s) \delta_s \,, \end{aligned} \tag{11}$$

where δ_s contains condition that the sum of all momentum must result in the partonic center-of-mass energy of the hard process equal to s. This formula is illustrated in Fig. 1.

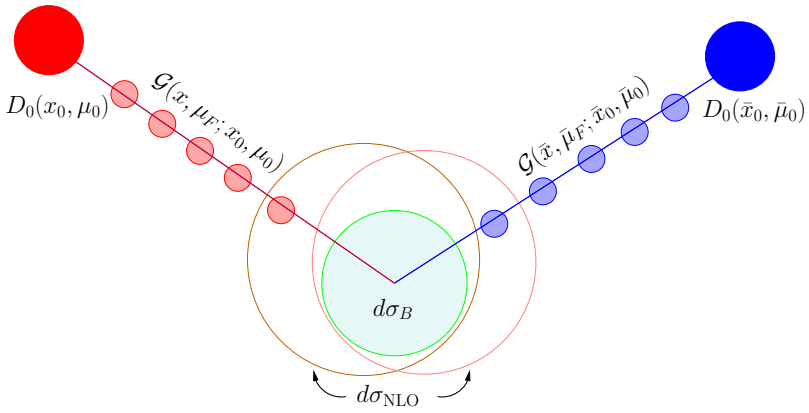

Fig. 1. The solid blobs represent the non-perturbative proton structure. Each of the smaller blobs represents a kernel in the evolution operator, $\mathcal{G}$. The central blob is the Born level hard process and the two overlapping circles contain the factors which must be considered at next-to-leading order.

3. Matching NLO matrix element

We now turn our attention to improving Eq. (11) to give the full next-to-leading order (NLO) prediction. As the evolution operators provide a full coverage of phase space we can re-weight configurations produced in the evolution to give the correct NLO distribution. In this way we wish to compute a function, $\beta(k)$, such that the gluon with momentum k is distributed according to the NLO distribution. In the standard factorization theorem we have the NLO prediction

$$d\sigma_{\rm HH}^{\rm NLO} = \int dx d\bar{x} f(x, \mu_{\rm F}^2) f(\bar{x}, \mu_{\rm F}^2) d\hat{\sigma}(x\bar{x}S; \mu_{\rm F}^2)\,, \tag{12}$$

where $f(x, Q^2)$ is the standard collinear pdf which is the solution to the DGLAP [1] evolution equation. The hat on the partonic cross section implies that the IR divergences have been treated according to the KLN [13] and factorization theorems. If we treat our Monte Carlo solution $D(x, q^2)$ in the same way we must find a function β such that

$$\begin{aligned} d\sigma_{\rm HH}^{\rm NLO} \;=\; & \int dx_0 d\bar{x}_0\, \mathcal{G}(x, q^2; x_0, q_0^2)\, D_0(x_0, q_0^2) \\ & \times \mathcal{G}(\bar{x}, \bar{q}^2; \bar{x}_0, \bar{q}_0^2)\, D_0(\bar{x}_0, \bar{q}_0^2)\, d\sigma_{\rm B}(s) \beta(s, k) \delta_s, \end{aligned} \tag{13}$$

is able to reproduce Eq. (12) with $f(x, Q^2)$ interchanged with $D(x, q^2)$. We write the β function as

$$\beta(s, k) = \beta_0(s) + \beta_0^{\rm col}(s) + \beta_1^{\rm col}(s) + \beta_1(s, k)\,, \tag{14}$$

where β_0 and $\beta_0^{\rm col}$ re-weight the Born level kinematics and β_1 and $\beta_1^{\rm col}$ re-weight the kinematics with an extra emission. We start by defining a value of the evolution variable at which to cut the evolution operator. We denote this as ξ. Remember as well that each hemisphere is separated by some fixed rapidity, η^*, which translates into a maximum value of the evolution operator, q^* and $\bar{q}^*$. This gives

