

CONTROLLING DIRECTED TRANSPORT IN INERTIA RATCHETS VIA ADAPTIVE BACKSTEPPING CONTROL

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We present a mechanism for controlling directed transport of particles in inertia ratchets. We study a parameter regime where two attractors — each transporting particles in different directions co-exist in phase space; and show that a proper control of direction of transport can be achieved by using adaptive backstepping based synchronisation technique.

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1. Introduction

Transport phenomena and in particular directed transport are at the heart of many problems in physics, chemistry and biology. Renewed interest in the field of directed transport stems from the theme of ratchet physics where unbiased, noise-induced transport emerges away from thermal equilibrium as a result of the action of Brownian motors [1–3]. Similarly, deterministic directed transport can result in time-dependent driven systems that exhibit a symmetry breaking of either spatial (for instance, the periodic ratchet potential) or of dynamic origin. Research activities in this area is partly motivated by the challenge to model and control some biological processes at both micro and macro scales as found in transport of ion channels and muscle operations respectively [4]. Another source of motivation is the potential for technological applications aimed at devising mechanisms for sorting, separating, pumping and controlling tiny particles at nanoscales and micro scales (see Refs. [2, 5] and references therein). Outstanding experimental realization of some of these devices have been carried out. Specifically, the control of motion of vortices in superconductors [6], particles in asymmetric silicon pores [7], charged particles through artificial pores [8], among others, have been reported recently.

Several attempts have been made to understand the generation of unidirectional motion from nonequilibrium fluctuations. The vast majority of the models described in the literature considers the overdamped cases in which the effect of the inertial term is neglected [1, 4]. Recently, ratchet models wherein the inertial term is considered have been extensively investigated since it was first studied by Jung *et al.* [9]. These ratchets possess, in general, a classical chaotic dynamics that modifies significantly the transport properties [9, 10]. For instance, current reversal and multiple current reversals have been attributed to changes in the bifurcation structure. In addition, the implication of chaotic dynamics in deterministic ratchets have been recently addressed in the quantum domain, together with the possible connection with quantum chaos [11].

In a different context, Savel'ev *et al.* [12] examined the transport properties of binary mixture of interacting particles and showed that attracting or repelling interaction among identical particles can result in the amplification (inversion) of their net current. This is potentially useful for enhancing and regulating transport (*e.g.* through synthetic ion channels) and separation of repelling particles. Interaction among identical and non-identical particles can lead to synchronised dynamics when a threshold is reached. Synchronisation of two coupled chaotic ratchets have been recently investigated [13] and it is believed that the synchronisation of coupled ratchets could provide some information regarding the transport properties of inertia ratchets [13]. Synchronisation phenomena in coupled or driven nonlinear oscillator are in general of fundamental importance in nonlinear dynamics and have been extensively investigated both theoretically and experimentally since the seminal work of Pecora and Carroll in 1990 [14]. Chaos synchronisation is closely related to the observer problem in control theory [15]. The problem may be treated as the design of control law for full chaotic observer (the slave system) using the known information of the plant (the master system) so as to ensure that the controlled receiver synchronises with the plant. Hence, the slave chaotic system traces the dynamics of the master in the course of time.

Various techniques have been proposed for achieving stable synchronisation between identical and non-identical systems. Notable among these methods, the active control scheme [16] has received a considerable attention in the last few years due to its simplicity and robustness. Applications to various systems abound, some of which includes the Lorenz, Chen and Lü system [17], geophysical system [18], spatiotemporal dynamical system [19], the so-called unified chaotic attractor [20], electronic circuits, which model a third-order “jerk” equation [21], the Bloch equation [22]; and most recently in RCL-shunted Josephson junction [23] and the Lorenz–Stenflo equations modelling acoustic gravity waves [24]. Another method, the recursive back-

stepping design which forms the building block for adaptive control of chaotic systems [25, 26], was recently extended to the synchronisation of identical chaotic systems by Tan *et al.* [27], in which the Lorenz system, Chua's circuit and Duffing oscillator were used as typical models to illustrate the efficiency of the technique. Backstepping design has been employed recently to control hydraulic servo systems [28], permanent magnet reluctance machine [29], Duffing oscillators [30] and a third-order phase locked loop [31]. The method is a recursive procedure that skillfully interlaces the choice of a Lyapunov function with the control. Indeed, backstepping control can guarantee global stability, tracking and transient performance for a broad class of strict-feedback systems [25, 26].

