DYNAMICAL FLUCTUATIONS AND LEVY STABILITY IN 14.5A GeV/c ^{28}Si NUCLEUS INTERACTIONS

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Presence of intermittency and Levy stability in 14.5A GeV/c ^{28}Si nucleus collisions is investigated using Monte Carlo approach. Our experimental data reveals the presence of intermittency. Levy stability analysis is carried out; values of Levy index for the experimental and simulated data are found to be 1.511 ± 0.061 and 1.491 ± 0.041, respectively. An attempt is also made to obtain Renyi Dimensions also called self-similar dimensions, D_q, and multifractal spectrum, $f(\alpha)$. The Renyi dimensions, D_q, are observed to decrease with increasing order of the moment, q. The self-similar multifractal spectrum is found to be a convex curve with a maximum around $q = 0$. Simulation technique is used and an analytical continuation is applied to find the multifractal spectrum for the fractional values of q.

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1. Introduction

Systematic and thorough study of fluctuations in multiparticle production in high energy hadronic interactions started since the observation of a relatively high multiplicity cosmic ray event known as JACEE[1] event. In order to explain the observed fluctuations in the rapidity distribution for this event, Bialas and Peschanski proposed [2,3] scaled factorial moments (SFMs) approach to investigate dynamical fluctuations in multiparticle production in high energy nuclear collisions. The q-th order factorial moment, F_q, is defined [2] as

$$F_q = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m(n_{m-1}) \, \ldots \, (n_m - q + 1) \rangle}{\langle n_m \rangle^q} \,, \tag{1}$$

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where M is the partition number in the available rapidity space into which a given rapidity interval is binned and n_m is the number of particles falling in the m-th bin and the symbol "..." represents vertical average obtained for the entire event. If the averaging is done over the whole data sample, the average value of q-th order factorial moment is calculated from:

$$\langle F_q \rangle = \frac{M^{q-1}}{N_{\rm evt}} \sum_{N_{\rm evt}} \sum_{m=1}^{M} \frac{n_m(n_{m-1}) \ldots (n_m + q - 1)}{\langle N \rangle^q} , \tag{2}$$

where $N_{\rm evt}$ denotes the total number of events in the sample.

Many experiments have reported that $\langle F_q \rangle$ exhibit anomalous scaling behaviour [4] exemplified by

$$\langle F_q \rangle \propto \left[\frac{\Delta\eta}{\delta\eta} \right]^{\phi_q} , \qquad (\delta\eta \to 0) , \tag{3}$$

where a small pseudorapidity interval $\Delta\eta$ is partitioned into M bins each of equal size $\delta\eta = \Delta\eta/M$. It is of interest to note that "intermittency" in particle physics refers to the power-law behaviour exhibited by F_q with decreasing bin size. Since the occurrence of fluctuations or power-law behaviour [2,3] in relativistic nuclear collisions has been quite frequently observed [2–7], it therefore, suggests that multi-hadronic final states in these collisions possess self-similar fractal. Hence one should carry out a study of Levy stability [8] and multifractal spectrum analysis for the fractal systems. The Levy stability analysis [8] is regarded as a vital tool for characterising the non-linear behaviour of dynamical fluctuations in high energy collisions.

2. Details of the data

A random sample comprising of 555 interactions having $n_s \geq 2$, where n_s represents the number of charged particles produced in an event with relative velocities, $\beta \geq 0.7$, produced in 14.5A GeV/c ^{28}Si emulsion collisions, is used for carrying out the present analysis. The emission angles of all the relativistic charged particles were measured and their pseudorapidities were determined. All other relevant details about the stacks used, criteria employed for selecting the events and the method of measuring the emission angles may be found elsewhere [9]. Furthermore, for comparing the experimental results with the corresponding values predicted by the Lund model, FRITIOF [10], a sample consisting of 5 000 events, identical to the experimental ones were simulated.

3. Results and discussion

The variations of $\ln\langle F_q\rangle$ with $-\ln\delta\eta$ for the experimental and FRITIOF data along with possible statistical errors are depicted in Fig. 1. It is observed that $\ln\langle F_q\rangle$ increases linearly with $-\ln\delta\eta$ for both the data samples.