$$\begin{aligned} d\sigma_{\rm HH}^{\rm NLO} &= \int dx d\bar{x} \left[\int dx' \mathcal{G}(x,q^*;x',\xi) D(x',\xi) D(\bar{x},\bar{q}^*) \theta(q^*-\xi) \right. \\ &\quad \left. + \int d\bar{x}' D(x,q^*) \mathcal{G}(\bar{x},\bar{q}^*;\bar{x}',\xi) D(\bar{x}',\xi) \theta(\bar{q}^*-\xi) \right] d\sigma_{\rm B}(s) \beta(k(\xi)) \,. \end{aligned} \tag{15}$$

We now extract the $\mathcal{O}(\alpha_{\rm S})$ contribution from the two evolution operators. We find the virtual and real contributions at $\mathcal{O}(\alpha_{\rm S})$ (modulus a negative sign in the virtual contribution) are given as

$$\rho_{PS}^{\rm V} = \int_{\xi^2}^{q^2} \frac{dq'^2}{q'^2} \int dz' \frac{\alpha_{\rm S}(q',z') \mathcal{K}(z')}{(1-z')} \, d\sigma_{\rm B} \tag{16}$$

$$\rho_{PS}^{\rm R} = \frac{\alpha_{\rm S}(q,z)\mathcal{K}(z)}{q^2(1-z)} dz dq^2 d\sigma_{\rm B} \,, \tag{17}$$

with $x' = zx$ and similarly for the backward hemisphere. Here we have defined

$$P(z,q) = \frac{\alpha_{\rm S}(z,q)\mathcal{K}(z)}{(1-z)} \,. \tag{18}$$

Thus when we expand Eq. (13) to $\mathcal{O}(\alpha_{\rm S})$ we find

$$\begin{aligned} d\sigma_{\rm HH}^{\rm NLO} &= \int dx d\bar{x} \left[\int dx' D(x',\xi) D(\bar{x},\bar{q}) \left(1 - \rho_{PS}^{\rm V} + \rho_{PS}^{\rm R}(k)\right) \theta(\eta_k - \eta^*) \right. \\ &\quad \left. + \int d\bar{x}' D(x,q) D(\bar{x}',\xi) \left(1 - \bar{\rho}_{PS}^{\rm V} + \bar{\rho}_{PS}^{\rm R}(k)\right) \theta(\eta^* - \eta_k) \right] \\ &\quad \times \left[\left(\beta_0^{(0)} + \alpha_{\rm S}\beta_0^{(1)}\right) \delta^{(4)}(k) + \alpha_{\rm S}\beta_1^{(0)} \right] d\sigma_{\rm B}(s) \,. \end{aligned} \tag{19}$$

Comparing this to Eq. (12) we find

$$\beta_0^{(0)} = 1 \,, \tag{20}$$

$$\beta_0^{(1)} = \frac{d\sigma^{\rm V} + \rho_{PS}^{\rm V} + \bar{\rho}_{PS}^{\rm V}}{d\sigma_{\rm B}} \,, \tag{21}$$

$$\beta_1^{(0)} = \frac{d\sigma^{\rm R} - \rho_{PS}^{\rm R} - \bar{\rho}_{PS}^{\rm R}}{d\sigma_{\rm B}} \,. \tag{22}$$

We must be careful with the collinear singularities. In the standard factorization theorem, the differential cross section has the collinear singularities removed by absorbing them into the non-perturbative part of the pdf. We will do the same thing and the details will be presented in Section 5.

We see that ξ acts in the same manner as the factorization scale, μ_{F}, in Eq. (12). For a given event, the logical choice for ξ is the value of q_i for a given emission. It is argued [14] that the optimal choice for the subleading terms is the emission with highest $p_\perp$. For reasons of ordering, the emission chosen must also be the last emission in the ladder. In `KrCMC` we do not order in $p_\perp$ but rather in rapidity, thus the last emission and the hardest emission are not equivalent; they correspond in about 60% of the events. If one wanted to strictly adhere to the hardest emission then the events where this is violated can be thrown away and the efficiency loss is not too large. As our goal is exact NLO matching, we instead choose the hardest emission between the last one of each ladder. This is illustrated in Fig. 1.

