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CHIRAL LOW-ENERGY CONSTANTS*

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The progress in determining the coupling constants of mesonic chiral Lagrangians is reviewed, with emphasis on the work performed in three successive European Networks (Eurodaphne I and II, Euridice). Reliable estimates of those constants are essential for making full use of next-to-next-to-leading-order calculations in chiral perturbation theory. The precision in the values of the strong coupling constants of $O(p^4)$ has been increasing steadily over the years. The situation is less satisfactory in the nonleptonic weak sector where further phenomenological input and more theoretical work are needed. A lot of progress has recently been achieved for electromagnetic coupling constants occurring in radiative corrections for mesonic processes at low energies.

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1. Introduction

Chiral low-energy constants (LECs) are the coupling constants of effective chiral Lagrangians. They are independent of the light quark masses by construction and they describe the influence of all "heavy" degrees of freedom that are not contained explicitly in the effective Lagrangians. The construction of effective Lagrangians is based on symmetry considerations only, so that a lot of information is lost in going from the underlying Standard Model to the effective theory. As a consequence, effective Lagrangians contain many LECs, especially at higher orders in the chiral expansion. Progress in chiral perturbation theory (CHPT) depends on realistic estimates of chiral LECs.

My task in this talk was to review the progress in determining or estimating chiral LECs since 1993 when the first Eurodaphne Network got started. Most of this progress is in fact due to work performed in the three European Networks Eurodaphne I, Eurodaphne II and Euridice. I will only consider the meson sector here. The corresponding effective chiral Lagrangian is given in Table I.

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TABLE I

Effective chiral Lagrangian in the meson sector. The numbers in brackets refer to the number of LECs for chiral SU(3).

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	Loop order
$\mathcal{L}_{p^2}(2)$ + $\mathcal{L}^{\text{odd}}_{p^4}(0)$ + $\mathcal{L}^{\Delta S=1}_{G_F p^2}(2)$ + $\mathcal{L}^{\text{em}}_{e^2 p^0}(1)$ + $\mathcal{L}^{\text{emweak}}_{G_8 e^2 p^0}(1)$	$L = 0$
+ $\mathcal{L}_{p^4}(10)$ + $\mathcal{L}^{\text{odd}}_{p^6}(23)$ + $\mathcal{L}^{\Delta S=1}_{G_8 p^4}(22)$ + $\mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28)$	$L \leq 1$
+ $\mathcal{L}^{\text{em}}_{e^2 p^2}(13)$ + $\mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14)$ + $\mathcal{L}^{\text{leptons}}_{e^2 p}(5)$	
+ $\mathcal{L}_{p^6}(90)$	$L \leq 2$

Information on chiral LECs is obtained either from phenomenology or with additional input from theory.

- Extraction from experiment
  Some LECs are associated with terms in the Lagrangian that contribute to amplitudes even in the chiral limit. They govern the momentum dependence of amplitudes and are at least in principle accessible experimentally. The other class involves chiral symmetry breaking terms that specify the quark mass dependence of amplitudes. The related LECs are much more difficult to determine phenomenologically but they are accessible in lattice simulations.

- Input from theory
  - Large-N_c methods match CHPT with QCD by bridging the gap $M_K \lesssim E \lesssim 1.5$ GeV with resonance exchange.
  - Lattice QCD.

2. Strong interactions

At lowest order in the chiral expansion, there are only two LECs B and F. B depends on the QCD renormalization scale and always appears multiplied by quark masses in CHPT amplitudes. The products Bm_q and the constant F can be expressed in terms of meson masses and of the pion decay constant F_π. Those relations involve LECs of $O(p^4)$ or higher to be discussed subsequently.

The strong chiral Lagrangians of $O(p^4)$ contain 7 measurable LECs l_i for chiral SU(2) [1] and 10 LECs L_i for chiral SU(3) [2]. The current phenomenological values are based on calculations to $O(p^6)$ in most cases, sometimes supplemented by dispersive methods. Although I restrict the discussion here to SU(3), the most precise determinations have been obtained for the SU(2) LECs l_1, l_2, l_4 by combining CHPT to $O(p^6)$ with Roy equations [3]. This information can also be used for some of the SU(3) LECs. The relations between the l_i and the L_i are however only known [2] to $O(p^4)$, which is not sufficient for the present purpose. Work in progress by the Bern group [4] will soon provide those relations to p^6 accuracy.