In a recent letter, we demonstrated the synchronisation of two identical chaotic ratchets and explored the property of active control to achieve the control of directed transport in inertia ratchets [32]. With this work, we extend our previous studies with the object of centering on the possibility of controlling at will, the directed transport arising from difference in initial conditions of the inertia ratchets by using another technique, the adaptive backstepping control proposed in Ref. [27]. In the next section, we describe our model and present the backstepping design for chaos synchronisation in Section 3. Section 4 deals with numerical simulation results and Section 5 concludes the paper.

2. The chaotic ratchet model

Let us consider the one-dimensional problem of a particle driven by a periodic time-dependent external force under the influence of an asymmetric potential of the ratchet type [9, 10, 13, 32]. The time average of the external force is zero. In the absence of stochastic noise, the dynamics is exclusively deterministic. The dimensionless equation of motion for a particle of unit mass moving in the ratchet potential $V(x)$ is given by (see Ref. [10] for instance):

$$\ddot{x} + b\dot{x} + \frac{dV(x)}{dx} = a \cos(\omega_D t), \quad (1)$$

where time t has been normalised in the unit of ω_0^{-1} , ω_0 being the frequency of the linear motion around the minima of $V(x)$; b is the damping parameter, while a and ω_D are the amplitude and frequency of the driving force respectively; $V(x)$ is the dimensionless potential given by

$$V(x) = C - \frac{1}{4\pi^2\delta} [\sin 2\pi(x - x_0) + 0.25 \sin 4\pi(x - x_0)]. \quad (2)$$

The constant $C \simeq 0.0173$ and $\delta \simeq 1.600$. The potential is shifted by a value x_0 in order that the minimum of $V(x)$ is located at the origin (see Fig. 1). We

note that apart from its periodicity, the ratchet potential (2) has an infinite number of potential wells; so that the orbits transports particles from one well to another. Thus, in the Poincaré section representation, one can utilise this periodicity to collapse the dynamics to a unit cell within a phase space region for which $-0.5 \leq x \leq 0.5$.

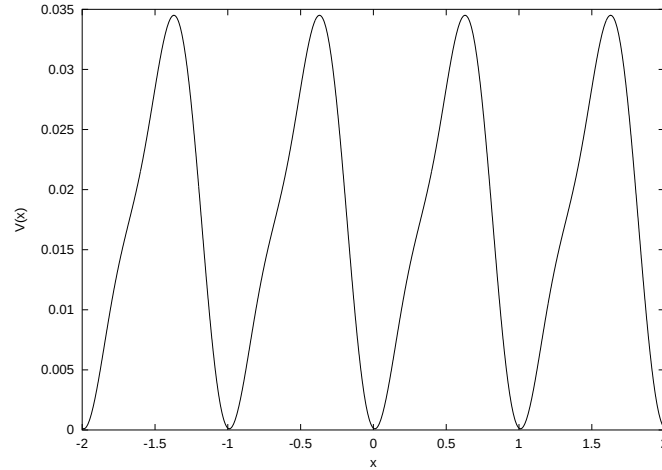


Fig. 1. The dimensionless ratchet potential.

The extended phase space in which the dynamics is taking place is three-dimensional, since we are dealing with an inhomogeneous differential equation with an explicit time dependence. Eq. (1) can be expressed in strict-feedback form and then solved numerically using the fourth-order Runge–Kutta algorithm. Since the equation is nonlinear, its solution therefore allows the possibility of periodic and chaotic orbits. System (1) exhibits rich varieties of dynamical behaviour including the co-existence of attractors. Here, we are particularly interested in the parameter regime where two attractors co-exist in phase space.