4. Treatment of NLO matrix element

We have defined the Monte Carlo weight which we can associate to a given emission to re-weight that emission to NLO. We see from Eqs. (21), (22) that the real and virtual contributions are separated. This means that the IR singularities must be treated using some counterterm. In this section we propose such a counterterm and in the next section we will study this term and show why it is an ideal choice to use in Eqs. (21), (22).

Inspecting Eq. (21) we find that if we choose a counterterm that matches ρ^{V}_{PS} away from the singularities, then our definition of β will be greatly simplified. We define this counterterm in $D = 4 - 2\epsilon$ dimensions and use the same hemisphere separation given by the Monte Carlo. Thus we use the D dimensional kernel and extend our range of q and z to all values, in order to contain the singularities. It turns out that when the regulators for the Monte Carlo are removed, it is more convenient to use the y and $\bar{y}$ variables than the q and z ones. In the counterterm, rather than using the hadron momenta we rescale these by $\sqrt{\tau/S}$, with τ a free parameter. This will be chosen to exactly match the singularities of the NLO calculation. The separation of hemispheres is given by

$$\begin{aligned} \eta^* &= \frac{1}{2}\ln\frac{y}{\bar{y}}, \\ \theta(\xi - q^*) &= \theta\left(y - \bar{y}e^{2\eta^*}\right), \\ \theta(\bar{q}^* - \xi) &= \theta\left(\bar{y} - ye^{-2\eta^*}\right). \end{aligned}$$

The form of the counterterm is

$$d\sigma^{(\mathrm{F})}_{\mathrm{V,ct}} = \int_0^1 dy \int_0^{ye^{-2\eta^*}} d\bar{y}\alpha_{\mathrm{S}}\mu_{\mathrm{R}}^{2\epsilon}\frac{\tau^{-\epsilon}(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)}\mathcal{K}(y,\epsilon)(y\bar{y})^{-1-\epsilon}d\sigma_{\mathrm{B}}, \tag{23}$$

for the forward hemisphere and $(y \leftrightarrow \bar{y}, \eta^* \to -\eta^*)$ in the backward hemisphere.

If we consider the Altarelli–Parisi [1] splitting function for $q \to qg$ we have

$$\mathcal{K}(y,\epsilon) = \frac{C_{\mathrm{F}}}{2\pi}\left(1+(1-y)^2-\epsilon y^2\right), \tag{24}$$

which gives

$$d\sigma^{(\mathrm{F})}_{\mathrm{V,ct}} = \left(\frac{2}{\epsilon^2}+\frac{3+4\eta^*}{\epsilon}+4\eta^{*2}+6\eta^*+8\right)\alpha_{\mathrm{S}}\,\mu_{\mathrm{R}}^{2\epsilon}\,\frac{(4\pi)^{\epsilon}}{(4\pi)^2}\frac{\tau^{-\epsilon}C_{\mathrm{F}}d\sigma_{\mathrm{B}}}{\Gamma(1-\epsilon)} \tag{25}$$

and $(\eta^* \to -\eta^*)$ for the backward hemisphere. Together these two components give

$$d\sigma_{\mathrm{V,ct}} = 4\pi\alpha_{\mathrm{S}}\mu_{\mathrm{R}}^{2\epsilon}\tau^{-\epsilon}C_{\mathrm{F}}N(\epsilon)H(\epsilon)d\sigma_{\mathrm{B}}\left(\frac{4}{\epsilon^2}+\frac{6}{\epsilon}+8\eta^{*2}+16\right), \tag{26}$$

$$N(\epsilon) = \frac{(4\pi)^{\epsilon}}{(4\pi)^2}\Gamma(1+\epsilon), \tag{27}$$

$$H(\epsilon) = \left(1-\frac{\pi^2}{6}\epsilon^2+\mathcal{O}(\epsilon^4)\right). \tag{28}$$

The term proportional to η^{*2} can be traced back to the effect of the hemisphere separation in the Sudakov factors. This is compensated for by the real emissions. As this is an unphysical object it should have no effect on physical predictions. This has been shown [12] for the distribution of the gluon momentum with no hard matrix element (*e.g.* $d\sigma_{\mathrm{B}} = 1$).