The present values of the renormalized SU(3) LECs $L_i(M_\rho)$ are shown in the second column of Table II. The first column contains the values originally obtained in Ref. [2]. No drastic changes have occurred although the mean values have generally decreased in absolute magnitude. For L_1, L_2, L_3 the information from $\pi\pi$ scattering [3] will be very useful once the relations between the SU(3) and SU(2) LECs will be available at the p^6 level [4]. The LECs $L_1, \ldots, L_4$ have also been extracted from πK scattering, based on a dispersive analysis [6] applied to a CHPT calculation of $O(p^4)$. The results are displayed in the third column in Table II where only experimental errors are shown.

TABLE II

Phenomenological values and theoretical estimates for the SU(3) LECs $L_i(M_\rho)$ in units of 10^{-3}. The first column shows the original values of Ref. [2], the second displays the present values taken from Ref. [5] and references therein. The third column is based on an analysis of πK scattering [6]. The fourth column contains recent lattice results from the MILC Collaboration [7]. The fifth column shows the resonance saturation results of Ref. [8] and the last column reproduces a systematic estimate of resonance contributions to lowest order in $1/N_c$ [13]. The entries marked with ‡ were taken as input in Ref. [8].

i	$O(p^4)$	$O(p^6)$	πK	Lattice	Ref. [8]	Ref. [13]
1	0.7 ± 0.3	0.43 ± 0.12	1.05 ± 0.12		0.6	0.9
2	1.3 ± 0.7	0.73 ± 0.12	1.32 ± 0.03		1.2	1.8
3	-4.4 ± 2.5	-2.35 ± 0.37	-4.53 ± 0.14		-3.0	-4.3
4	-0.3 ± 0.5	~ 0.2	0.53 ± 0.39	-0.2 ± 0.4	0	0
5	1.4 ± 0.5	0.97 ± 0.11		1.2 ± 0.4	$1.4^\ddagger$	2.2
6	-0.2 ± 0.3	~ 0.0		0.1 ± 0.2	0	0
7	-0.4 ± 0.2	-0.31 ± 0.14			-0.3	-0.3
8	0.9 ± 0.3	0.60 ± 0.18		0.7 ± 0.2	$0.9^\ddagger$	0.8
9	6.9 ± 0.7	5.93 ± 0.43			$6.9^\ddagger$	7.2
10	-5.5 ± 0.7	-5.09 ± 0.47			-6.0	-5.4

With most of the chiral LECs of $O(p^4)$ reasonably well known, can we understand the specific values with additional theory input? Lattice QCD has come a long way to determine some of the constants directly from QCD. The fourth column in Table II shows the most recent results of the MILC Collaboration [7] with three dynamical light (staggered) quarks. The agreement with the phenomenological values in the second column is indeed "staggering".

A different approach makes use of the properties of QCD at large N_c where amplitudes can be expressed in terms of (stable) resonance exchange. To illustrate the main features of this approach [8, 9], let us consider elastic meson–meson scattering, specializing to a channel with $s \leftrightarrow u$ symmetry (*e.g.*: $\pi^+\pi^0 \to \pi^+\pi^0$). From axiomatic field theory (Froissart theorem) we know that the scattering amplitude $A(\nu, t)$ satisfies a once-subtracted forward dispersion relation in $\nu = (s-u)/2$:

$$A(\nu, t=0) = A(0,0) + \frac{\nu^2}{\pi} \int\limits_0^\infty d\nu'^2 \frac{\text{Abs } A(\nu', 0)}{\nu'^2 \left(\nu^2 - \nu'^2\right)} \,. \tag{1}$$