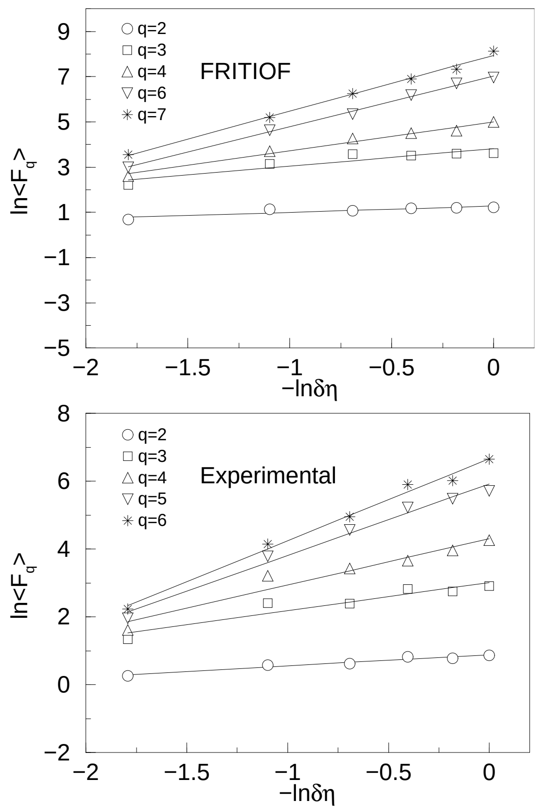

Fig. 1. Variations of $\ln\langle F_q\rangle$ with $-\ln\delta\eta$.

In order to investigate the dependence of factorial moments on the target mass, variation of $\ln\langle F_q\rangle$ with $-\ln\delta\eta$ is examined separately for CNO, emulsion and AgBr groups of target nuclei; these variations are plotted in Fig. 2. The solid lines in Figs. 1 and 2 are obtained by the method of least squares fitting to the data.

It is seen that for the three categories of interactions, CNO, emulsion and AgBr, a linear rise in the SFMs with decreasing bin width $\delta\eta$, is observed. This behaviour supports the presence of intermittency in these collisions. It is also clear from the figures that the values of SFMs are relatively higher

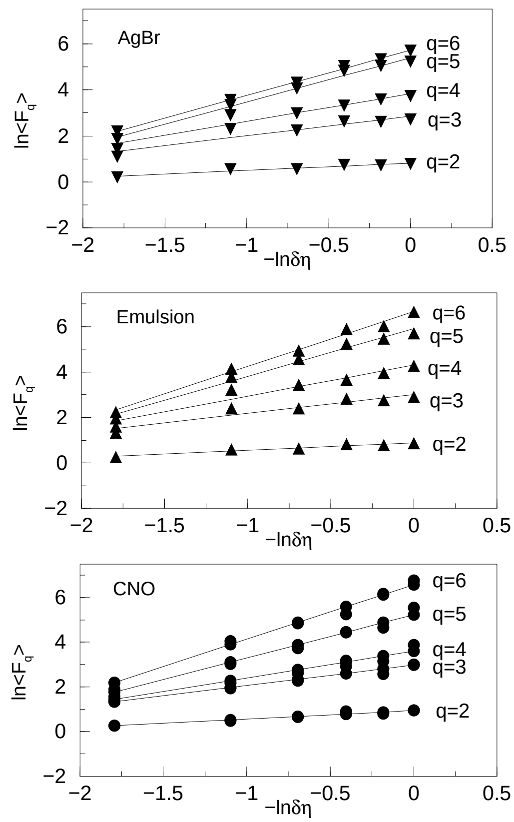

Fig. 2. Variations of $\ln\langle F_q\rangle$ with $-\ln\delta\eta$.

for the interactions due to lighter targets, CNO, in comparison to those for the interactions due to the heavy targets. This might happen due to low multiplicities in the interactions due to CNO in comparison to those for AgBr targets [11].

The intermittency indices, ϕ_q, are correlated with the strength of the intermittency. The values of ϕ_q can be calculated from the observed linear rise of $\ln\langle F_q\rangle$ with $-\ln\delta\eta$. It may be mentioned that $\phi_q^{'}s$ are the slopes of the fits to $\ln\langle F_q\rangle$ *versus* $-\ln\delta\eta$ plots. Given in Table I are the values of ϕ_q for the collisions of 14.5A GeV ^{28}Si nuclei with CNO, emulsion and AgBr targets.

Fig. 3 shows the variations of ϕ_q with q for the experimental and simulated data. Statistical errors are also indicated in the plots. In Fig. 3 a linear increase is clearly discernible in the value of ϕ_q with the order of the moments, q. It may be stressed that increase in ϕ_q with q is of, approximately, the same nature for both the data sets. However, from Table I it is clearly evident that amongst the three groups of targets, the values of $\phi_q^{'}s$ are relatively higher for the collisions due to CNO targets.