We define that the virtual counterterm is just minus the integral of the real counterterm. Thus we have

$$d\sigma_{\mathrm{R,ct}} = -\frac{\alpha_{\mathrm{S}}\,\mu_{\mathrm{R}}^{2\epsilon}(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)}(y\bar{y})^{-1-\epsilon}\left(\mathcal{K}(y,\epsilon)\theta(\xi-q^*)+\mathcal{K}(\bar{y},\epsilon)\theta(\bar{q}^*-\xi)\right)\frac{dy\,d\bar{y}\,d\phi}{2\pi}. \tag{29}$$

With such a definition we know that if the singularities are canceled in the virtual term, they must also be canceled in the real contributions.

With these definitions of the counterterms we define the regularized cross sections as

$$d\hat{\sigma}_{\mathrm{V}} = d\sigma_{\mathrm{V}} + d\sigma_{\mathrm{V,ct}}, \tag{30}$$

$$d\hat{\sigma}_{\mathrm{R}} = d\sigma_{\mathrm{R}} + d\sigma_{\mathrm{R,ct}}. \tag{31}$$

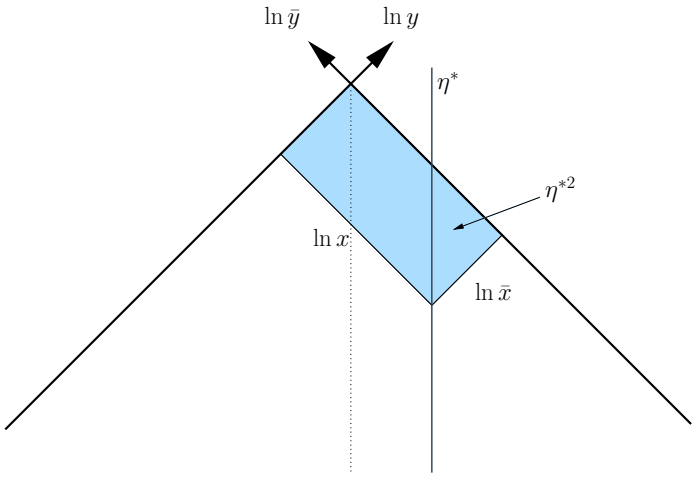

Fig. 2. The shaded region is the region of phase space absent for the corresponding choice of η^*, used to divide the hemispheres.

Having regularized our NLO contributions we turn our attention to the structure of these counterterms and why they are ideally suited for the constrained Monte Carlo.

5. Connection to scheme dependence

The implementation of the evolution equation in `KrCMC` provides an unintegrated pdf. This differs from the standard collinear pdf as the momentum in the transverse plane are unintegrated. This means that we depart slightly from the strict theoretical backing of the collinear factorization. If we are to include the condition that the transverse momentum entering the hard process are exactly zero, we would recover exactly the standard factorization theorem. We have proposed [12] a factorization formula (given in Eq. (11)) which reduces to the standard factorization theorem when we "downgrade" our Monte Carlo to be that of the standard collinear factorization with pdfs evolved according to the leading-log DGLAP evolution equation.

Due to the connection to the standard pdf, we feel we are on solid ground to be able to define a factorization scheme by which we can treat the collinear singularities of the hard matrix element. This will, in the end, be equivalent to the collinear factorization schemes and will only apply in the strictly collinear region.