We fix $b = 0.1$, $\omega = 0.67$ and $a = 0.156$ throughout the paper and show (Fig. 2) in a Poincaré section, two co-existing attractors as reported in Ref. [33]. One is a chaotic attractor and generates positive current while the other is a periodic attractor and generate negative current. This scenario can be considered as a mixture of non-identical particles studied in [12]. Of course the mixing properties is revealed by the complex way the basins associated with these attractors are intermingled with fractal basin boundaries (see Fig. 5, Ref. [33]). It is noteworthy that co-existing attractors (the so-called “battle of attractors”) can be observed in ratchet models subjected to thermal noise, when the driving amplitude is increased to some critical value [34]. It has been reported by Macura *et al.* [34], that the particles in

this state burn energy for both barrier crossing and intra-well oscillator — a behaviour reflected in an enormous enhancement of the effective diffusion (see Ref. [34] and references therein). The goal here is to design a control force using adaptive backstepping technique, that will drive the chaotic attractor to transport particles in a non-chaotic manner; hence generating negative current.

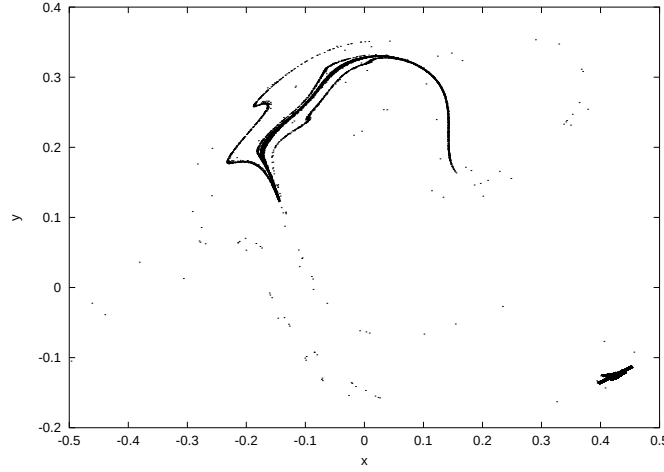


Fig. 2. Co-existing attractors: a chaotic attractor and a periodic attractor for $b = 0.1, \omega = 0.67$ and $a = 0.156$.

3. Adaptive backstepping design

The goal of this section is to design a control force using adaptive backstepping technique, that will synchronise two non-identical ratchet dynamics evolving from different initial states, such that in their long time run, the two systems are identical in dynamics. To treat this problem, let the drive ratchet be given by

$$\begin{aligned}\dot{x}_1 &= y_1, \\ \dot{y}_1 &= -by_1 + a \cos(\omega_1 t) + f(x_1),\end{aligned}\tag{3}$$

and the response ratchet be given by

$$\begin{aligned}\dot{x}_2 &= y_2, \\ \dot{y}_2 &= -by_2 + a \cos(\omega_2 t) + f(x_2) + u,\end{aligned}\tag{4}$$

where

$$f(x_1) = \frac{1}{4\pi\delta} [2 \cos 2\pi(x_1 - x_0) + \cos 4\pi(x_1 - x_0)],\tag{5}$$

$$f(x_2) = \frac{1}{4\pi\delta} [2 \cos 2\pi(x_2 - x_0) + \cos 4\pi(x_2 - x_0)] , \quad (6)$$

and u is the control that is required to drive system (4) (the response) to a synchronised state with system (3) — the driver. To achieve this goal, we define the error state between (3) and (4) as

$$e_x = x_2 - x_1 \text{ and } e_y = y_2 - y_1 . \quad (7)$$

Subtracting Eq. (3) from Eq. (4) and using definition (7), we have the following error dynamics equation for the drive-response system:

$$\begin{aligned} \dot{e}_x &= e_y \\ \dot{e}_y &= -be_y + f(x_2) - f(x_1) + a[\cos(\omega_2 t) - \cos(\omega_1 t)] + u . \end{aligned} \quad (8)$$

In the absence of u , Eq. (8) would have an equilibrium at $(0, 0)$. If a u can be chosen such that the equilibrium $(0, 0)$ remains unchanged, then the problem of synchronisation between the drive-response system can be transformed to that of realizing asymptotic stabilisation of system (8). Thus, the goal is to find a control law u such that system (8) is stabilised at the origin.