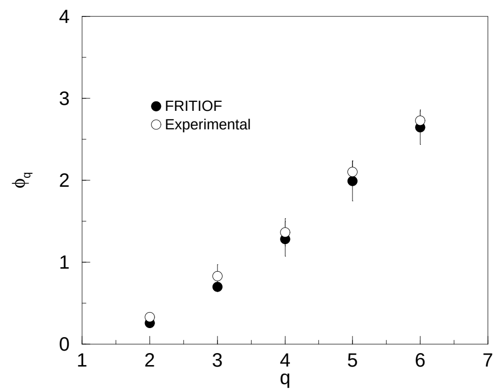

Fig. 3. Variations of intermittency indices with q.

TABLE I

Calculated values of intermittency indices, ϕ_q, for the interactions of 14.5A GeV/c ^{28}Si nuclei with CNO, emulsion and AgBr groups of targets.

Order of the moment q	ϕ_q (CNO)	ϕ_q (Em)	ϕ_q (AgBr)
2	0.382 ± 0.046	0.331 ± 0.038	0.315 ± 0.042
3	0.915 ± 0.133	0.833 ± 0.138	0.752 ± 0.164
4	1.403 ± 0.143	1.368 ± 0.169	1.278 ± 0.160
5	2.136 ± 0.151	2.005 ± 0.133	1.922 ± 0.042
6	2.744 ± 0.142	2.628 ± 0.117	2.473 ± 0.172

Fig. 4 exhibits the variations of anomalous dimensions [11], $d_q(=\frac{\phi_q}{q-1})$, with the order of the moments, q; the value of d_q is found to increase with q. This increasing trend in the value of d_q with q is observed for all the three categories of interactions. Fig. 5 shows a comparison of variations of anomalous dimensions with q for the experimental and FRITIOF data. It is interesting to note that exactly similar trends of variations of d_q with q are observed for the experimental, as well as FRITIOF, data.

The generalised dimensions $D_q(= 1 - d_q)$ are calculated and plotted against $\ln q/(q-1)$ in Fig. 6. The slope of the best fit to the data defines the specific heat of the system of multiparticle final state in relativistic nuclear collisions. A thermodynamical interpretation [12–14] is suggested by

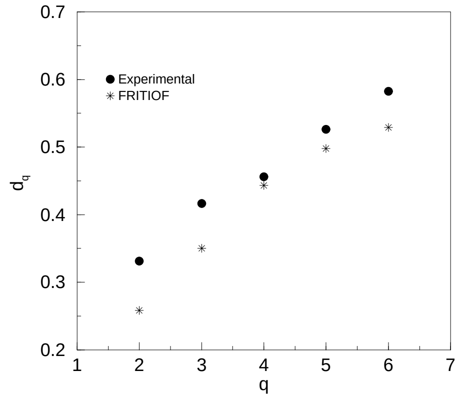

Fig. 4. Variations of d_q with q.

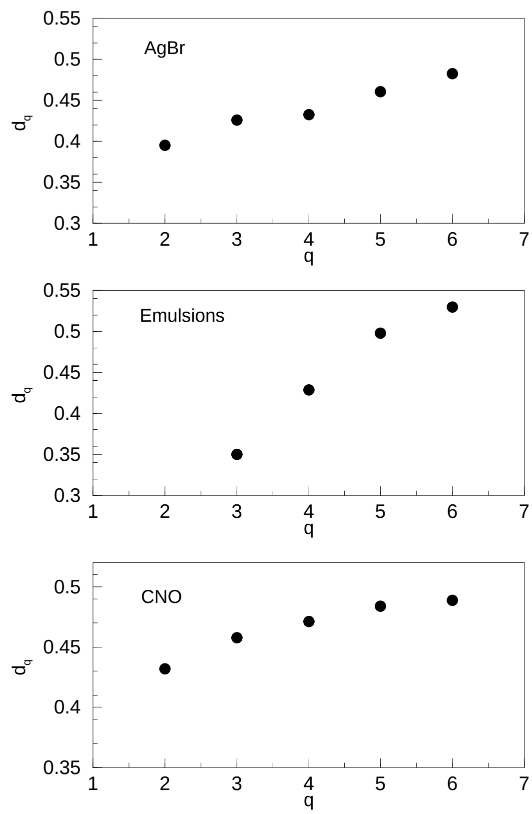

Fig. 5. Variations of d_q with q for the three group of target nuclei.