Exchange of a resonance (R) generates the absorptive part

$$\text{Abs } A(\nu, 0) = \pi c_{\rm R} {M_{\rm R}}^4 \delta\left(\nu^2 - {M_{\rm R}}^4\right) \,, \tag{2}$$

where the constant $c_{\rm R}$ is related to the partial decay width $\mathit{\Gamma}(R \to \pi\pi)$ in this case. Therefore, Eq. (1) gives rise to

$$A(\nu, 0) = A(0,0) + \frac{c_{\rm R}\nu^2}{\nu^2 - {M_{\rm R}}^4} \,. \tag{3}$$

On the other hand, resonance exchange on the basis of a chiral resonance Lagrangian produces an amplitude of the general form

$$A_{\rm R}(\nu, 0) = \frac{P_{\rm R}(\nu^2)}{\nu^2 - {M_{\rm R}}^4} \,, \tag{4}$$

with a polynomial $P_{\rm R}(\nu^2)$ satisfying the on-shell condition $P_{\rm R}({M_{\rm R}}^4) = c_{\rm R}{M_{\rm R}}^4$. Decomposing the polynomial $P_{\rm R}(\nu^2)$ as

$$P_{\rm R}(\nu^2) = P_{\rm R}({M_{\rm R}}^4) + \left(\nu^2 - {M_{\rm R}}^4\right) \overline{P_{\rm R}}(\nu^2) \,, \tag{5}$$

the condition $A_{\rm R}(\nu, 0) = A(\nu, 0)$ requires $\overline{P_{\rm R}}(\nu^2)$ to be a constant,

$$\overline{P_{\rm R}}(\nu^2) = A(0,0) + c_{\rm R} \,, \tag{6}$$

which will not be the case for a general resonance Lagrangian. Therefore, the short-distance constraint embodied in the once-subtracted dispersion relation (1) demands that in general an appropriate polynomial $P_c(\nu^2)$ be added to $A_{\rm R}(\nu,0)$:

$$A_{\rm R}(\nu,0) = P_c(\nu^2) + \overline{P_{\rm R}}(\nu^2) + \frac{P_{\rm R}({M_{\rm R}}^4)}{\nu^2 - {M_{\rm R}}^4}\,. \tag{7}$$

The counterterm polynomial $P_c(\nu^2)$ is fixed by the short-distance constraint to satisfy

$$P_c(\nu^2) + \overline{P_{\rm R}}(\nu^2) = A(0,0) + c_{\rm R}\,, \tag{8}$$

ensuring at the same time the correct low-energy behaviour of the resonance exchange amplitude:

$$A_{\rm R}(\nu,0) = A(\nu,0) = A(0,0) - \frac{c_{\rm R}}{{M_{\rm R}}^4}\nu^2 + O(p^8)\,. \tag{9}$$

The coefficient of ν^2 depends only on the mass and on the partial decay width of the resonance and it defines the resonance contribution to a certain combination of the L_i.

With the proper choice of resonance fields, such counterterm polynomials are not needed at $O(p^4)$ for the exchange of $V(1^{--})$, $A(1^{++})$, $S(0^{++})$ and $P(0^{-+})$ mesons [10], but they are unavoidable for $T(2^{++})$ and $A(1^{+-})$ exchange [11, 12].

The example of elastic meson scattering raises the legitimate question: why should one bother at all with resonance Lagrangians? It may seem like an unnecessary detour to use the couplings of a resonance Lagrangian that have to be corrected by short-distance constraints after all. The alternative is to study Green functions directly with a large-N_c inspired ansatz in the first place. The main advantages of a Lagrangian approach are first of all that chiral symmetry is automatically guaranteed for the generated Green functions and amplitudes and there is no need to impose chiral Ward identities. At least as important from a practical point of view is the possibility to integrate out the resonances once and for all in the generating functional of Green functions (always to leading order in $1/N_c$), thereby generating all LECs of a given order. Of course, the short-distance analysis still remains to be done.

The fifth column in Table II shows the original resonance estimates of Ref. [8]. The last column contains more recent systematic estimates of resonance contributions to lowest order in $1/N_c$ [13]. Remembering that the renormalization scale is not fixed at leading order in $1/N_c$, the agreement between the resonance exchange contributions and the phenomenological values in Table II is more than satisfactory. Attempts to include corrections of next-to-leading order in $1/N_c$ have also been made.