We start by looking at the virtual counterterm. This choice is for notational simplicity as the bounds of the integrals are explicit. The same treatment applies to the real counterterm and both will be used to define the procedure.

The virtual counterterm can be written in the q, z space of the evolution for each hemisphere separately. As we have seen the treatment of the hemisphere separation appears in the maximum evolution time. For the forward hemisphere we find

$$d\sigma^{\mathrm{F}}_{\mathrm{V,ct}} = \int_0^{q^{*2}} \frac{dq'^2}{(q'^2)^{1+\epsilon}} \int_0^1 dz' \alpha_{\mathrm{S}} \mu_{\mathrm{R}}^{2\epsilon} \frac{\tau^{-\epsilon}(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \frac{\mathcal{K}(z',\epsilon)}{[(1-z')x]^{1+2\epsilon}} \, d\sigma_{\mathrm{B}} \,. \tag{32}$$

We can divide this integral into three separate regions, as illustrated in Fig. 3. These are

$$\rho^{\mathrm{V}}_{C} = \int_0^{q_n^2} \frac{dq'^2}{(q'^2)^{1+\epsilon}} \int_0^1 dz' \alpha_{\mathrm{S}} \mu_{\mathrm{R}}^{2\epsilon} \frac{\tau^{-\epsilon}(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \frac{\mathcal{K}(z',\epsilon)}{[(1-z')x]^{1+2\epsilon}} \, d\sigma_{\mathrm{B}} \,, \tag{33}$$

$$\rho^{\mathrm{V}}_{PS} = \int_{q_n^2}^{q^{*2}} \frac{dq'^2}{(q'^2)^{1+\epsilon}} \int_0^{1-\varepsilon_z} dz' \alpha_{\mathrm{S}} \mu_{\mathrm{R}}^{2\epsilon} \frac{\tau^{-\epsilon}(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \frac{\mathcal{K}(z',\epsilon)}{[(1-z')x]^{1+2\epsilon}} \, d\sigma_{\mathrm{B}} \,, \tag{34}$$

$$\rho^{\mathrm{V}}_{S} = \int_{q_n^2}^{q^{*2}} \frac{dq'^2}{(q'^2)^{1+\epsilon}} \int_{1-\varepsilon_z}^1 dz' \alpha_{\mathrm{S}} \mu_{\mathrm{R}}^{2\epsilon} \frac{\tau^{-\epsilon}(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)} \frac{\mathcal{K}(z',\epsilon)}{[(1-z')x]^{1+2\epsilon}} \, d\sigma_{\mathrm{B}} \,. \tag{35}$$

As indicated by the notation, the second term is exactly that which is given in the parton shower. In this case there are no singularities and we can take $\epsilon \to 0$. We have now used ε_z to represent the cutoff in the parton shower that defines the "resolvability" of an emission. One may consider several different choices for this cutoff [11] but we treat it here as an abstract quantity. We will see that the exact meaning of this does not change the matching method presented.

We look first at Eq. (35). As can be seen in Fig. 3 this corresponds to the soft region. This is the region of phase space which is considered "unresolved" in the parton shower. As this is the soft region we can safely ignore this and it should not affect any IR safe observables.

The quantity of real interest is ρ_C. We make use of the "plus" prescription in the z variables and define the q prescription in the q integration

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{1-z} \,, \tag{36}$$

$$\int_0^{q^2} dq'^2 \frac{f(q'^2)}{(q'^2)_q} = \int_0^{q^2} dq'^2 \frac{f(q'^2)-f(0)}{q'^2} \,. \tag{37}$$