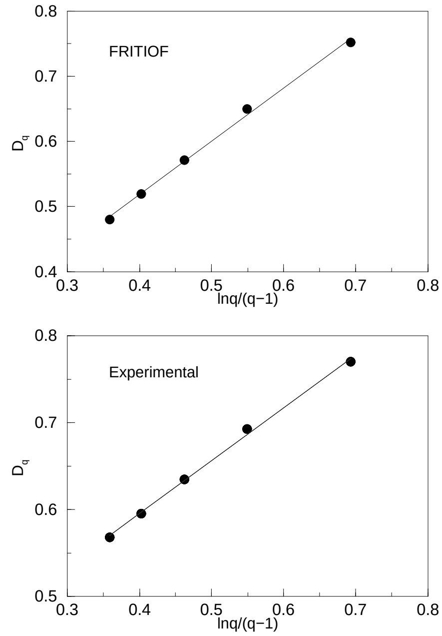

Fig. 6. Variations of d_q with $\ln q/(q-1)$.

considering q as inverse of temperature, $q = T^1$. The science of thermodynamics has established that in many important cases the specific heat of gases and solids is constant over a considerable range of temperature interval [14]. It can be shown [15] that generalised dimension and specific heats are related as:

$$D_q = (a-c) + c\frac{\ln q}{q-1}\,, \tag{4}$$

where a and c are constants, and c is termed as constant specific heat.

In the present study, the values of the specific heats, c, for the experimental and FRITIOF data are found to be 0.607 ± 0.017 and 0.816 ± 0.206, respectively.

According to Levy stability, as a self-similar fractal system, the multifractal final state in high energy nuclear collisions can be described in terms of Levy stability index [16], μ. It is used to investigate the existence of multifractality as a self-similar system in relativistic nuclear collisions. This parameter is envisaged to provide some useful information about elementary fluctuations around the tail [11] of the pseudorapidity distribution. It is also envisaged to be a measure [16] of the degree of multifractality. Furthermore, it has been proposed that μ=0 corresponds to mono-fractals and the situations characterised by $\mu < 1$ would define "calm" singularities, whereas wild singularities correspond to the condition $\mu > 1$.

As discussed earlier, there exists a possible relationship between anomalous dimension, d_q, and intermittency index, ϕ_q. Anomalous dimensions may also be expressed in terms of the Levy index μ [4] as

$$d_q = \left(\frac{C_1}{\mu - 1}\right) \frac{q^\mu - q}{q - 1}, \tag{5}$$

where C_1 is a constant. In terms of Levy index, μ, the intermittency indices, ϕ_q, may be written [4] as:

$$\phi_q = \phi_2 \frac{q^\mu - q}{2^\mu - 2}. \tag{6}$$

According to Levy stability theory the Levy index should have any value lying in the interval $0 \leq \mu \leq 2$, and the central limit is approached as $\mu \to 2$. Under this limiting condition, Eq. (6) would become

$$\phi_q = \phi_2 \frac{q(q-1)}{2}. \tag{7}$$

It may be stressed that Eq. (6) is valid for any real value of the order of the moment, $q \geq 0$, while this equation in its present form is not applicable for $q < 0$. An analytical continuation of Eq. (6), which is defined for $q < 0$, leads to the following expression:

$$\phi_q = \phi_2 \frac{(q^2)^{\frac{\mu}{2}} - q}{2^\mu - 2}. \tag{8}$$

ϕ_q are exhibited in Fig. (7) for both the data and FRITIOF data. Fitting is done using Eq. (7) and is shown as solid curve in the figure. Also, the curve shown in the figure by the dashed lines is fitted using Eq. (6), which corresponds to the central limiting case.

The value of Levy index μ for the produced hadronic system in 14.5A GeV ^{28}Si nucleus collisions obtained from the fit turns out to be $\mu = 1.511 \pm 0.061$ and for the FRITIOF data its value has been found to be $\mu = 1.491 \pm 0.042$. Interestingly, the two values of μ lie in the interval: $0 < \mu < 2$. This, incidentally, is in good accord with the Levy stability condition according to which μ should be greater than unity and the multifractals should correspond to "wild" singularities.