To improve the precision of LECs of $O(p^4)$, realistic estimates for some LECs of $O(p^6)$ are also needed. Several results have already been obtained by members of the Euridice Collaboration [14]. All possible resonance contributions of the standard variety $V(1^{--})$, $A(1^{++})$, $S(0^{++})$, $P(0^{-+})$ have recently been presented in Ref. [15] that also contains an up-to-date bibliography. The short-distance analysis remains to be done in many cases of interest.

The odd-intrinsic-parity Lagrangian of $O(p^4)$ is given by the Wess–Zumino–Witten Lagrangian $\mathcal{L}_{p^4}^{\rm odd}(0)$ [16] in Table II. After several conflicting results in the literature there is now a consensus that the corresponding Lagrangian of $O(p^6)$ has 23 LECs [17]. Only partial results are available for the numerical values of those constants, but the most promising approach is again based on a short-distance analysis with or without chiral resonance Lagrangians [14, 18].

3. Nonleptonic weak interactions

The chiral Lagrangian of lowest order, $O(G_{\rm F}p^2)$, contains two LECs g_8, g_{27} to describe nonleptonic weak decays of kaons. Especially the value of the octet coupling g_8 is very sensitive to chiral corrections [19]. Isospin breaking corrections are potentially important for the 27-plet coupling constant g_{27}. The present status is presented in Table III. Although different isospin breaking contributions are sizeable the overall corrections are small for both LECs.

TABLE III

Octet and 27-plet couplings g_8, g_{27} at lowest order, $O(G_{\rm F}p^2)$, and at next-to-leading order, $O(G_{\rm F}p^4)$, without (IC) and with (IB) isospin breaking [20].

	IC $O(G_{\rm F}p^2)$	IC $O(G_{\rm F}p^4)$	IB $O(G_{\rm F}p^4)$
g_8	5.09 ± 0.01	3.67 ± 0.14	3.65 ± 0.14
g_{27}	0.294 ± 0.001	0.297 ± 0.014	0.303 ± 0.014

The LECs of $O(G_{\rm F}p^4)$ (22 couplings N_i in the octet and 28 couplings D_i in the 27-plet Lagrangians) are much less known than their strong counterparts at $O(p^4)$. The most recent phenomenological analysis of those combinations that occur in the dominant $K \to 2\pi, 3\pi$ decays can be found in Ref. [21]. Many more LECs appear in rare K decays and a phenomenological update is definitely needed here.

Resonance saturation of weak LECs [22] suffers from the obvious drawbacks that the weak resonance couplings are unknown and that short-distance constraints are missing. Nevertheless, resonance saturation provides at least a possible parametrization of the LECs. The most systematic approach is based on factorization (valid to leading order in $1/N_c$) [20, 23] but higher-order corrections in $1/N_c$ may well be sizeable.

If the situation of the LECs of $O(G_{\rm F}p^4)$ is already unsatisfactory, even less is known about higher orders. However, the leading (double) chiral logs of $O(G_{\rm F}p^6)$ are known [24].

4. Dynamical photons

Radiative corrections at low energies involve the effective Lagrangians in Table I with superscript "em" or "emweak".

In the presence of photons as dynamical degrees of freedom the chiral counting is different from the purely strong or nonleptonic weak cases. The lowest-order Lagrangian for electromagnetic corrections to strong processes is of $O(e^2p^0)$ with a single LEC [8] that can be determined from the $\pi^+ - \pi^0$ mass difference. The next-to-leading-order Lagrangian of $O(e^2p^2)$ with 13 LECs K_i was constructed by Urech [25]. By convoluting pure QCD n-point functions ($n \leq 4$) with the photon propagator, sum rule representations were derived for all the K_i [26]. Numerical estimates for the K_i are obtained by saturating the sum rules with resonance exchange. Since the LECs K_i are difficult to determine phenomenologically, the systematic work of Ref. [26] is especially important for controlling radiative corrections to strong processes at low energies.