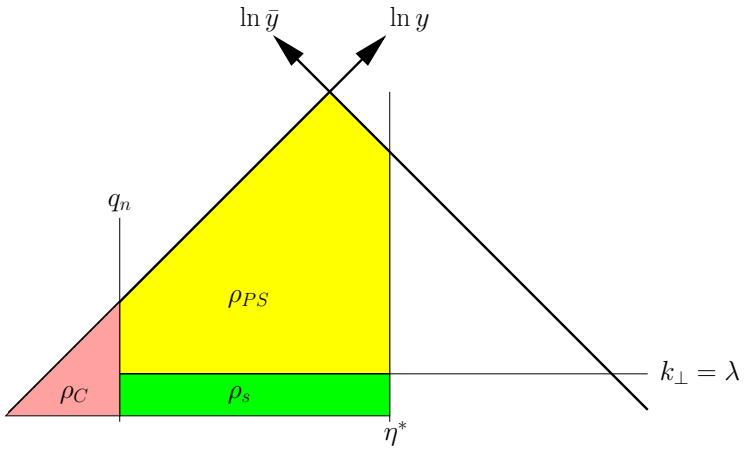

Fig. 3. Plot in the Sudakov plane. The decomposition of the counterterm is given by the sum of the three shaded areas. Each area is labeled according to the text. The line of minimum $k_\perp = \lambda$ is an example of a choice of ϵ_z.

Using these prescriptions we find

$$\rho_C^{\mathrm{V}} = \int_0^{q^{*2}} dq'^2 \int_0^1 dz \left[\frac{\delta(q'^2)\delta(1-z)}{2\epsilon^2} - \frac{\delta(q'^2)}{\epsilon} \frac{1}{(1-z)_+} - \frac{\delta(1-z)}{2\epsilon} \frac{1}{(q'^2)_q} + \frac{1}{(1-z)_+} \frac{1}{(q'^2)_q} - \frac{\delta(1-z)}{2} \left(\frac{\log q'^2}{q'^2} \right)_q - \delta q'^2 \left(\frac{\log(1-z)}{1-z} \right)_+ \right] \times \alpha_{\mathrm{S}} \, \mu_{\mathrm{R}}^{2\epsilon} \frac{\tau^{-\epsilon}(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \mathcal{K}(z', \epsilon) \, d\sigma_{\mathrm{B}} \, . \tag{38}$$

To first order in α_{S} we can treat the scale of the running coupling constant with respect to q and thus the q prescription yields 0. The result is

$$\rho_C^{\mathrm{V}} = \int_0^{q^{*2}} dq'^2 \int_0^1 dz \left[\frac{\delta(q'^2)\delta(1-z)}{2\epsilon^2} - \frac{\delta(q'^2)}{\epsilon} \frac{1}{(1-z)_+} - \delta(q'^2) \left(\frac{\log(1-z)}{1-z} \right)_+ \right] \times \alpha_{\mathrm{S}} \, \mu_{\mathrm{R}}^{2\epsilon} \frac{\tau^{-\epsilon}(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \mathcal{K}(z, \epsilon) \, d\sigma_{\mathrm{B}} \, . \tag{39}$$

By definition ρ_C^{R} is minus the integrand of ρ_C^{V} and after expanding in ϵ we find:

$$\frac{\rho_C^{\mathrm{R}}+\rho_C^{\mathrm{V}}}{d\sigma_{\mathrm{B}}} = \left(\frac{1}{\epsilon}+\ln 4\pi-\gamma_E+\log\frac{\tau}{\mu_{\mathrm{R}}^2}\right)\mathcal{P}(z)+\frac{\alpha_{\mathrm{S}}C_{\mathrm{F}}}{2\pi}\left(\frac{\delta(1-z)}{2}-1+z\right) + \alpha_{\mathrm{S}}\left(\frac{\log(1-z)}{(1-z)_+}\mathcal{K}(z)+\frac{7}{4}\frac{C_{\mathrm{F}}}{2\pi}\delta(1-z)\right), \tag{40}$$

$$\mathcal{P}(z) = \frac{\alpha_{\mathrm{S}}C_{\mathrm{F}}}{2\pi}\left(\frac{\mathcal{K}(z)}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right). \tag{41}$$