The method of multifractal analysis has been widely used in many areas of physics in particular and Science [17], in general. The two very useful parameters for studying multifractality are: Renyi dimensions, $D_q = 1 - d_q$, and multifractal spectrum, $f(\alpha)$. In terms of the intermittency indices, ϕ_q,

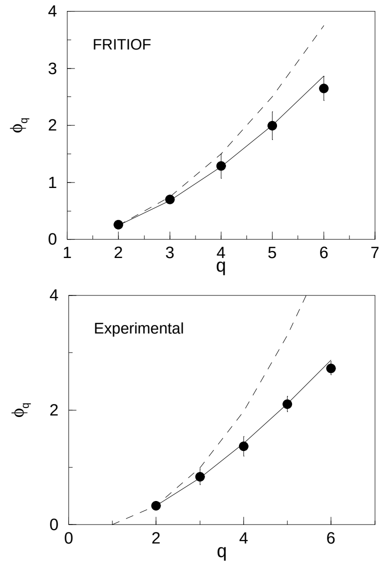

Fig. 7. Fittings to ϕ_q *versus* q plots according to Levy stability.

the multifractal spectral function f(α) and the Renyi dimensions, D_q, may be obtained [19] from:

$$D_q = \frac{T_q}{q-1}\,, \tag{9}$$

and

$$f(\alpha) = q\alpha - T_q\,, \tag{10}$$

where

$$T_q = q - 1 - \phi_q\,, \tag{11}$$

and

$$\alpha = \frac{dT_q}{dq}\,. \tag{12}$$

By taking some values of ϕ_q, calculated for the experimental data, a continuous spectrum of ϕ_q is simulated using Eq. (7). From these values multifractal spectrum, $f(\alpha)$, is obtained. The Renyi dimensions, D_q, and multifractal spectrum $f(\alpha)$ are calculated using Eqs. (8) and (9), respectively, for the values of q lying in the interval: 1.0 to 9.0, in a step 0.2. In Figs. (8) and (9) D_q *versus* q and $f(\alpha)$ *versus* α plots are, respectively, exhibited for both the data sets. It may be of interest to mention that the Renyi dimensions, D_q, decrease with the order of the moments q, indicating thereby the presence of multifractality in multiparticle production in relativistic nuclear collisions. The $f(\alpha)$ plotted against α is observed to have a maximum around $\alpha = 5$ for experimental as well as simulated data. However, it may be noticed that

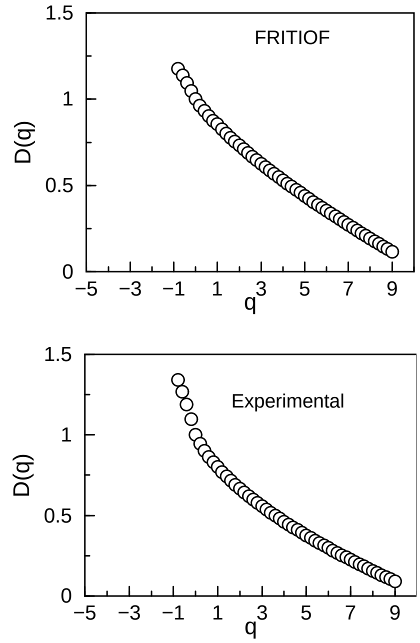

Fig. 8. Variations of Renyi dimensions D_q with q.

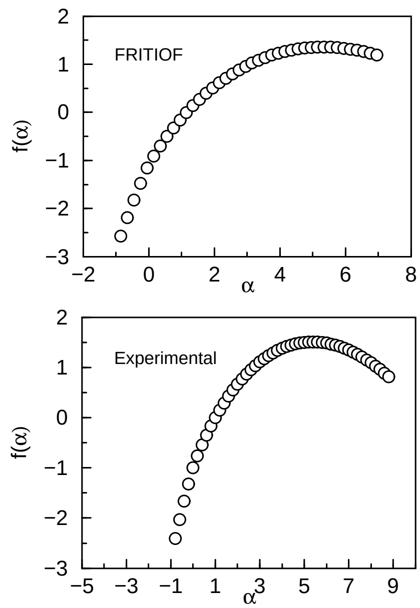

Fig. 9. Variations of $f(\alpha)$ with α.

it extends up to $\alpha = 9$ for the experimental data and up to $\alpha = 7$ for the FRITIOF data. Incidentally, similar results have been observed by Liu Lianshou *et al.* [16].

4. Conclusions

Results obtained in the present study reveal the presence of intermittency and multifractality in the data on 14.5A GeV/c ^{28}Si nucleus interactions. Moreover, analyses for the experimental and simulated data show good compatibility. Present results are in good agreement with the previously reported results and are vital to understand the occurrence of intermittency and multifractaly in relativistic nuclear collisions. Author is a working along with Aligarh Muslim University group in ALICE Collaboration and wish to carry-out similar analysis for first LHC data at an energy never attained before in heavy-ion collisions.

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