The situation is much less favourable for electromagnetic corrections to nonleptonic weak processes. Although the single LEC of lowest order, $O(G_8e^2p^0)$, related to the electromagnetic penguin contribution [27], is reasonably well known, the 14 additional LECs of $O(G_8e^2p^2)$ [28] are only known to leading order in $1/N_c$ (factorization). In this way, the LECs can be expressed in terms of Wilson coefficients, the strong LECs L_5, L_8 and the electromagnetic LECs K_i [20, 23].

5. Dynamical photons and leptons

Radiative corrections for semileptonic weak decays require the incorporation of leptons as dynamical degrees of freedom. The leading-order Lagrangian $\mathcal{L}^{\rm leptons}_{e^2p}(5)$ in Table I contains five LECs X_i [29].

With a two-step matching procedure (Standard Model $\leftrightarrow$ Fermi theory $\leftrightarrow$ CHPT), Descotes-Genon and Moussallam have recently established integral representations for all the X_i [30]. One important application is in K_{l3} decays, still the best source for extracting the CKM matrix element V_{us}.

As a consistency check, the isospin violating ratio

$$r_{+0} := \left(\frac{2\,\Gamma(K^+_{e3(\gamma)})\, M^5_{K^0}\, I_{K^0}}{\Gamma(K^0_{e3(\gamma)})\, M^5_{K^+}\, I_{K^+}} \right)^{1/2} = \frac{|f_+^{K^+\pi^0}(0)|}{|f_+^{K^0\pi^-}(0)|} \qquad (10)$$

has been considered [31] that depends essentially only on X_1. With the result for X_1 from Ref. [30], the theoretical prediction $r_{+0} = 1.024 \pm 0.003$ is now in perfect agreement with the most recent K_{l3} data [32]. The agreement also indicates that higher-order corrections to the theoretical prediction for r_{+0} of $O[(m_u - m_d)p^4, e^2\,p^4]$ behave as expected from chiral power counting.

6. Outlook

Since 1993, when Eurodaphne got started, substantial progress has been made in the understanding of low-energy constants, both from phenomenology and from theory (lattice QCD and large-N_c approaches).

In the strong sector, most of the machinery is now ready for a precision determination of the LECs of $O(p^4)$. This endeavour involves also LECs of $O(p^6)$ where we are still in the exploratory stage. We need reliable estimates for some of those LECs to make full use of next-to-next-to-leading-order calculations. In the nonleptonic sector, improvements both in phenomenology and in theory are needed. The most impressive progress in recent years has happened for electromagnetic LECs. As a consequence, radiative corrections in the meson sector at low energies are now under control. Semileptonic K_{l3} decays are one prime example of phenomenological relevance.

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REFERENCES

[1] J. Gasser, H. Leutwyler, *Ann. Phys.* **158**, 142 (1984).

[2] J. Gasser, H. Leutwyler, *Nucl. Phys.* **B250**, 465 (1985).

[3] G. Colangelo, J. Gasser, H. Leutwyler, *Nucl. Phys.* **B603**, 125 (2001) [hep-ph/0103088].

[4] J. Gasser, C. Haefeli, M.A. Ivanov, M. Schmid, `hep-ph/0706.0955`.

[5] J. Bijnens, *Prog. Part. Nucl. Phys.* **58**, 521 (2007) [`hep-ph/0604043`].

[6] P. Büttiker, S. Descotes-Genon, B. Moussallam, *Eur. Phys. J.* **C33**, 409 (2004) [`hep-ph/0310283`].

[7] C. Bernard *et al.*, `hep-lat/0611024`.

[8] G. Ecker, J. Gasser, A. Pich, E. de Rafael, *Nucl. Phys.* **B321**, 311 (1989).

[9] S. Peris, M. Perrottet, E. de Rafael, *J. High Energy Phys.* **9805**, 011 (1998) [`hep-ph/9805442`]; M. Knecht, S. Peris, M. Perrottet, E. de Rafael, *Phys. Rev. Lett.* **83**, 5230 (1999) [`hep-ph/9908283`]; A. Pich, in *Phenomenology of Large-N_C QCD*, R.F. Lebed ed., World Scientific, Singapore 2002 [`hep-ph/0205030`]; E. de Rafael, *Nucl. Phys. Proc. Suppl.* **119**, 71 (2003) [`hep-ph/0210317`].