Considering the stability of system (9):

$$\dot{e}_x = e_y , \quad (9)$$

and regarding e_y as a virtual control. An estimate stabilising function $\alpha_1(e_x)$ can be designed for the virtual control e_y . Choose a Lyapunov function

$$V_1(e_x) = \frac{1}{2}e_x^2 . \quad (10)$$

The derivative of $V_1(e_x)$ is

$$\dot{V}_1(e_x) = e_x \dot{e}_x . \quad (11)$$

For $V_1(e_x)$ to be negative definite, then $\dot{e}_x = -e_x$. That is

$$\dot{V}_1(e_x) = -e_x^2 < 0 . \quad (12)$$

It follows that $\alpha_1(e_x) = -e_x$. Now, let the error state e_z be defined by

$$e_z = e_y - \alpha_1(e_x) . \quad (13)$$

Considering the (e_x, e_z) subspace given by

$$\begin{aligned} \dot{e}_x &= e_z - e_x , \\ \dot{e}_z &= (1 - b)e_z + (b - 1)e_x + f(x_2) - f(x_1) \\ &\quad + a[\cos(\omega_2 t) - \cos(\omega_1 t)] + u , \end{aligned} \quad (14)$$

and the Lyapunov function

$$V_2(e_x, e_z) = V_1(e_x) + \frac{1}{2}e_z^2. \quad (15)$$

The derivative of Eq. (15) is

$$\begin{aligned} \dot{V}_2(e_x, e_z) = & -e_x^2 + e_z[(1-b)e_z + (b-1)e_x \\ & + f(x_2) - f(x_1) + a(\cos(\omega_2 t) - \cos(\omega_1 t)) + u]. \end{aligned} \quad (16)$$

If we choose

$$\begin{aligned} u = & -e_z - (1-b)e_z - (b-1)e_x \\ & -f(x_2) + f(x_1) - a(\cos(\omega_2 t) - \cos(\omega_1 t)), \end{aligned} \quad (17)$$

then, $\dot{V}_2(e_x, e_z) = -e_x^2 - e_z^2 < 0$ is negative definite and according to LaSalle–Yoshizawa theorem [25, 26], the error dynamics e_x, e_z will converge to zero as $t \rightarrow \infty$, while the equilibrium $(0, 0)$ remains global asymptotically stable. Thus, the synchronisation problem between the drive-response ratchets is solved.

4. Numerical results

In the numerical simulations that follows, we set the system parameters as in Fig. 2 and make the following choice of initial conditions: $x_1(0) = 0.45, y_1(0) = -0.12, x_2(0) = -0.1$ and $y_2(0) = 0.25$. With these set of initial conditions, we display the trajectories associated with the chaotic and the periodic attractors in Fig. 3 when the control is switched off. In Fig. 3, the driver system corresponds to the periodic attractor transporting particles in the negative direction while the response system is the chaotic attractor that transport particle in the positive direction. The reader may wish to refer to Fig. 4 in Ref. [33] where the motion of the attractors is in the uncontrolled state.

When the control u is switched on, the response ratchet is driven to a synchronised state with the drive ratchet. In this synchronised state, the error dynamics $[e_x, e_y] \rightarrow 0$ as $t \rightarrow \infty$ and the two systems assume identical dynamics. Considering Fig. 4, where we have plotted the trajectories of the two systems when the control is switched on at $t = 50$, we find that the direction of transport for the chaotic attractor has been reversed to follow the direction of transport of the periodic attractor, both now generating negative current.