The reader will recognize the standard $\overline{\mathrm{MS}}$ terms used to renormalize the bare parton distribution. We denote these by $\kappa_{\overline{\mathrm{MS}}}$. Thus

$$\frac{\rho_C^{\mathrm{R}}+\rho_C^{\mathrm{V}}}{d\sigma_{\mathrm{B}}} = \kappa_{\overline{\mathrm{MS}}}(z)+\alpha_{\mathrm{S}}\left(\frac{\log\tau/\mu_{\mathrm{R}}^2}{(1-z)_+}+\frac{\log(1-z)}{(1-z)_+}\right)\mathcal{K}(z) + \frac{\alpha_{\mathrm{S}}C_{\mathrm{F}}}{2\pi}\left(\frac{\delta(1-z)}{2}-1+z\right)+\frac{\alpha_{\mathrm{S}}C_{\mathrm{F}}\delta(1-z)}{2\pi}\left(\frac{3}{2}\log\frac{\tau}{\mu_{\mathrm{R}}^2}+\frac{7}{4}\right). \tag{42}$$

We can see the coefficients of $\mathcal{O}(\epsilon)$ in the splitting function lead a finite effect in the collinear region. As this term is universal we propose to renormalize the bare parton density with

$$\kappa_{\mathrm{CPS}} = \left(\frac{1}{\epsilon}+\ln 4\pi-\gamma_E\right)\mathcal{P}(z,\epsilon), \tag{43}$$

$$\mathcal{P}(z,\epsilon) = \frac{\alpha_{\mathrm{S}}C_{\mathrm{F}}}{2\pi}\left[\frac{\mathcal{K}(z,\epsilon)}{(1-z)_+}+\left(\frac{3}{2}+\frac{\epsilon}{2}\right)\delta(1-z)\right], \tag{44}$$

were the acronym CPS is for "constrained parton shower". Thus

$$\frac{\rho_C^{\mathrm{R}}+\rho_C^{\mathrm{V}}}{d\sigma_{\mathrm{B}}} = \kappa_{\mathrm{CPS}}(z)+\alpha_{\mathrm{S}}\left(\frac{\log\tau/\mu_{\mathrm{R}}^2}{(1-z)_+}+\frac{\log(1-z)}{(1-z)_+}\right)\mathcal{K}(z) + \frac{\alpha_{\mathrm{S}}C_{\mathrm{F}}\delta(1-z)}{2\pi}\left(\frac{3}{2}\log\frac{\tau}{\mu_{\mathrm{R}}^2}+\frac{7}{4}\right). \tag{45}$$

It should also be obvious that if the free parameter μ_{R}^2 is chosen as $\tau(1-z)$ for both the real and virtual counterterm then

$$\rho_C^{\mathrm{R}}+\rho_C^{\mathrm{V}} = \kappa_{\mathrm{CPS}}\, d\sigma_{\mathrm{B}}. \tag{46}$$

We now turn out attention back to Eqs. (21), (22). Our regularized terms can be expressed as

$$d\hat{\sigma}_{\mathrm{V}} = d\sigma_V+\rho_C^{\mathrm{V}}+\rho_{PS}^{\mathrm{V}}+\rho_S^{\mathrm{V}}+\bar{\rho}_C^{\mathrm{V}}+\bar{\rho}_{PS}^{\mathrm{V}}+\bar{\rho}_S^{\mathrm{V}}, \tag{47}$$

$$d\hat{\sigma}_{\mathrm{R}} = d\sigma_{\mathrm{R}}+\left(-\rho_{PS}^{\mathrm{R}}+\rho_C^{\mathrm{R}}+\rho_S^{\mathrm{R}}\right)\theta_{\mathrm{F}}+\left(-\bar{\rho}_{PS}^{\mathrm{R}}+\bar{\rho}_C^{\mathrm{R}}+\bar{\rho}_S^{\mathrm{R}}\right)\theta_B. \tag{48}$$