[10] G. Ecker *et al.*, *Phys. Lett.* **B223**, 425 (1989).

[11] D. Toublan, *Phys. Rev.* **D53**, 6602 (1996); Erratum, *Phys. Rev.* **D57**, 4495 (1998) [`hep-ph/9509217`].

[12] G. Ecker, C. Zauner, `hep-ph/0705.0624`.

[13] R. Kaiser, `hep-ph/0502065`.

[14] B. Moussallam, *Nucl. Phys.* **B504**, 381 (1997) [`hep-ph/9701400`]; M. Knecht, A. Nyffeler, *Eur. Phys. J.* **C21**, 659 (2001) [`hep-ph/0106034`]; J. Bijnens, E. Gamiz, E. Lipartia, J. Prades, *J. High Energy Phys.* **0304**, 055 (2003) [`hep-ph/0304222`]; V. Cirigliano *et al.*, *Phys. Lett.* **B596**, 96 (2004) [`hep-ph/0404004`]; *J. High Energy Phys.* **0504**, 006 (2005) [`hep-ph/0503108`]; J. Portoles, *Acta Phys. Pol. B* **38**, 2789 (2007), these proceedings.

[15] V. Cirigliano *et al.*, *Nucl. Phys.* **B753**, 139 (2006) [`hep-ph/0603205`].

[16] J. Wess, B. Zumino, *Phys. Lett.* **B37**, 95 (1971); E. Witten, *Nucl. Phys.* **B223**, 422 (1983).

[17] T. Ebertshauser, H.W. Fearing, S. Scherer, *Phys. Rev.* **D65**, 054033 (2002) [`hep-ph/0110261`]; J. Bijnens, L. Girlanda, P. Talavera, *Eur. Phys. J.* **C23**, 539 (2002) [`hep-ph/0110400`].

[18] P.D. Ruiz-Femenia, A. Pich, J. Portoles, *J. High Energy Phys.* **0307**, 003 (2003) [`hep-ph/0306157`].

[19] J. Kambor, J. Missimer, D. Wyler, *Phys. Lett.* **B261**, 496 (1991).

[20] V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, *Eur. Phys. J.* **C33**, 369 (2004) [`hep-ph/0310351`].

[21] J. Bijnens, F. Borg, *Eur. Phys. J.* **C40**, 383 (2005) [`hep-ph/0501163`].

[22] G. Ecker, J. Kambor, D. Wyler, *Nucl. Phys.* **B394**, 101 (1993); G. D'Ambrosio, J. Portoles, *Nucl. Phys.* **B533**, 494 (1998) [`hep-ph/9711211`].

[23] E. Pallante, A. Pich, I. Scimemi, *Nucl. Phys.* **B617**, 441 (2001) [`hep-ph/0105011`].

[24] M. Büchler, *Eur. Phys. J.* **C44**, 111 (2005) [`hep-ph/0504180`].

[25] R. Urech, *Nucl. Phys.* **B433**, 234 (1995) [`hep-ph/9405341`].

[26] B. Moussallam, *Nucl. Phys.* **B504**, 381 (1997) [`hep-ph/9701400`]; B. Ananthanarayan, B. Moussallam, *J. High Energy Phys.* **0406**, 047 (2004) [`hep-ph/0405206`].

[27] J. Bijnens, M. B. Wise, *Phys. Lett.* **B137**, 245 (1984).

[28] G. Ecker *et al.*, *Nucl. Phys.* **B591**, 419 (2000) [hep-ph/0006172].

[29] M. Knecht, H. Neufeld, H. Rupertsberger, P. Talavera, *Eur. Phys. J.* **C12**, 469 (2000) [hep-ph/9909284].

[30] S. Descotes-Genon, B. Moussallam, *Eur. Phys. J.* **C42**, 403 (2005) [hep-ph/0505077].

[31] V. Cirigliano, H. Neufeld, H. Pichl, *Eur. Phys. J.* **C35**, 53 (2004) [hep-ph/0401173].

[32] H. Neufeld, private communication and to be published.