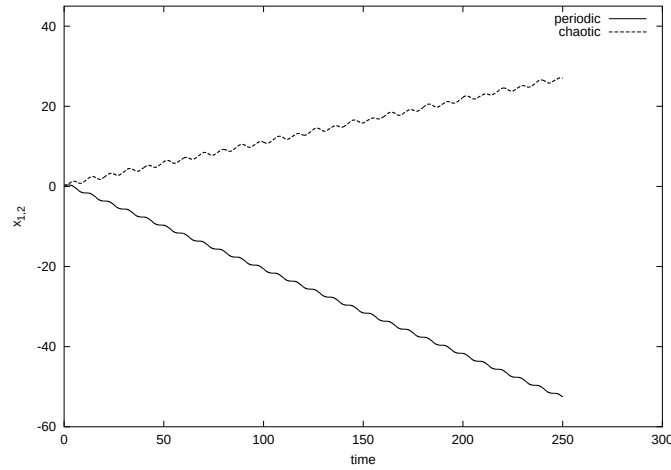


Fig. 3. Trajectories of the two chaotic attractors shown in Fig. 2: Positive current (dashed line) generated by the chaotic attractor; negative current (solid line) generated by the periodic attractor. Here control has been switched off.

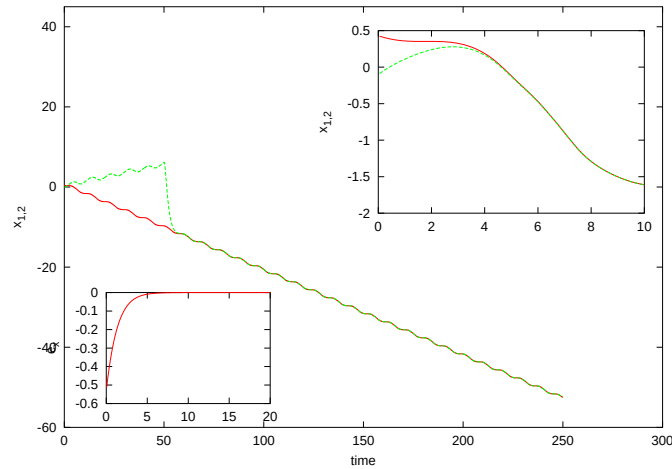


Fig. 4. Synchronisation dynamics of the master-slave ratchets for initial conditions: $x_1(0) = 0.43$, $y_1(0) = -0.12$, $x_2(0) = -0.10$, $y_2(0) = 0.25$, and $b = 0.1$, $\omega_D = 0.67$. Control of particle transport to the negative direction in a non-chaotic fashion is achieved when control is switched on at $t = 50$. Inset (bottom-left) shows the transient behaviour of the error dynamics (e_x versus t) when control is activated at $t = 0$; while the inset (top-right) is the zoom of the initial transient for X_1 and x_2 when control is activated $t = 0$.

5. Concluding remarks

Conclusively, we have applied the recursive backstepping synchronisation scheme to a more concrete physical system of interest in non-equilibrium physics and have shown that the method can be used to control directed transports of particle arising from the co-existence of two non-identical attractors in phase space. The method is simple, singularity free, gives flexibility to construct a control law and the closed-loop system is globally stable. Unlike the method of active control that was employed in our earlier study, the present method has some advantages. Although the active control is more easier to design, there are however, two controller functions required for the design, while only one controller is needed for the backstepping. Thus, the controllers obtained using active control method are more complex for practical implementation. Comparing the equation of motion for the inertia ratchet and the control equations for active control (see Eqs. (7), (9) and (10)) in Ref. [32], it is obvious that the controller is more complex than the system to be controlled. Moreover the flexibility in the choice of control laws for recursive backstepping design gives room for further improvement in its performance.

The problem of controller complexity is a very crucial issue in the practical implementation of control techniques [35]. Two fundamental issues in this direction are (i) the cost implication and the density requirement for designing controllers and (ii) the need to make the complexity of the controller to be, at least comparable to, or less than, the device being controlled, if the controlling technique is desired to achieve a useful end far beyond scientific curiosity. Hence, the entire concept of control mechanism would become untenable if a simple chaotic system requires a massively complex controller.

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