We can now see why this counterterm is ideal for the CMC as

$$\beta_0^{(1)} = \frac{d\hat{\sigma}^{\mathrm{V}}}{d\sigma_{\mathrm{B}}} - \frac{\rho_C^{\mathrm{V}} + \rho_S^{\mathrm{V}} + \bar{\rho}_C^{\mathrm{V}} + \bar{\rho}_S^{\mathrm{V}}}{d\sigma_{\mathrm{B}}}\,, \tag{49}$$

and

$$\beta_1^{(0)} = \frac{d\hat{\sigma}^{\mathrm{R}}}{d\sigma_{\mathrm{B}}} - \frac{(\rho_C^{\mathrm{R}} + \rho_S^{\mathrm{R}})\theta_{\mathrm{F}} + (\bar{\rho}_C^{\mathrm{R}} + \bar{\rho}_S^{\mathrm{R}})\theta_B}{d\sigma_{\mathrm{B}}}\,. \tag{50}$$

We have already stated that the soft pieces can be safely neglected as they will not effect the IR safe observables. Formally, this means we have integrated the soft phase space at NLO.

As per the factorization theorem we can absorb the singularity and some finite pieces in the parton distribution function. After this operation, the last term in Eq. (49) (for each hemisphere) is defined as β_0^{col} and the last term of Eq. (50) (again for each hemisphere) is β_1^{col} and depends only on one variable, $z(\bar{z})$. This corresponds to the contributions to the NLO calculation from emissions in the purely collinear region. This term is universal and factorization scheme specific. We also see that with a judicious choice of the renormalization scale and the factorization scheme we can set these terms to 0.

Readers familiar with MC@NLO [15] will recognize these finite collinear contributions to be that of the $\tilde{2}$ configuration. In that work these terms are treated like the Born level kinematics after a longitudinal boost to compensate for the z value. For both MC@NLO and the matching scheme prescribed here, these are collinear emissions that effect the kinematics but are not included in the parton shower contributions.

To summarize the results of this section we present the formula for the β function in the two different factorization schemes discussed. In the results below we use a renormalization scale of $\tau(1-z)$. This argument is proportional to the transverse momentum of the emitted gluon. In either scheme we have

$$\beta_0^{(1)} = \frac{d\hat{\sigma}^{\mathrm{V}}}{d\sigma_{\mathrm{B}}}\,, \tag{51}$$

$$\beta_1^{(0)} = \frac{d\hat{\sigma}^{\mathrm{R}}}{d\sigma_{\mathrm{B}}}\,. \tag{52}$$

In the $\overline{\mathrm{MS}}$ scheme we find

$$\beta_0^{\mathrm{col}} = \frac{\alpha_{\mathrm{S}} C_{\mathrm{F}}}{4\pi}\delta(1-z)\,, \tag{53}$$

$$\beta_1^{\mathrm{col}} = -\frac{\alpha_{\mathrm{S}} C_{\mathrm{F}}}{2\pi}(1-z)\frac{d\sigma_{\mathrm{B}}(zs)}{d\sigma_{\mathrm{B}}(s)}\,, \tag{54}$$

while in the CPS scheme these collinear terms are zero.

6. Conclusion

We have presented here a proposal of how to match a NLO matrix element with the parton shower implementation known as KrCMC [12]. This method uses a specially designed counterterm to simultaneously avoid double counting with the parton shower and to treat the singularities of the NLO result. This counterterm has been analyzed in detail and its relation to standard factorization schemes has been discussed. The choice of factorization scheme and renormalization scale can lead to a non-zero, finite remainder in the collinear region which is compensated for by an appropriate weight.

In the future this method will be applied to the process $pp \to W^+W^-$ at NLO. This process is chosen as it contains a rich structure to test all aspects of the matching. In particular this process has NLO contributions from $q\bar{q} \to W^+W^-$, $gq \to W^+W^-$ and $g\bar{q} \to W^+W^-$ as well as a richer virtual structure due to box diagrams. For hadron-hadron to colorless objects, this process presents many technical hurdles which serve as a cross check of the method.

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